What have we learned from market design?

Update to Roth (2008): September 20, 2010

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After a market has been designed, adopted, and implemented, it has a continuing life of its own. For those involved directly in the market, it is useful to continue to monitor it to make sure it is functioning well. For those of us involved in market design, it is also good to check how things are going, as a way to find out if there are unanticipated problems that still need to be addressed. Finally, the design and operation of new marketplaces also raises new theoretical questions, which sometimes lead to progress in economic theory. In this update, I'll briefly point to developments of each of these kinds, since the publication of Roth (2008), *What have we learned from market design?*. I'll discuss theoretical results only informally, to avoid having to introduce the full apparatus of notation and technical assumptions.

Medical labor markets

One of the longstanding empirical mysteries regarding the medical labor market clearinghouse is why it works as well as it does in connection with helping couples find pairs of jobs. The story actually began sometime in the 1970's, when for the first time the percentage of women medical graduates from U.S. medical schools rose above 10% (it is now around 50%). With this rise in women doctors came a growing number of graduating doctors who were married to each other, and wished to find two residency positions in the same location. Many of these couples started to defect from the match. As noted in Roth (1984), not only doesn't the deferred acceptance algorithm produce a matching that is stable when couples are present (even when couples are allowed to state preferences over *pairs* of positions), but when couples are present it is possible that no stable matching exists. The following simple example from Klaus and Klijn (2005) makes this clear. (This version is from Roth 2008b.)

Example 1*--market with one couple and no stable matchings (Klaus and Klijn 2005): Let c=(s1,s2) be a couple, and suppose there is another single student s3, and two hospitals h1 and h2. Suppose that the acceptable matches for each agent, in order of preference, are given by*

c: (h1,h2)[1]*;   s3: h1, h2,*

h1: s1, s3;   h2: s3, s2

[1] Couple c submits a preference list over pairs of positions, and specifies that only a single pair, h1 for student s1 and h2 for student s2 is acceptable. Otherwise couple c prefers to remain unmatched. For a couple, this could make perfect sense, if e.g. h1 and h2 are in a different city than the couple now resides, and they will move only if they find two good jobs.

Then no individually rational matching μ (i.e. no μ that matches agents only to acceptable mates) is stable. We consider two cases, depending on whether the couple is matched or unmatched.

Case 1: μ(c)=(h1,h2). Then s3 is unmatched, and s/he and h2 can block μ, because h2 prefers s3 to μ (h2)=s2.

Case 2: μ (c)=c (unmatched). If μ (s3)=h1, then (c, h1,h2) blocks μ. If μ (s3)=h2 or μ (s3)=s3 (unmatched), then (s3,h1) blocks μ.

The new algorithm designed for the National Resident Matching Program by Roth and Peranson (1999) allows couples to state preferences over pairs of positions, and seeks to find stable matchings. The empirical puzzle is why it almost never fails to find a stable matching, in the several dozen annual labor markets in which it has now been employed for over a decade (see Roth 2008b for a recent list). Some recent insight into this, reported in Kojima, Pathak and Roth (2010), connects the success in finding stable matchings that include couples to other recent results about the behavior of large markets.

Roth and Peranson initiated a line of investigation into large markets by showing computationally that, if as a market gets large, the number of places that a given applicant interviews (and hence the size of his rank order list) does not grow, then the set of stable matchings becomes small (when preferences are strict). Immorlica and Mahdian (2005) showed analytically that in a 1-1 marriage model with uncorrelated preferences, the set of people who are matched to different mates at different stable matchings grows small as the market grows large in this way, and that therefore the opportunities for profitable manipulation grow small. Kojima and Pathak (2009) substantially extended this result to the case of many-to-one matching, in which opportunities for employers to profitably manipulate can occur even when there is a unique stable matching, and in which employers can manipulate capacities as well as preferences. They show that as the size of a market grows towards infinity in an appropriate way, the proportion of employers who might profit from (any combination of) preference or capacity manipulation goes to zero in the worker proposing deferred acceptance algorithm. Kojima et al. (2010) showed that when couples are present, if the market grows large in a sufficiently regular way that makes couples a small part of the market, then the probability that a stable matching exists converges to one. That is, in big enough markets with not too many couples we should not be surprised that the algorithm succeeds in finding a stable matching so regularly.

