Low Scale Seesaw and Neutrinoless Double Beta Decay

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Outline

- Introduction: neutrino masses and mixing
- Seesaw mechanism for neutrino mass generation
- Low energy signatures of type I seesaw scenario
- Neutrinoless double beta ($0\nu\beta\beta$) decay
- Radiative corrections to neutrino masses
- Heavy neutrino contribution to $0\nu\beta\beta$ decay rate

Neutrino masses and mixing

Compelling experimental evidence of physics beyond the Standard Model

atmospheric neutrinos:

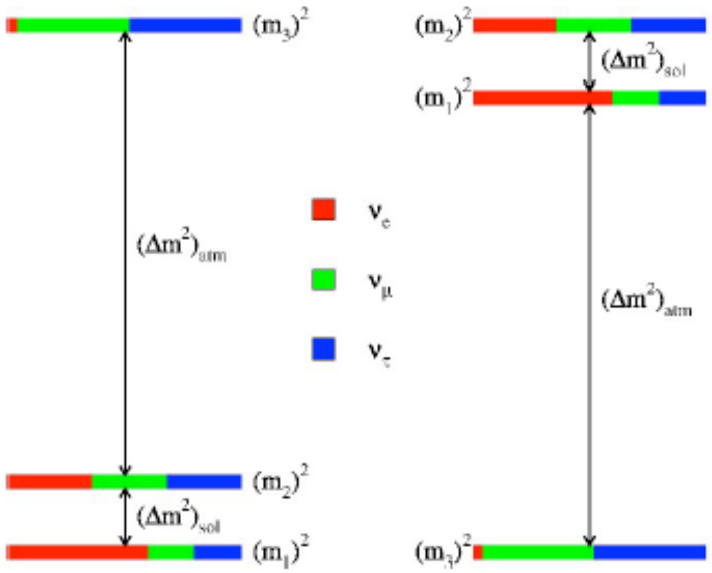
Super-Kamiokande: $|\Delta m_A^2| \sim O(10^{-3}\ \mathrm{eV}^2)$ and $\theta_{23} \cong \pi/4$

solar neutrinos:

SNO, SK and KamLAND: $\Delta m_S^2 \sim O(10^{-5}\ \mathrm{eV}^2)$ and $\theta_{12} \cong \arcsin(\sqrt{0.3})$

reactor and accelerator neutrinos:

Daya Bay, RENO, T2K, MINOS, Double CHOOZ: $\theta_{13} \neq 0$ at 10σ, $\theta_{13} \sim 0.15$



1. at least two massive neutrinos ν_j with masses $m_j \neq 0$
2. existence of neutrino mixing:

$$\nu_{\ell \mathrm{L}}(x) = \sum_j (U_{\mathrm{PMNS}})_{\ell j}\, \nu_{j\mathrm{L}}(x), \quad \ell = e, \mu, \tau$$

$$U_{\mathrm{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \mathrm{diag}(1, e^{i\frac{\alpha_{21}}{2}}, e^{i\frac{\alpha_{31}}{2}})$$

Neutrino masses and mixing

- From data on the invisible Z decay width: 3 flavour active neutrinos $\nu_{\ell L}$, $\ell = e, \mu, \tau$

- The number of mass eigenstate ν_j can be larger than 3 (*sterile neutrinos ?*), *but* at least 3 of the ν_j should be "light":

$$m_{1,2,3} < 1 \text{ eV and } m_1 \neq m_2 \neq m_3$$

- ^{3}H $\beta-$decay experiments and astrophysical observations

$$m_j \lesssim 0.5 \text{ eV} \qquad m_j / m_{\ell,q} \lesssim 10^{-6}$$

- Important questions:

  1. *Are neutrinos Majorana or Dirac particles ?*
  2. *What is the mass ordering ?*
  3. *Is there CP violation in the neutrino sector ?*
  4. *Is there a new fundamental mass scale* Λ *in particle physics ?*

Neutrino masses and mixing

Why do neutrinos have a non-zero tiny mass ?

Symmetry principles give us an answer, even if the dynamics involved is not understood

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Electroweak Theory + Quantum Chromodynamics:

- Lorentz invariance
- Gauge invariance: SU(3)xSU(2)xU(1)
- Renormalizability

The Standard Model is not "complicated" enough to violate baryon and lepton number conservation (except for tiny quantum effects unobservable at the temperature of the present universe)

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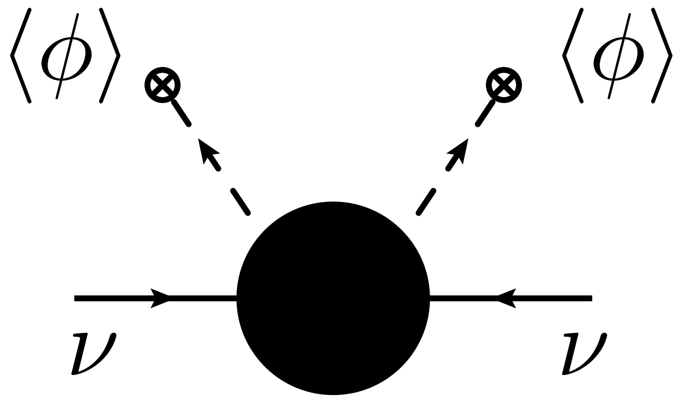
We can relax the renormalizability requirement and introduce all the possible interaction operators which are allowed by gauge and Lorentz symmetries (*effective field theories*).
In this case we introduce a *new mass scale* in the theory, suppressing the new interactions.

