Collective neutrino oscillations in two spatial dimensions

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July 17, 2015

Motivation

- Core-collapse supernovae are one of the most intense sources neutrinos
- 10^{53} ergs (or 10^{58} neutrinos) are released in an interval of 10 seconds
- Core-collapse supernovae are one of the most favored sites for R-process nucleosynthesis
- Question: Do we understand neutrino flavor oscillations in the interior of core-collapse supernovae?

Neutrino Oscillations

$$\rho = \begin{pmatrix} \langle \psi^*_{\nu_e} \psi_{\nu_e} \rangle & \langle \psi^*_{\nu_e} \psi_{\nu_\mu} \rangle \\ \langle \psi^*_{\nu_\mu} \psi_{\nu_e} \rangle & \langle \psi^*_{\nu_\mu} \psi_{\nu_\mu} \rangle \end{pmatrix} \quad \bar{\rho} = \begin{pmatrix} \langle \bar{\psi}^*_{\nu_e} \bar{\psi}_{\nu_e} \rangle & \langle \bar{\psi}^*_{\nu_e} \bar{\psi}_{\nu_\mu} \rangle \\ \langle \bar{\psi}^*_{\nu_\mu} \bar{\psi}_{\nu_e} \rangle & \langle \bar{\psi}^*_{\nu_\mu} \bar{\psi}_{\nu_\mu} \rangle \end{pmatrix}$$

$$\rho(L) = e^{-iHL} \rho(0) e^{iHL}$$

$$H = \frac{1}{2} \begin{pmatrix} -\omega \cos(2\theta_{\mathrm{v}}) & \omega \sin(2\theta_{\mathrm{v}}) \\ \omega \sin(2\theta_{\mathrm{v}}) & \omega \cos(2\theta_{\mathrm{v}}) \end{pmatrix} \quad \omega = \frac{m_2^2 - m_1^2}{2E}$$

$$P(\nu_e \to \nu_\mu) = sin^2(2\theta_{\mathrm{v}}) sin^2 \left(\frac{\omega}{2} L \right)$$

Hamiltonian

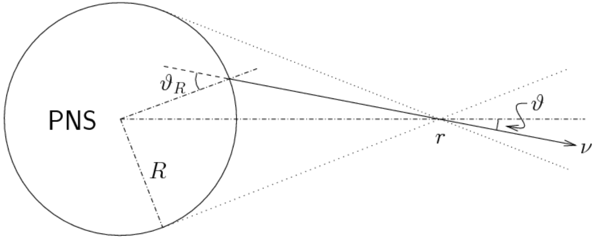
$$H = H_{vac} + H_{mat} + H_{self}$$

$$\begin{aligned}
H_{vac} &= \frac{1}{2}\begin{pmatrix} -\omega\cos(2\theta_{\mathrm{v}}) & \omega\sin(2\theta_{\mathrm{v}}) \\ \omega\sin(2\theta_{\mathrm{v}}) & \omega\cos(2\theta_{\mathrm{v}}) \end{pmatrix} \\
H_{mat} &= \begin{pmatrix} \sqrt{2}G_F n_e & 0 \\ 0 & 0 \end{pmatrix} \\
H_{self} &= \sqrt{2}G_F \int d^3p'(1 - v \cdot v')(\rho_{p'} - \bar{\rho}_{p'})
\end{aligned}$$

Neutrino oscillations in a medium

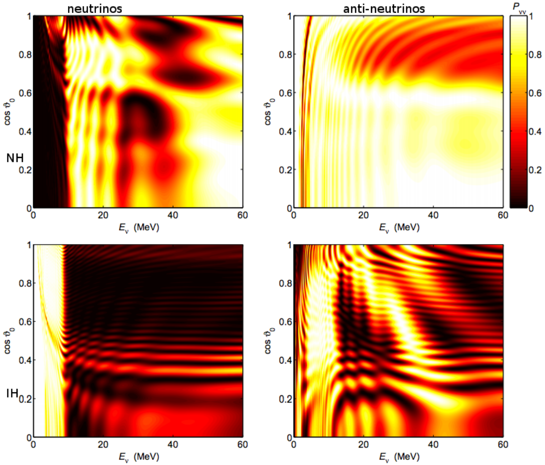
- Neutrino flavor oscillations can be modified in presence of matter when at least one flavor experiences a potential that is different from the potential experienced by other flavors
- The potential may be a result of matter or due to presence of neutrino gas
- The modification of neutrino flavor oscillation due to neutrino gas is different from the effect of matter in two crucial ways:
  1. Neutrino-neutrino interaction leads to non-linear effect, while the matter effect is linear
  2. Neutrino-neutrino potential is dependent on the relative angle between the neutrinos

Neutrino bulb model



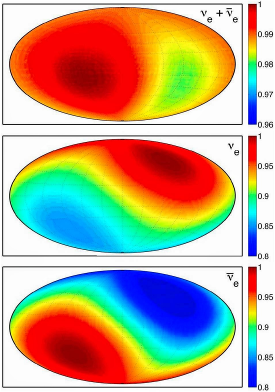
H. Duan, G. M. Fuller and Y. Z. Qian, Ann. Rev. Nucl. Part. Sci. **60**, 569 (2010) [arXiv:1001.2799 [hep-ph]].