A key element of the proofs is that if the market is large, but no applicant can apply to more than a small fraction of positions, then, even though there may be more applicants than positions, it is a high probability event that there will be a large number of hospitals with vacant positions after the centralized clearinghouse has found a stable matching. This result is of interest independently from helping in the proofs of the results described above: it means that stable clearinghouses are likely to leave both people unmatched and positions unfilled, even when the market grows very large. Most clearinghouses presently have a secondary, post-match market, often called a “scramble,” at which these unmatched people and positions can find one another. The newly developing theory of large markets suggests that post-match marketplaces will continue to be important in markets in which stable centralized clearinghouses are used.

On an operational note, in the 2008 paper I noted that the gastroenterology match had gotten off to a successful start with participation of 121 fellowship programs in the match for 2007 fellows. It seems to have established itself as a reliable marketplace; in the match for 2010 fellows, 153 certified fellowship programs participated. This suggests that the policies adopted to decrease the frequency and effectiveness of exploding offers have been effective (cf. Niederle and Roth 2009a,b).[2]

Kidney transplantation

Perhaps the most dramatic recent change in kidney exchange is that, following the publication of Rees et al.'s (2009) report on the first Non-simultaneous Extended Altruistic Donor (NEAD) chain in the *New England Journal of Medicine*, there has been a small explosion of such chains, not only by established exchange networks, but also by transplant centers of all sorts around the United States. (See e.g. the various chains reported at http://marketdesigner.blogspot.com/search/label/chains, or the more detailed report of chains conducted by the Alliance for Paired Donation in Rees et al. (2010).) Simulations by Ashlagi et al. (2010) using clinical data from the APD suggest that such chains can play an important role in increasing the number of live donor transplants.

The passage into law of what became the `Charlie W. Norwood Living Organ Donation Act' [Public Law 110-144, 100th Congress] in December, 2007 has set in motion plans that may eventually become a national kidney exchange network, but this is still moving slowly, and the issues involved with providing the right incentives for transplant centers to fully participate have not yet been resolved. Ashlagi and Roth (2010) explore some of these incentive issues in large markets, and show that the cost of making it safe for hospitals to participate fully is low, while the cost of failing to do so could be large if that causes hospitals to match their own internal patient-donor pairs when they can, rather than making them available for more efficient exchanges. That is, guaranteeing hospitals that patients who they can transplant internally will receive transplants will not be too costly in terms of the overall number of transplants that can be accomplished in large markets. (See also Unver, 2010, for a discussion of dynamic kidney exchange in large markets.)

While kidney exchange is growing quickly (in 2005 there were 27 reported transplants from exchange, in 2007 there were 121, and in 2009 there were 304)[3] it is still a very small part of the number of transplants, and the growth is not yet enough to halt the growth of the waiting list for deceased-donor kidneys. This has led to continued discussion about ways to recruit more donors, and to continued interest in assessing views on whether kidneys might, in an appropriately regulated environment, under

[2] The job market for some other medical subspecialties continues to unravel, and Orthopedic surgeons have recently taken steps to organize a centralized match, see Harner et al. (2008).
[3] See Roth (2010), "Kidney Exchange Time Series," http://marketdesigner.blogspot.com/2010/05/kidney-exchange-time-series.html, May 5.

some circumstances be bought and sold, or whether donors could in some way be compensated. The whole question of compensation for donors remains an extremely sensitive subject.

For example, two recent surveys published in the surgical literature showed that public opinion and patient opinion both reflected a willingness to consider payment for organs (Leider and Roth, 2010, and Herold 2010, respectively). However the journal that published those surveys also published an editorial (Segev and Gentry, 2010) expressing the opinion that it was a waste of resources even considering the opinions of anyone other than physicians, and expressing the view that physicians were unalterably opposed to any change from current law prohibiting any “valuable consideration” for transplant organs. (This view of physician opinion seems not to be quite accurate, based on available surveys of physician opinion, and on the letters to the editor the journal received in reply to what seems to be a fringe view.) Nevertheless, it is an indication that this remains a controversial subject, with views ranging widely from those who might contemplate a fairly unregulated market (cf. Becker and Elias 2007), to those who favor a moderately regulated market like the one in Iran (described in Fatemi, 2010), to those who would consider less direct forms of donor compensation (cf. Satel 2009), to those like the editorialists mentioned above who consider the issue to be beyond discussion except insofar as it impacts physicians.