When we do experiments to detect neutrino oscillations or proton decay, what we are measuring are the *non-renormalizable effective interactions* added to the renormalizable part of the Standard Model

Seesaw mechanisms

Why do neutrinos have a non-zero tiny mass ?



Weinberg, PRD 22 (1980) 1694

$$\frac{c_{\ell\ell'}}{\Lambda}\left(\overline{\psi^c_{\ell L}}\tilde{\phi}^*\right)\left(\tilde{\phi}^\dagger\psi_{\ell' L}\right)$$

Λ is a new physical scale responsible for tiny neutrino masses: $m_\nu \approx c\,\langle \phi^2 \rangle / \Lambda$

Seesaw mechanisms

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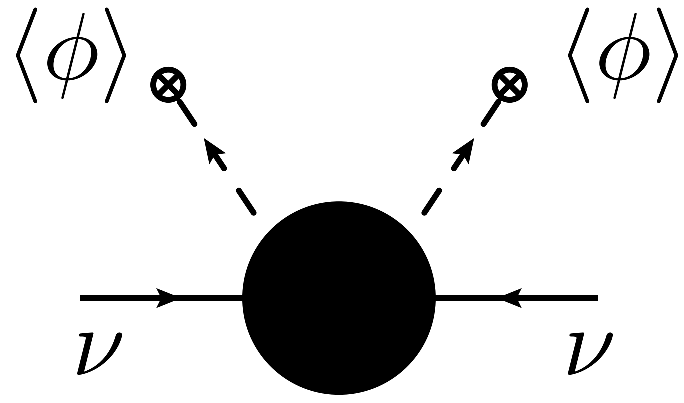

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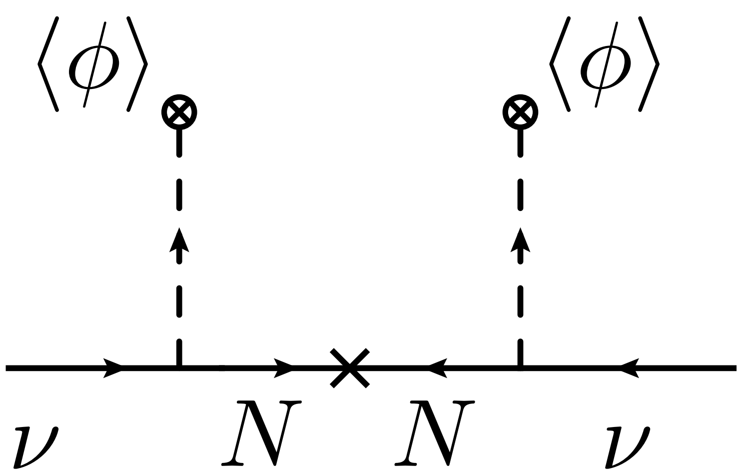
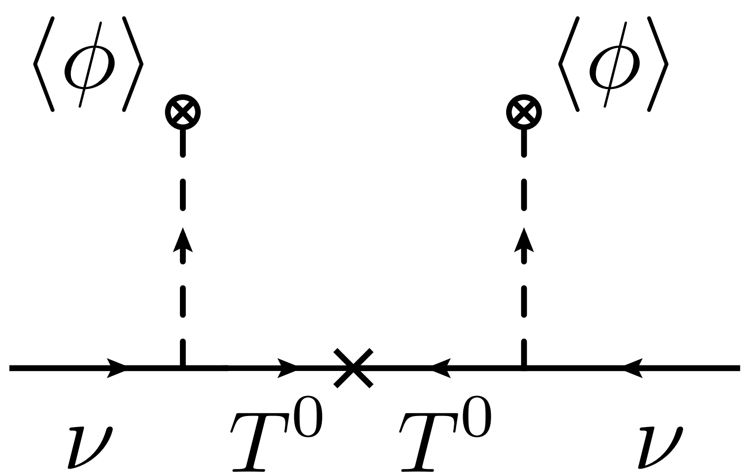
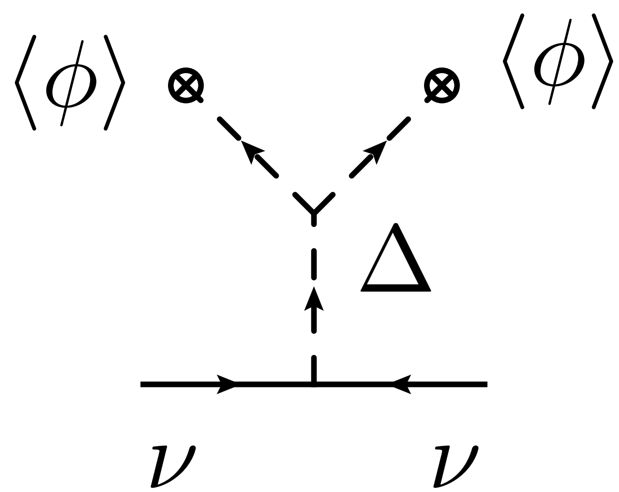


type I



type III



type II

SEESAW MECHANISMS

Type I seesaw scenario

Minkowski, 1977
Yanagida, 1979
Gell-Mann, Ramond, Slansky, 1979
Mohapatra, Senjanovic, 1980