Neutrino oscillations in the bulb model



H. Duan, G. M. Fuller, J. Carlson and Y. Z. Qian, Phys. Rev. Lett. **97**, 241101 (2006) [astro-ph/0608050].

Assumptions of the neutrino bulb model revisited

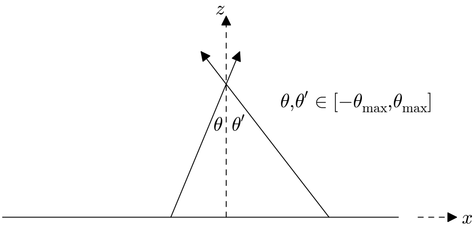


I. Tamborra, F. Hanke, H. T. Janka, B. Mller, G. G. Raffelt and A. Marek, Astrophys. J. **792**, 96 (2014) [arXiv:1402.5418 [astro-ph.SR]].

Scrutiny of spherical symmetry

- How is the neutrino flavor oscillation modified if the assumption of spherical symmetry is removed?
- A model of neutrino flavor oscillations in three spatial dimensions is difficult to formulate
- We consider a two dimensional toy model to investigate the collective neutrino oscillations in multi dimensional space

Neutrino Line model



- At each point (x, z) the neutrino flavor structure in the direction θ is given by a 2×2 density matrix for neutrinos and anti-neutrino.
- We assume mono-energetic neutrinos of electron-type emanating from each point x.

Neutrino Line model

If two points on the line start with almost identical flavor will they remain almost identical? We can quantify this effect in two different ways,

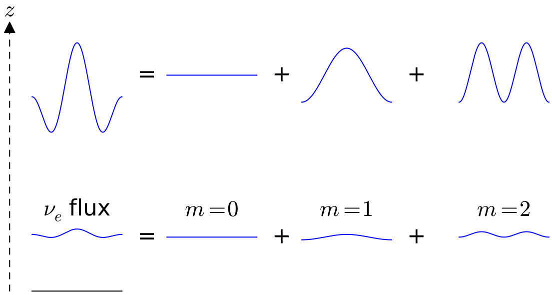
$$\rho_\theta(x,z) = \begin{pmatrix} 1 & \epsilon_\theta(x,z) \\ \epsilon^*_\theta(x,z) & 0 \end{pmatrix}$$

or in terms of Fourier modes,

$$\rho_{m,\theta}(z) \quad = \frac{1}{L}\int_0^L e^{\frac{i2\pi mx}{L}} \rho_\theta(x,z)dx = \begin{pmatrix} \delta_{m,0} & \epsilon_{m,\theta}(z) \\ \epsilon^*_{-m,\theta}(z) & 0 \end{pmatrix}$$

We have very similar expressions for anti-neutrinos.

Fourier modes



Fourier modes

- All Fourier modes m except $m = 0$ encode the spatial dependence of flavor in the x direction
- In the linear regime (off-diagonal element of $\rho_m \ll 1$) the equation of motion for all modes is decoupled

$$
\begin{aligned}
i\cos\theta\partial_z\epsilon_{m,\theta}(z) &= \frac{2\pi m}{L}\sin\theta\epsilon_{m,\theta}(z) - \omega\eta\epsilon_{m,\theta}(z) \\
&+ \mu(1-\alpha)\int_{-\theta_{\max}}^{\theta_{\max}}\epsilon_{m,\theta}(z)(1-\cos(\theta-\theta'))d\theta' \\
&- \mu\int_{-\theta_{\max}}^{\theta_{\max}}(\epsilon_{m,\theta'}(z) - \alpha\bar{\epsilon}_{m,\theta'}(z))(1-\cos(\theta-\theta'))d\theta'
\end{aligned}
$$

Linear stability analysis

$$i\partial_z \begin{pmatrix} \epsilon_{m,\theta_1} \\ \bar{\epsilon}_{m,\theta_1} \\ \epsilon_{m,\theta_2} \\ \bar{\epsilon}_{m,\theta_2} \\ \vdots \\ \epsilon_{m,\theta_N} \\ \bar{\epsilon}_{m,\theta_N} \end{pmatrix} = \begin{pmatrix} & & \\ & 2N \times 2N & \\ & & \end{pmatrix} \begin{pmatrix} \epsilon_{m,\theta_1} \\ \bar{\epsilon}_{m,\theta_1} \\ \epsilon_{m,\theta_2} \\ \bar{\epsilon}_{m,\theta_2} \\ \vdots \\ \epsilon_{m,\theta_N} \\ \bar{\epsilon}_{m,\theta_N} \end{pmatrix}$$

What does instability mean?

