# MS&E 214, Revenue Management Example from Advertising

## Problem 1: Adwords numerical example

A search engine company needs to decide which advertisement to show on the search results page each time an internet user searches for a keyword. Assume there are 2 keywords (Keyword 1 and Keyword 2) and 2 advertisers (Advertiser 1 and Advertiser 2):

- The number of queries for Keyword 1 and 2 is estimated to be 100 and 200, respectively.
- Advertisers 1 and 2 have budgets $B_1$ and $B_2$ of 200 and 100, respectively.
- Advertiser $a$ is willing to pay a price $v_{a,w}$ for an advertisement shown on the search results page for keyword $w$, with $v_{1,1} = 2$, $v_{1,2} = 3$, $v_{2,1} = 5$, and $v_{2,2} = 1$.

Consider the simple case where only one advertisement can be shown on each search results page. The search engine company needs to decide, for each $(a, w)$, the number (can be fractional) of times to display the ad from advertiser $a$ to an internet user who searched keyword $w$.

a) Formulate this as a linear program, where the search engine company seeks to maximize revenue. Solve using excel or by inspection.

b) Determine the dual of the problem and give an interpretation of the dual variables. Solve the dual problem manually without calculator or excel and without looking at the sensitivity sheet from part (a).

## Problem 2: Adwords

Adwords: Consider the marketing of Adwords to retailers. Assume search engine shows (at most one) ad each time a search is made. The query is called an "adword" or just a "word". Each advertiser has specified a price it is willing to pay for its ad to appear against every adword. The search engine is free to display any ad, but can only charge what the advertiser is willing to pay.

Suppose that for each word $w$ and advertiser $a$ we know: $n_w$ the number of times the word $w$ will appear, $B_a$ the budget of advertiser $a$, and $v_{a,w}$ the willingness of advertiser $a$ to pay for word $w$ (per appearance).

Let $x_{a,w}$ be the number of appearances of word $w$ to assign to advertiser $a$. Then the LP to determine the number of each word appearance to assign to each advertiser in order to maximize profit can be formulated as follows:

$$\begin{aligned} \max \quad & \sum_{a,w} v_{a,w} x_{a,w} \\ s.t. \quad & \forall w : \sum_{a} x_{a,w} \leq n_w \\ & \forall a : \sum_{w} v_{a,w} x_{a,w} \leq B_a \\ & \forall a, w : \; x_{a,w} \geq 0 \end{aligned}$$

**a** Formulate the dual of this problem. Let $\alpha_w$ be the dual variable corresponding to the number of words constraint in the primal, and let $\beta_a$ be the dual variable corresponding to the budget constraint for advertiser $a$ in the primal.

**b** Using the complementarity conditions write down the implications of each of the following:

a) $x_{a,w} = 0$,

b) $\beta_a = 0$,

c) and $\alpha_w = 0$.

**c** Suppose the $(1-\beta_a)$ values were to be known without solving the LP. Using parts (a) and (b) explain how we can determine which advertisers to sell each adword to without solving either the primal or the dual LP. [Note: you only have to determine which advertisers to sell each word to. You do not have to determine how much of each word these advertisers get.]

In practice this analysis is conducted in an online fashion instead of using the static formulation given here (the $1-\beta_a$ values are known as budget throttling factors); sometimes, historical data is used to compute the throttle factors. Also, the same ideas work when the advertising is being targeted by demographic parameters, and when it is triggered by other kinds of media (eg. YouTube videos) as opposed to search keywords. Furthermore, in practical settings, one complication is that search engines and other platforms run some form of a second price auction, whereas here we just charged an advertiser what it is willing to pay. If you are interested, this paper by Amin Saberi describes how this is done: `http://dl.acm.org/citation.cfm?id=1284321`.