

Randomized Algorithms

CME309/CS365, Winter 2012-2013, Stanford University

Instructor: Ashish Goel

Handout 4: Homework 2. Due 2/14/2013 @ 11:00am.

Collaboration policy: Limited collaboration is allowed you can discuss the problem with other students in this class but cannot collaborate on writing the actual final answer. Please do not look at someone else's solution and do not share your solution with anyone else.

Late HW policy: You are allowed one late day for any one HW (i.e. submit the HW on Friday as opposed to Thursday).

Problem numbers refer to the text-book (Motwani and Raghavan).

Non-letter grade students: please do any three problems. If you do more, we will grade any three.

1. Problem 2.9
2. Problem 2.10
3. Problem 4.6
4. Problem 4.11
5. Problem 4.13

6. Uniform sampling from a stream

- (a) Suppose you are given a stream of data a_1, a_2, \dots, a_k . You want to generate samples x_1, x_2, \dots, x_k such that x_i is a uniform random sample from $\{a_1, a_2, \dots, a_i\}$. A simple way to do this is to let

$$x_{i+1} = \begin{cases} a_{i+1} & \text{with probability } \frac{1}{i+1} \\ x_i & \text{otherwise} \end{cases}$$

Bound the number of times x_i changes.

- (b) Imagine N cars, each of which travels at a different maximum speed. Initially, the cars are queued in random order at the starting point of a semi-infinite, one lane highway. Each car drives at the minimum of its maximum speed and the speed at which the car in front of it is driving. The cars will form "clumps". How many such clumps will we have in expectation?