The continued shortage of kidneys (and other organs) for transplant therefore underlines the importance of continuing to try to expand deceased donation. Kessler and Roth (2010) report on possibilities of increasing donation by changing organ allocation policy to give increased priority to people who have been long time registered donors. (This is an element of Singapore’s organ allocation policy, and proposals have been made to incorporate it into Israel’s policy.)

School choice

School assignment systems face different problems in different cities. In New York City, high school assignment had a strong resemblance to the problems facing labor markets for medical school graduates. In both cases, a large number of people have to be matched with a large number of positions at around the same time. And in both cases, the “positions” are in fact strategic players: NYC high school principals, like directors of medical residency programs, have preferences over who they match with, and have some strategic flexibility in meeting their goals. So it made sense to think of the New York City high school assignment process as a two-sided matching market that needed to reach a stable matching in order to damp down some of the strategic behavior that made it hard for the system to work well.

However there is an important difference between labor markets and school choice. In a labor market like the one for medical graduates, assuming that the parties have strict preferences (and requiring them to rank order each other) probably doesn’t introduce much distortion into the market. But in a school choice setting, schools in many cases have (and are often required to have) very large indifference classes, i.e. very many students between whom they can’t distinguish. So the question of tie-breaking arises: when there are enough places in a given school to admit only some of a group of otherwise equivalent students, who should get the available seats?

How to do tie-breaking was one of the first questions we confronted in the design of the New York City high school match, and we had to make some choices among ways to break ties by lottery. In particular, we considered whether to give each student a single number to be used for tiebreaking at every school (single tiebreaking), or to assign numbers to each student at each school (multiple tiebreaking). Computations with simulated and then actual submitted preferences indicated that single tiebreaking had superior welfare properties. Subsequent theoretical and empirical work have clarified the issues involved in tie-breaking. A simple example with just one-to-one matching is all that will be needed to explain, but first it will be helpful to look at how the deferred acceptance algorithm works. (For a description of how the algorithm is adapted to the complexities of the NYC school system, see Abdulkadiroğlu, Pathak, and Roth, 2009.)

The basic deferred acceptance algorithm with tie-breaking proceeds as follows:

- Step 0.0: students and schools **privately** submit preferences (and school preferences may have ties, i.e. schools may be indifferent between some students).
- Step 0.1: arbitrarily break all ties in preferences
- Step 1: Each student "applies" to her first choice. Each school tentatively assigns its seats to its applicants one at a time in their priority order. Any remaining applicants are rejected.

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- Step k: Each student who was rejected in the previous step applies to her next choice if one remains. Each school considers the students it has been holding together with its new applicants and tentatively assigns its seats to these students one at a time in priority order. Any remaining applicants are rejected.
- The algorithm terminates when no student application is rejected, and each student is assigned her final tentative assignment.

Notice that (just as Gale and Shapley 1962 showed,) the matching produced in this way is stable, not just with respect to the strict preferences that follow step 0.1, but with respect to the underlying preferences elicited from the parties, which may have contained indifferences. That is, there can't be a student and a school, not matched to one another, who would prefer to be. The reason is that, if a student prefers some school to the one she was matched with in the algorithm, she must have already applied to that school and been rejected. This applies to the original preferences too, which may not be strict, since tie breaking just introduces more blocking pairs; so any matching that is stable with respect to artificially strict preferences is also stable with respect to the original preferences. But those additional blocking pairs are constraints, and these additional constraints can harm welfare. A simple 1-1 ("marriage market") matching example is sufficient to see what's going on.

Example 2 (Tie breaking can be inefficient): Let M = {m_1, m_2, m_3} and W = (w_1, w_2, w_3} be the sets of students and schools respectively, with preferences given by :

$P(m_1) = w_2, w_1, w_3$	$P(w_1) = [m_1, m_2, m_3]$
$P(m_2) = w_1, w_2, w_3$	$P(w_2) = m_3, m_1, m_2$
$P(m_3) = w_1, w_2, w_3$	$P(w_3) = m_1, m_2, m_3$

The brackets around w_1's preferences indicate that w_1 is indifferent between any of [m_1, m_2, m_3] while, in this example, everyone else has strict preferences. Since there is only one place at w_1, but w_1 is the first choice of two students (m_2 and m_3), some tie-breaking rule must be used.