$$\mathcal{L}^{\rm seesaw}(x) = \mathcal{L}_{\rm Y}(x) + \mathcal{L}_{\rm M}^{\rm N}(x)$$

$$\begin{aligned} \mathcal{L}_{\rm Y}(x) &= -\lambda_{\ell i}\, \overline{\psi}_{\ell L}(x)\, \widetilde{H}(x)\, N_{iR}(x) - h_\ell\, \overline{\psi}_{\ell L}(x)\, H(x)\, \ell_R(x) + \text{h.c.} \\ \mathcal{L}_{\rm M}^{\rm N}(x) &= -\frac{1}{2}\, M_i\, \overline{N_i}(x)\, N_i^C(x)\,, \qquad i \geq 2 \end{aligned}$$

At energies below the lightest N_i mass, the heavy Majorana fields are integrated out $\Longrightarrow$ *Majorana mass term* for the LH flavour neutrinos at $E \sim M_Z$:

$$m_\nu = -v^2 \lambda\, M^{-1}\, \lambda^T = U^*_{\rm PMNS}\, Diag(m_1, m_2, m_3)\, U^\dagger_{\rm PMNS}$$

taking $|\lambda| \sim 1$ and $m_\nu \sim 10^{-2}$ eV $\Rightarrow M \sim 10^{14}$ GeV

$\Lambda \simeq M$ *is not related to the EWSB scale and can, in principle, take arbitrary values up to the Planck mass! Testing the see-saw mechanism???*

Low energy effects of RH neutrinos

$$m_\nu \simeq -m_D M^{-1} m_D^T \qquad m_D \simeq \lambda v$$

$$\text{naively for } M = 1 \text{ TeV} \curvearrowright m_D \approx 10^{-4} \text{ GeV} \Rightarrow \lambda \approx 10^{-6}$$

low energy effects very suppressed:

- tiny EDMs
- tiny lepton radiative decays
- tiny deviations from EW precision observables
- production cross-section at colliders is suppressed (except when RH neutrino has additional interactions, e.g. $U(1)_{B-L}$)

conversely, testing the seesaw mechanism at colliders and/or from low energy observables requires large Yukawa couplings. Again naively,

$$\lambda = 0.1, \qquad M = 1 \text{ TeV} \quad \Rightarrow \quad m_\nu \approx 0.1 \text{ GeV}$$

is it possible to have seesaw models at low scale consistent with light neutrino masses and sizeable Yukawa couplings ?

Mohapatra, '86
Mohapatra, Valle, '86
Pilaftsis, '92;'95
Pilaftsis, Underwood, 2005
de Gouvea, 2007
Kersten, Smirnov, 2007
...

RH neutrinos and large Yukawa couplings

There is a continuous family of Dirac masses compatible with neutrino data

Consider for simplicity the case of 2 RH neutrinos in the basis where charged lepton Yukawa and RH neutrino mass matrices are diagonal:

$$m_D = i \overbrace{U^*_{PMNS}\sqrt{\hat{m}}}^{\text{low energy "measurable"}} \overbrace{O\sqrt{\hat{M}}}^{\text{high energy free parameters}}$$

Casas, Ibarra, 2001

$$O \equiv \begin{pmatrix} 0 & 0 \\ \cos\hat{\theta} & \pm\sin\hat{\theta} \\ -\sin\hat{\theta} & \pm\cos\hat{\theta} \end{pmatrix} \quad \text{for normal hierarchy}$$

$$O \equiv \begin{pmatrix} \cos\hat{\theta} & \pm\sin\hat{\theta} \\ -\sin\hat{\theta} & \pm\cos\hat{\theta} \\ 0 & 0 \end{pmatrix} \quad \text{for inverted hierarchy}$$

$$\hat{\theta} \equiv \omega - i\xi$$

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There is a continuous family of Dirac masses compatible with neutrino data

Consider for simplicity the case of 2 RH neutrinos in the basis where charged lepton Yukawa and RH neutrino mass matrices are diagonal:

$$m_D = i\, U^*_{PMNS} \sqrt{\hat{m}}\, O\, \sqrt{\hat{M}}$$

Casas, Ibarra, 2001

U^*_{PMNS}: $\mathcal{O}(0.1)$

$\sqrt{\hat{m}}$: $\sqrt{\mathcal{O}(10^{-10})\,\text{GeV}}$

$\sqrt{\hat{M}}$: $\sqrt{\mathcal{O}(10^{3})\,\text{GeV}}$

O: adjust O to generate large m_D

e.g. $m_D \approx 10$ GeV $\Rightarrow$ $|O| \approx 10^6$

$$O \equiv \begin{pmatrix} 0 & 0 \\ \cos\hat{\theta} & \pm\sin\hat{\theta} \\ -\sin\hat{\theta} & \pm\cos\hat{\theta} \end{pmatrix} \xrightarrow[\xi \gg 1]{\hat{\theta} \equiv \omega - i\xi} \frac{e^{i\omega} e^{\xi}}{2} \begin{pmatrix} 0 & 0 \\ 1 & \mp i \\ i & \pm 1 \end{pmatrix}$$

exponentially enhanced!
fixed flavour structure!

