MS&E 114/214 Autumn 2024 Homework 4

Handed out 11/19, due 12/3 at noon

No extensions beyond noon on 12/4 for any reason, since we don't want to delay handing out the solutions.

Problem 1

[**EC for both MS&E 114 and 214**] Does there exist a currency arbitrage opportunity, given the exchange rates below? You must use an LP solver and your LP must correspond to a min-cost-flow problem. If an arbitrage opportunity exists, you must report one.

Table 1: Exchange Rates (Currency A to B)

	USD	**EUR**	**GBP**	**JPY**	**CAD**
USD	1.000	0.900	0.770	140.000	1.350
EUR	1.111	1.000	0.860	155.000	1.500
GBP	1.300	1.163	1.000	180.000	1.750
JPY	0.007	0.0064	0.0056	1.000	0.010
CAD	0.741	0.667	0.571	100.000	1.000

Problem 2

Consider the following production problem. A furniture company builds tables and chairs using only wood and nails. Each table requires 4 planks of wood and 2 boxes of nails. Each chair requires 1 plank of wood and 2 boxes of nails. The company currently has 2000 planks of wood and 1200 boxes of nails. Each table you build can be sold for $100 and each chair can be sold for $40.

(a) Formulate an LP that will solve this problem. (Assume that you can build fractional tables and chairs.)

(b) Now formulate the dual for this problem.

(c) Using Excel, solve the dual LP. Write up your solution. Also print out and attach your Answer Report.

(d) Without solving the primal LP, what is the maximum revenue the company can make? How do you know?

(e) In one concise sentence, explain what the dual variables represent with respect to the original problem.

(f) The company realizes that it actually has one more plank of wood. Without solving another LP, what is the new maximum revenue?

(g) If the company instead had to pay $15 for an additional plank of wood, should it do so? What if it had to pay $15 more for an additional box of nails?

Problem 3

The cities of Palu and Donggala have been affected by floods and an international agency has decided to help. Suppose you are in charge of coordinating an aid campaign to bringing medicaments from three different cities (Shanghai, Delhi and Mumbai) to the cities of Palu and Donggala. The logistics group of the project determined that the best way to do it is in two phases. First, bring the medicaments from the origin cities to an intermediate transfer city (Jakarata, Surabaya, Bandung are the possible transfer cities), and then send them to the destination cities. Assume that there is only one type of medicament.

The origin city i ($i = 1, \dots, I$) has an availability of medicaments equal to S_i. You are not required to use up all the available supply at an origin city. The warehouse at transfer city j ($j = 1, \dots, J$) has a capacity T_j; that is, no more than T_j units of medicaments can go through transfer city j. Everything that goes into transfer city j must go out to some destination city. The destination city k ($k = 1, \dots, K$) has a demand D_k of medicaments, which must be satisfied. The unit transportation cost from city i to the transfer city j is c_{ij}, for $i = 1, \dots, I$ and $j = 1, \dots, J$. The unit transportation cost from the transfer city j to the destination city k is d_{jk}, for $j = 1, \dots, J$ and $k = 1, \dots, K$.

(a) Formulate a min-cost flow problem that minimizes the total transportation costs while satisfying the demand requirements of each city, without violating the availability and capacity constraints.

Hint: To model the capacity constraints of a node, one can construct some "dummy" nodes and edges. To model the fact that not all the supply has to be used up, you can use another dummy node that uses up the excess supply. But in this problem, instead of using dummy nodes, you should also feel free to modify the min-cost flow formulation so that (a) some of the conservation constraints are "$\leq$" constraints, to capture the fact not all the supply has to be used, and (b) There are additional constraints on the total incoming flow into a transfer city.

(b) Suppose the origin cities are Delhi with an availability of 1,200 (units are in thousands of medicaments), Shanghai with an availability of 1,200, and Mumbai with an availability of 1,200. There are three transfer cities: Jakarta with a capacity of 800, Surabaya with a capacity of 1,500 and Bandung with capacity of 800. The destination cities are: Palu and Donggala with demands of 1,500 and 1,500 respectively. The unit costs of transportation are given in the following tables (in thousands of dollars per thousands of medicaments):

	Jakarta	Surabaya	Bandung
Delhi	10	12	25
Shanghai	7	8	15
Mumbai	5	8	10

	Palu	Donggala
Jakarta	20	15
Surabaya	15	20
Bandung	10	16

Solve using a LP solver.

(c) **[EC for MS&E 114 and 214]** Use the values of the dual variables (on Excel's sensitivity sheet or using the dual variables from cvxpy) to answer the following. Suppose you can invest at the same cost in expanding the availability at one of the origin cities by one unit or expanding the capacity at one of the transfer cities by one unit. In which would you invest? Why? Suppose you have to provide an additional unit of medicament to one of the destination cities. Which is the most expensive one to provide it to? Why?

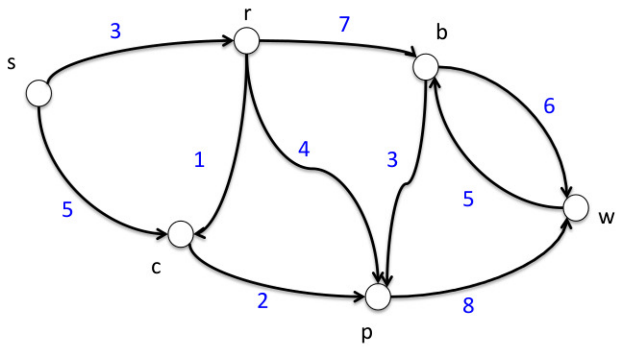
Hint: This part will be easier if you followed our suggestion in the hint of the first part of this problem to modify the min-cost flow LP as opposed to adding dummy nodes.

Problem 4

Consider a directed graph with the set of nodes V and the set of edges E where $c(v, w)$ is the cost of edge (v, w).. Suppose that a graph has one source node s and two possible destination nodes t_1 and t_2. Your goal is to find the shortest path starting at s and going to either t_1 or t_2.

(a) How will you rewrite this as a standard shortest path problem with a single source and a single destination? Note that we are not looking for an LP formulation.

(b) Consider the following network. Let the numbers on the edges represent the cost of traversing that edge. Your goal is to find the shortest path starting at s and ending at either p or w. Formulate this as a standard shortest path problem by drawing the new graph, specifying the source and the destination, and the edge costs. Solve using a LP solver.

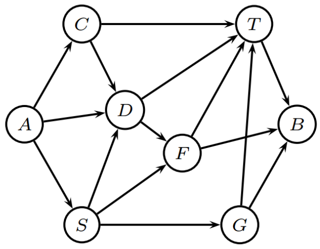


Problem 5

Consider the oil pipeline network in the figure below. The table below gives the capacity of each pipeline (in million gallons per hour), the cost per million gallons of sending oil over a pipeline, and the failure probability.

(a) Provide an LP and solve it to determine maximum amount of oil that can be sent from A to B.

(b) Formulate an LP to find the path from A to B that has the minimum failure probability, assuming that edges fail independently. Do not solve.



Edge	Capacity	Cost	Failure Prob.
AC	5	$7000	.4
AD	4	$4000	.9
AS	2	$1000	.1
CD	1	$2000	.2
CT	1	$4000	0
DT	3	$2000	.5
DF	1	$2000	.1
SD	1	$1000	.3
SF	2	$3000	.4
SG	2	$5000	.2
FB	1	$2000	.2
FT	4	$6000	.2
GT	5	$0	1
TB	2	$1000	.6
GB	2	$2000	.8