Suppose, at step 0 of the deferred acceptance algorithm, the ties in w_1's preferences are broken so as to produce the (artificial) strict preference $P(w_1) = m_1, m_2, m_3$. The deferred acceptance algorithm operating on the artificial strict preferences produces $\boldsymbol{\mu_M = [(m_1,w_1); (m_2,w_3); (m_3,w_2)]}$, at which m_1 and m_3 each receive their second choice (while m_2 receives his last choice). But note that the matching $\boldsymbol{\mu = [(m_1,w_2); (m_2,w_3); (m_3,w_1)]}$, is Pareto superior for the students, as m_1 and m_3 each receive their first choice, so they are both strictly better off than at μ_M, and m_2 is not worse off. If the preferences of school w_1 were in fact strict, the matching $\boldsymbol{\mu}$ would be unstable, because m_2 and w_1 would be a blocking pair. But w_1 doesn't really prefer m_2 to m_3; in fact $\boldsymbol{\mu}$ is stable with respect to the original, non-strict preferences. The pair (w_1, m_2) is not a blocking pair for μ, and only appeared to be in the deferred acceptance algorithm because of the arbitrary ways in which ties were broken to make w_1's preferences look strict.

So, there are costs to arbitrary or random tie breaking. Erdil and Ergin (2007, 2006), Abdulkadiroğlu, Pathak, and Roth (2009), and Kesten (2010) each explore this from different angles.[4]

Kesten notes that students are collectively better off at $\boldsymbol{\mu}$ than at $\boldsymbol{\mu_M}$ in Example 2 because, in the deferred acceptance algorithm, m_2's attempt to match with w_1 harms m_1 and m_3 without helping m_2. Kesten defines an *efficiency adjusted deferred acceptance mechanism* that produces $\boldsymbol{\mu}$ in Example 2 by disallowing the blocking pair (w_1, m_2) via a definition of "reasonable fairness" that generalizes stable matchings. But he shows that there is no mechanism that is Pareto efficient, reasonably fair, and strategy proof.

To understand Erdil and Ergin's approach, note that the Pareto improvement from μ_M to μ in Example 2 comes from an exchange of positions between m_1 and m_3. This exchange doesn't introduce any new blocking pairs, since, among those who would like to change their positions, m_1 and m_3 are

[4] In the computer science literature there has been a focus on the *computational* costs of non-strict preferences, which adds to the computational complexity of some calculations (but not others), see e.g. Irving (1994) and Irving, Manlove, and Scott (2000). When preferences aren't strict, not all stable matchings will have the same number of matched people, and Manlove, Irving, Iwama, Miyazaki and Morita (2002) show that the problem of finding a maximal stable matching is NP hard.

among the most preferred candidates of w_1 and w_2. Since there weren't any blocking pairs to the initial matching, this exchange can occur without creating any new blocking pairs.

Formally, Erdil and Ergin define a *stable improvement cycle* starting from some stable matching to be a cycle of students who each prefer the school that the next student in the cycle is matched to, and each of whom is one of the school's most preferred candidates among the students who prefer that school to their current match. They prove the following theorem.

Theorem 15 (Erdil and Ergin, 2008): If μ is a stable matching that is Pareto dominated (from the point of view of students) by another stable matching, then there is a stable improvement cycle starting from μ .

This implies that there is a computationally efficient algorithm that produces stable matchings that are Pareto optimal with respect to students. The initial step of the algorithm is a student-proposing deferred acceptance algorithm with arbitrary tie-breaking of non-strict preferences by schools. The output of this process (i.e. the student optimal stable matching of the market with artificially strict preferences) is then improved by finding and satisfying stable improvement cycles, until no more remain. Erdil and Ergin show, however, that this algorithm is not strategy-proof, i.e. unlike the student-proposing deferred acceptance algorithm, this deferred acceptance plus stable improvement cycle algorithm doesn't make it a dominant strategy for students to state their true preferences. They show in fact that no mechanism that always produces a stable matching that is Pareto optimal for the students can be strategy proof.