RH neutrinos and large Yukawa couplings

Sizable couplings of RH neutrinos to Standard Model leptons

Lagrangian mass terms:

$$\mathcal{L}_\nu \;=\; -\overline{\nu_{\ell L}}\,(m_D)^*_{\ell a}\,\nu_{aR} - \frac{1}{2}\,\overline{\nu^C_{aL}}\,(M)^*_{ab}\,\nu_{bR} \;+\; \text{h.c.}$$

$$M = V^*\hat{M}V^\dagger,\; \hat{M} \equiv \text{diag}(M_1, M_2),\; R^* \simeq m_D M^{-1}$$

Heavy Majorana Neutrino Interactions

$$\mathcal{L}^N_{CC} \;=\; -\frac{g}{2\sqrt{2}}\,\bar{\ell}\,\gamma_\alpha\,(RV)_{\ell k}(1-\gamma_5)\,N_k\,W^\alpha \;+\; \text{h.c.}$$

$$\mathcal{L}^N_{NC} \;=\; -\frac{g}{4c_w}\,\overline{\nu_{\ell L}}\,\gamma_\alpha\,(RV)_{\ell k}\,(1-\gamma_5)\,N_k\,Z^\alpha \;+\; \text{h.c.}$$

$$\mathcal{L}^N_{H} \;=\; -\frac{gM_k}{4\,M_W}\,\overline{\nu_{\ell L}}\,(RV)_{\ell k}\,(1+\gamma_5)\,N_k\,h \;+\; \text{h.c.}$$

Low energy effects are parametrized by (RV)

RH neutrinos and large Yukawa couplings

the flavour structure of the neutrino Yukawa couplings is determined by neutrino oscillation parameters:

NH:

$$RV \approx -\frac{e^{-i\,\omega}e^{\xi}}{2}\sqrt{\frac{m_3}{|M_1|}}\begin{pmatrix} \left(U_{e3}+i\sqrt{m_2/m_3}\,U_{e2}\right) & \pm i\left(U_{e3}+i\sqrt{m_2/m_3}\,U_{e2}\right)/\sqrt{M_2/M_1} \\ \left(U_{\mu3}+i\sqrt{m_2/m_3}\,U_{\mu2}\right) & \pm i\left(U_{\mu3}+i\sqrt{m_2/m_3}\,U_{\mu2}\right)/\sqrt{M_2/M_1} \\ \left(U_{\tau3}+i\sqrt{m_2/m_3}\,U_{\tau2}\right) & \pm i\left(U_{\tau3}+i\sqrt{m_2/m_3}\,U_{\tau2}\right)/\sqrt{M_2/M_1} \end{pmatrix}$$

Shaposhnikov, 2007
Raidal,Strumia,Turzynski, 2007
Kersten,Smirnov, 2009
Gavela,Hambye,Hernandez,Hernandez, 2009
Ibarra, EM, Petcov, 2010

IH:

$$m_{2,3} \to m_{1,2}$$
$$U_{\alpha 2,\alpha 3} \to U_{\alpha 1,\alpha 2}\,(\alpha = e,\mu,\tau)$$

It is convenient to parametrize the size of the couplings in terms of the highest neutrino Yukawa eigenvalue:

$$y^2 v^2 \equiv \max\left\{\mathrm{eig}\left(m_D m_D^\dagger\right)\right\} = \max\left\{\mathrm{eig}\left(\sqrt{\hat{m}}O\hat{M}O^\dagger\sqrt{m}\right)\right\} = \frac{1}{4}e^{2\,\xi}(m_2+m_3)(M_1+M_2)$$

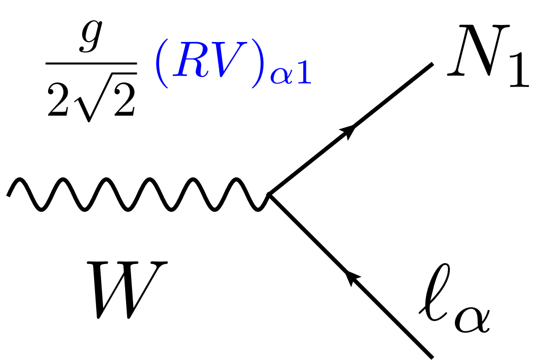
RH neutrinos and large Yukawa couplings

the flavour structure of the neutrino Yukawa couplings is fixed by neutrino oscillation data and *RV* can be calculated in term of few parameters:

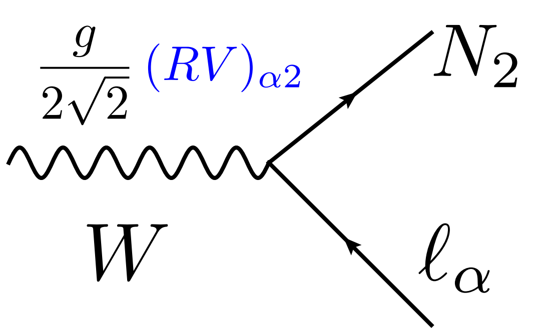
- maximum Yukawa coupling: y
- RH neutrino masses: M_1 and M_2
- a phase: ω