The solution of the off-diagonal term of the density matrix for each mode and angle $\epsilon_{m,\theta}$ is of the exponential form,

$$\epsilon_{m,\theta} \propto \exp(-i\Omega_m z).$$

Complex Ω_m for $m \neq 0$ implies that the one-dimensional system is qualitatively and quantitatively different from a two-dimensional system in the linear regime.

Region of instability

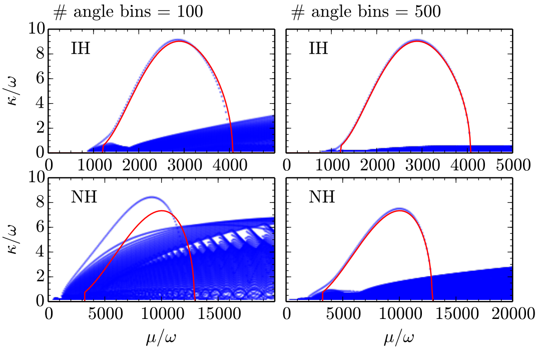


Figure : Growth rate ($\text{Im}(\Omega_m)$) for m=5000 and number of angle bins, N = 100 and 500 (first and second column respectively). The top row assumes inverted mass hierarchy while the bottom row assumes normal mass hierarchy. Here $\mu = \sqrt{2}G_F n_\nu / 2\theta_{max}$

Region of instability

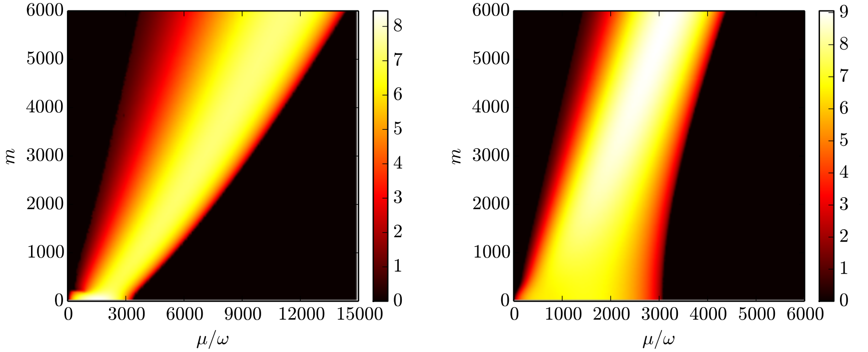


Figure : Region of instability for normal (left) and inverted hierarchy (right) with $n_{\bar{\nu}}/n_{\nu} = 0.8$, $L = 40\pi(\omega^{-1})$ and θ in the range $-\pi/6$ to $\pi/6$.
$\mu = \sqrt{2}G_F n_{\nu}/2\theta_{max}$

The matter effect

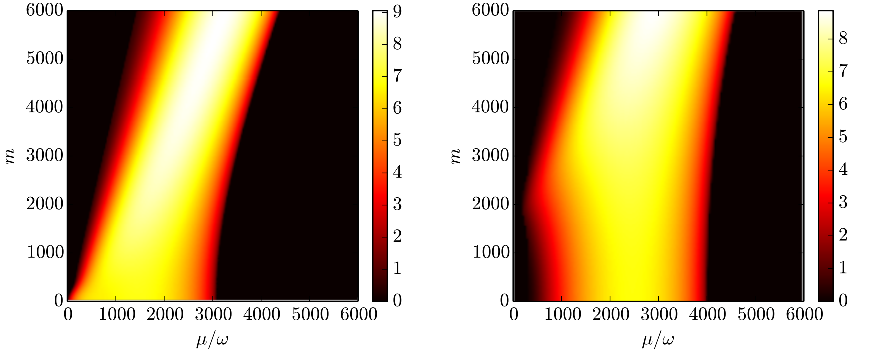


Figure : Region of instability for inverted hierarchy without matter (left) and with matter (right) with $n_{\bar{\nu}}/n_\nu = 0.8$, $L = 40\pi(\omega^{-1})$ and θ in the range $-\pi/6$ to $\pi/6$. $\mu = \sqrt{2}G_F n_\nu/2\theta_{max}$

Conclusion and Discussion

- In a multi-dimensional model of collective neutrino oscillations the instability can occur and much larger effective neutrino number density than in one dimensional model
- It there a region near the proto-neutron star where effect of both collisions and flavor instability can be seen simultaneously?
- At what length scale will we have to revisit the assumption of coherent forward scattering?