Abdulkadiroğlu, Pathak and Roth (2009) establish that no mechanism (stable or not, and Pareto optimal or not) that is better for students than the student proposing deferred acceptance algorithm with tie breaking can be strategy proof. Following the design of the New York and Boston school choice mechanisms, define a *tie-breaking rule* T to be an ordering of students that is applied to any school's preferences to produce a strict order of students within each of the school's indifference classes (that is, when a school is indifferent between two students, the tie breaking rule determines which is preferred in the school's artificial strict preferences). *Deferred acceptance with tie breaking rule T* is then simply the deferred acceptance algorithm operating on the strict preferences that result when T is applied to schools' preferences. One mechanism *dominates* another if for every profile of preferences the first mechanism produces a matching that is at least as good for every student as the matching produced by the second mechanism, and for some preference profiles the first mechanism produces a matching that is preferred by some students.

Theorem 16 (Abdulkadiroğlu, Pathak and Roth, 2009): For any tie-breaking rule T, there is no mechanism that is strategy proof for every student and that dominates student proposing deferred acceptance with tie-breaking rule T.

But Abdulkadiroğlu, Pathak and Roth also analyze the preferences submitted in recent New York City high school matches (under a deferred acceptance with tie breaking mechanism) and find that, *if* the preferences elicited from the strategy-proof mechanism could have been elicited by a stable improvement cycle mechanism, then about 1,500 out of about 90,000 New York City students could have gotten a more preferred high school. (In contrast, the same exercise with the preferences submitted in the Boston school choice system yield almost no improvements.) So a number of open questions remain, among them, what accounts for the difference between NYC and Boston, and to what extent could the apparent welfare gains in NY actually be captured? The potential problem is that, when popular schools are known, it's not so hard to find manipulations of stable improvement cycles mechanisms (which give families the incentive to rank popular schools more highly than in their true preferences, because of the possibility of using them as endowments from which to trade in the improvement cycles). Azevedo and Leshno (2010) show by example that at equilibrium such manipulations could sometimes be welfare decreasing compared to the (non Pareto optimal) outcome of the deferred acceptance algorithm with tie breaking.[5]

Economists and Lawyers: two markets worth watching

Coles et al. (2010) describe the recent experience of the market for new Ph.D. economists with the newly instituted "pre-market" signaling mechanism, and "post-market" scramble. From 2006 through 2009, the number of candidates who used the signaling mechanism remained roughly constant at around 1,000 per year. The evidence is suggestive if not conclusive that judicious signaling increases the probability of receiving an interview. The pattern of signals suggests something about what might constitute "judicious" signaling; when one compares the reputational "ranks" of the school a student is graduating from and those he signals to, very few signals are sent from lower to higher ranking schools. It appears that the signals play a coordination role in ameliorating congestion, with signals distributed across a very broad range of schools.

Participation in the post-market "scramble" has been more variable, with from 70 to 100 positions listed in each of the years 2006-10. It appears that at least 10% of these positions are filled each year through contact made in the scramble.

Further developments in the market for new Ph.D. economists will provide an ongoing window into the possibilities of dealing with congestion through signaling in a decentralized market, and in achieving thickness in the aftermarket.

A window of a different kind is being provided by several of the markets for new law graduates in the United States, which continue to suffer from problems related to the timing of transactions. The market for federal court clerks now appears to be nearing the end of the latest attempt to enforce a set of dates before which applications, interviews, and offers will not be made. (Avery et al. (2007) already reported a high level of cheating in that market, as judges accepted applications, conducted interviews and made offers before the designated dates.)[6] Roth and Xing (1994) reported on various ways that markets could

[5] There has been a blossoming of new theory on school choice, including reconsideration of some of the virtues of the Boston algorithm, new hybrid mechanisms, and experiments. See for example Abdulkadiroglu, Che, and Yasuda (2010a,b), Calsamiglia, Haeringer, and Klijn. (2010), Featherstone and Niederle (2010), Haeringer and Klijn 2009), Kojima and Unver (2010), Mirrales (2009).

[6] Presently the market for new associates at large law firms is also unraveling: see e.g. http://marketdesigner.blogspot.com/2010/07/unraveling-of-law-firm-interviews-of.html

fail through the unraveling of appointment dates, but the markets for lawyers have frequently offered the opportunity to observe new failures of this kind.

Conclusions:

The new marketplace designs reported in Roth (2008), for labor markets, for schools, and for kidney exchange, have continued to operate effectively. However in each of these domains, unsolved operational problems remain, which often raise new theoretical questions about how markets work, and how market failures can be avoided and repaired. Holmstrom, Milgrom and Roth (2002) quote Robert Wilson (1993) on this: *". . . for the theorist, the problems encountered by practitioners provide a wealth of topics.*"

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