NH:

$$(RV)_{\alpha 1} = -e^{i\,\omega} y\, v \sqrt{\frac{M_2}{M_2+M_1}} \sqrt{\frac{m_3}{m_2+m_3}} \left(U_{\alpha 3} + i\sqrt{m_2/m_3}\, U_{\alpha 2}\right)$$



$$(RV)_{\alpha 2} = \mp i\, e^{i\,\omega} y\, v \sqrt{\frac{M_1}{M_2+M_1}} \sqrt{\frac{m_3}{m_2+m_3}} \left(U_{\alpha 3} + i\sqrt{m_2/m_3}\, U_{\alpha 2}\right)$$



$$(RV)_{\alpha 2} = \pm i\, (RV)_{\alpha 1} \sqrt{\frac{M_1}{M_2}}$$

Generalized lepton charge

$$\mathcal{L}_\nu \;=\; -\,\overline{\nu_{\ell L}}\,(m_D)_{\ell a}\,\nu_{aR} - \frac{1}{2}\,\overline{\nu^c_{aL}}\,(M_R)_{ab}\,\nu_{bR} \;+\; \text{h.c.}$$

For an arbitrary number of RH neutrino fields ν_{aR}:

$$L' \;=\; \sum_{k=e,\mu,\tau,\ldots} (-1)^{n_k}\, a_k\, L_k\,,\;\; n_k = 0,1,\;\; a_k = 0,1,\;\; L_k \neq 0$$

$$\text{massive Dirac fermions}: \quad \min(n_+(L'), n_-(L'))$$

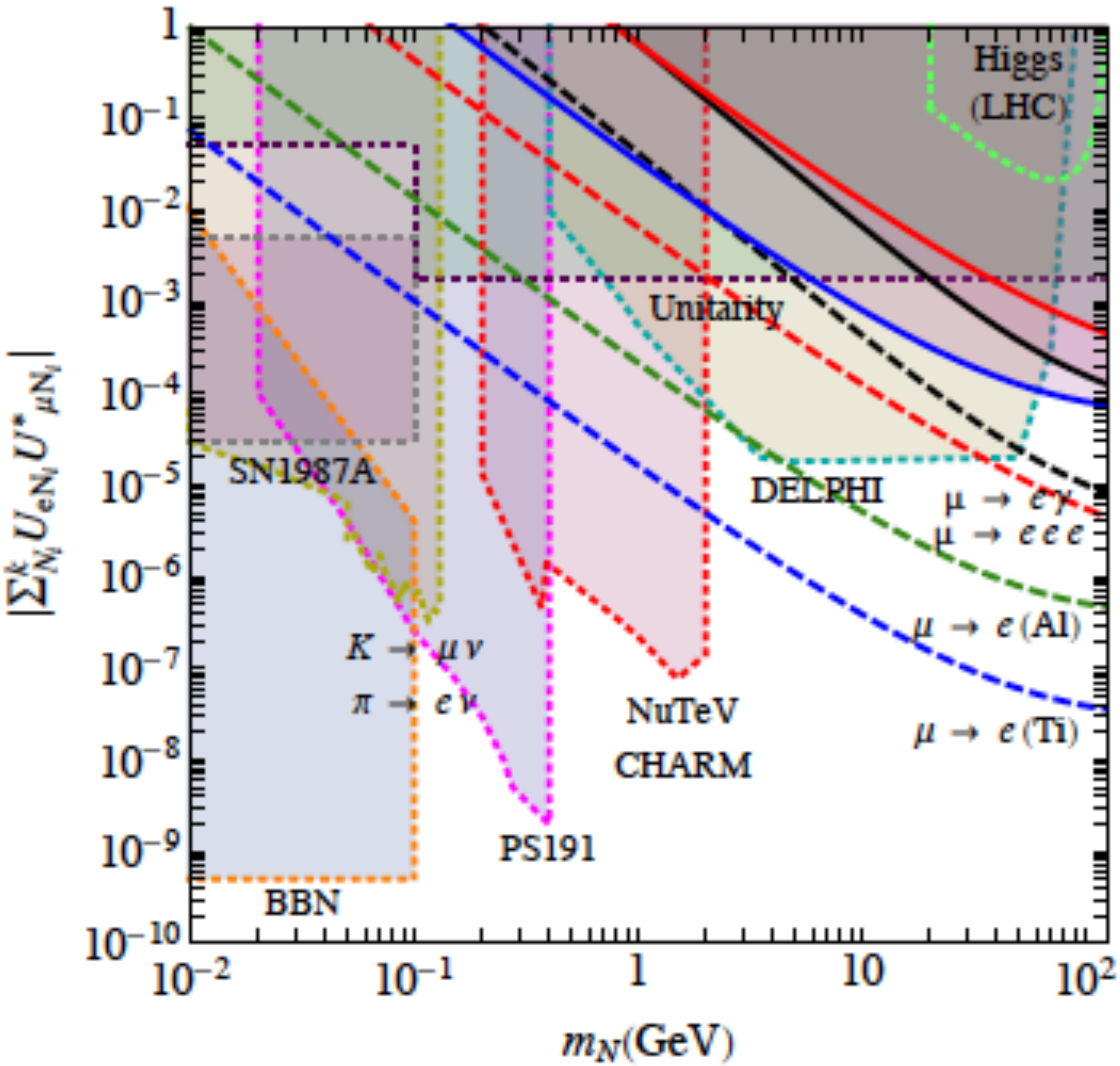
$$\text{massless fermions}: \quad |n_+(L') - n_-(L')|$$

Bilenky, Pontecorvo, 1981; Wolfenstein, 1981
Leung, Petcov, 1983; Wyler, Wolfenstein, 1983

L' softly broken $\Longrightarrow$ $|n_+(L')-n_-(L')|$ Majorana neutrinos with tiny masses and $\min(n_+(L'), n_-(L'))$ massive pseudo-Dirac fermions, corresponding to pairs of Majorana fermions almost degenerate in mass

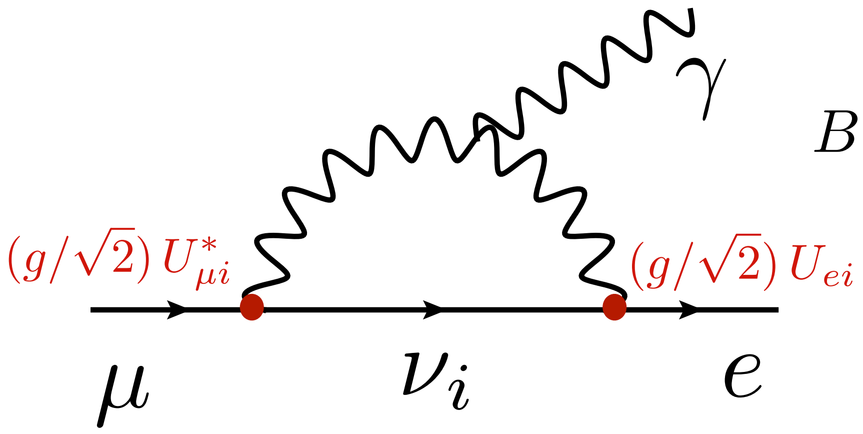
Constraints on low scale seesaw scenarios

Alonso, Dhen, Gavela, Hambye, 2012



Lepton flavour violation

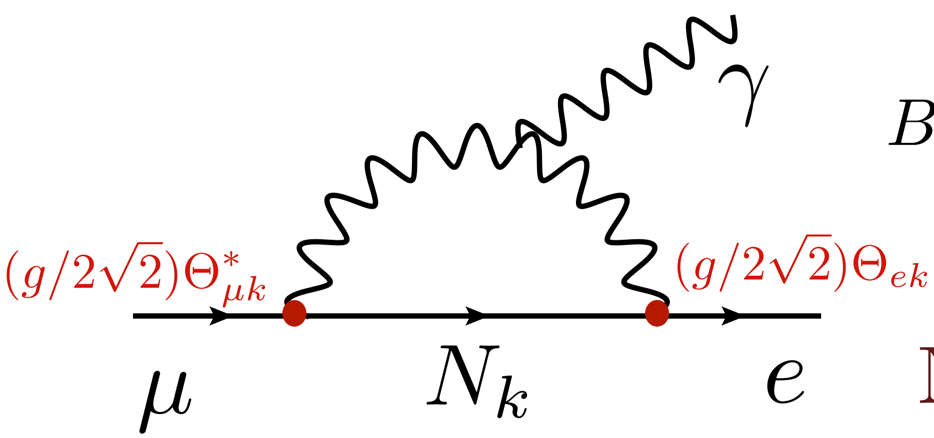
Standard contribution



$$BR(\mu \to e + \gamma) = \frac{3\alpha_{\rm em}}{32\pi}\left|\frac{\Delta m^2_{\rm sol}}{M_W^2}U_{e2}U^*_{\mu 2} + \frac{\Delta m^2_{\rm atm}}{M_W^2}U_{e3}U^*_{\mu 3}\right|^2 < 10^{-54}$$

Sizeable couplings, but strong GIM suppression, $\Delta m^2/M_W^2$

New contribution



$$BR(\mu \to e + \gamma) \; = \; \frac{3\alpha_{\rm em}}{8\pi}\left|\Theta^*_{\mu 1}\Theta_{e1}\right|^2 \left|G(M_1^2/M_W^2) - G(0)\right|^2$$

No GIM suppression, observable effects!

$$\Theta = RV$$

Lepton flavour violation

Present experimental bound:

$$\mathrm{BR}(\mu^+ \to e^+ \gamma) < 5.7 \times 10^{-13}$$ **MEG @ PSI**

Present experimental bound:

$$\mathrm{BR}(\mu^+ \to e^+ e^- e^+) < 1.0 \times 10^{-12}$$ **SINDRUM @ PSI**

Projected bounds:

$$\mathrm{BR}(\mu^+ \to e^+ e^- e^+) < 10^{-15}$$ **MuSIC facility @ Osaka University**

Present experimental bound:

$$\mathrm{CR}(\mu \mathrm{Ti} \to e \mathrm{Ti}) < 4.3 \times 10^{-12}$$ **SINDRUM II @ PSI**

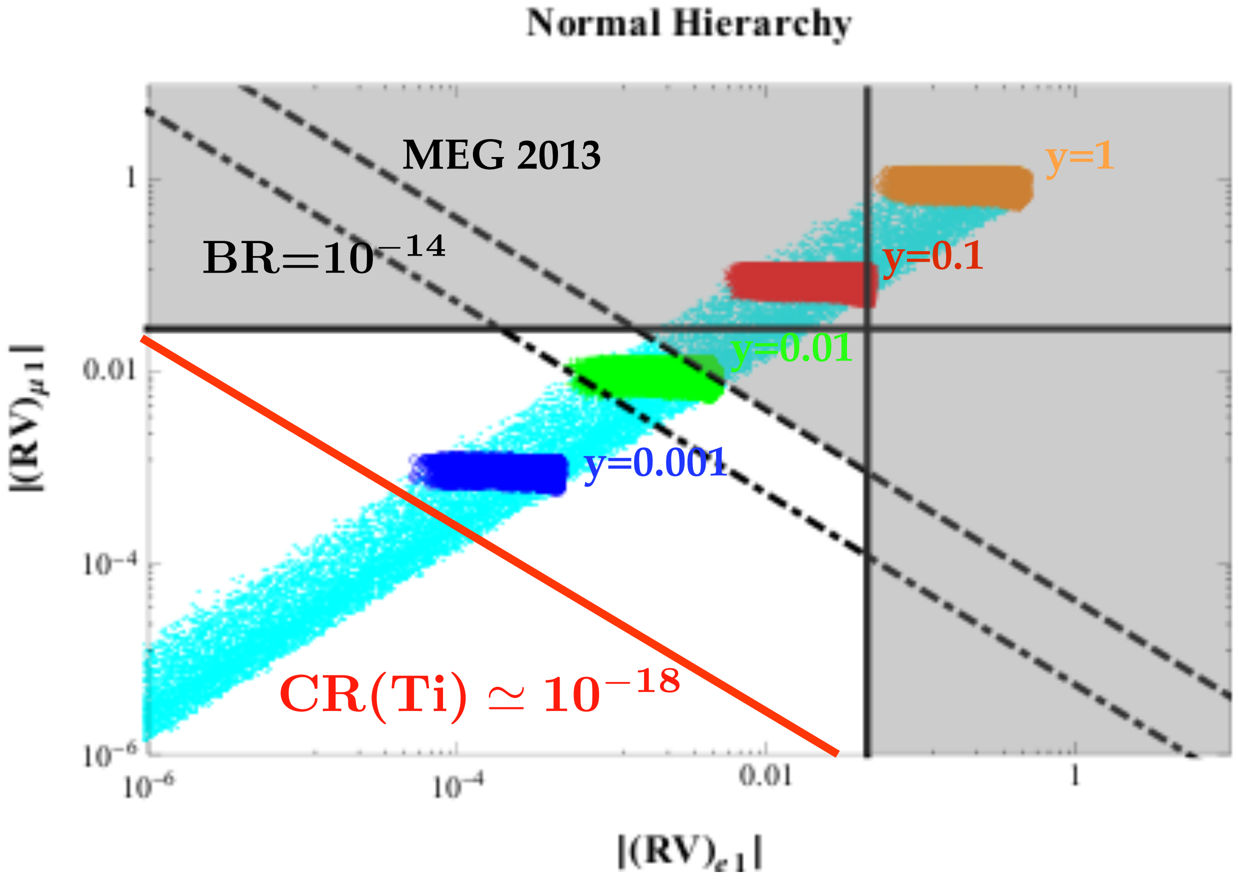
Projected bounds:

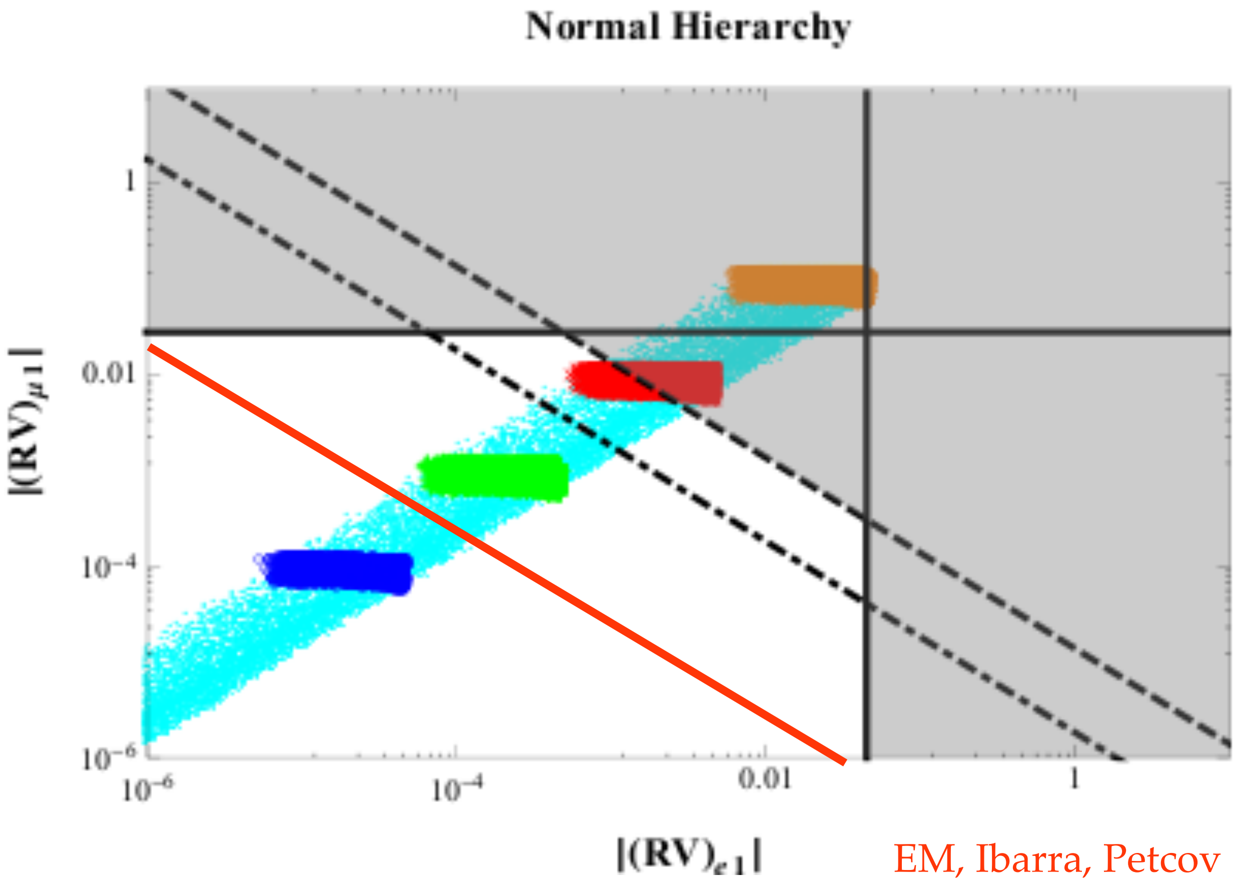
$$\mathrm{CR}(\mu \mathrm{Ti} \to e \mathrm{Ti}) \approx 10^{-18}$$ **PRISM/PRIME @ KEK, Project-X @ Fermilab**

$$\mathrm{CR}(\mu \mathrm{Al} \to e \mathrm{Al}) \approx 10^{-16}$$ **COMET @ KEK, Mu2e @ Fermilab**

Lepton flavour violation

$\mathbf{M_1} = \mathbf{100}$ GeV　　　　$\mathbf{M_1} = \mathbf{1000}$ GeV





EM, Ibarra, Petcov
Dinh, EM, Ibarra, Petcov

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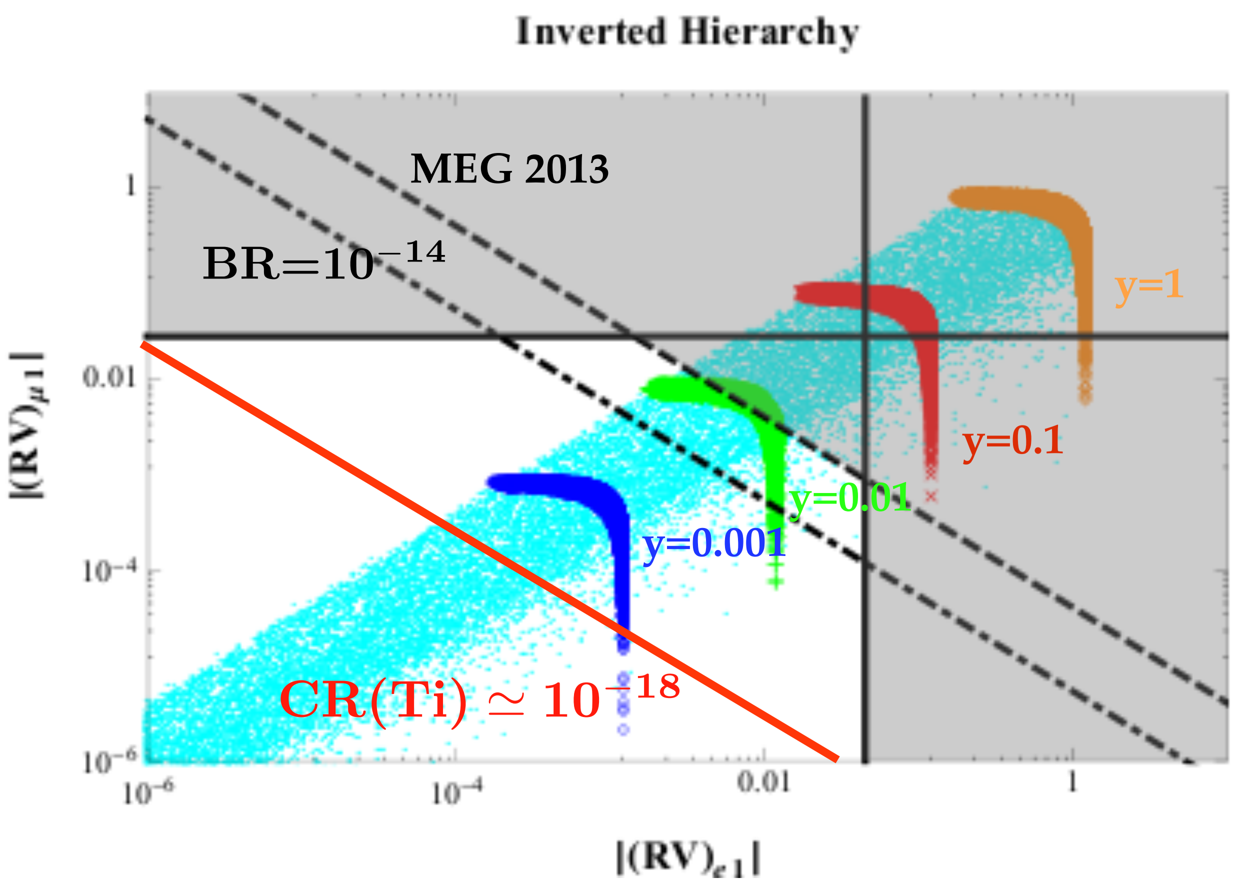
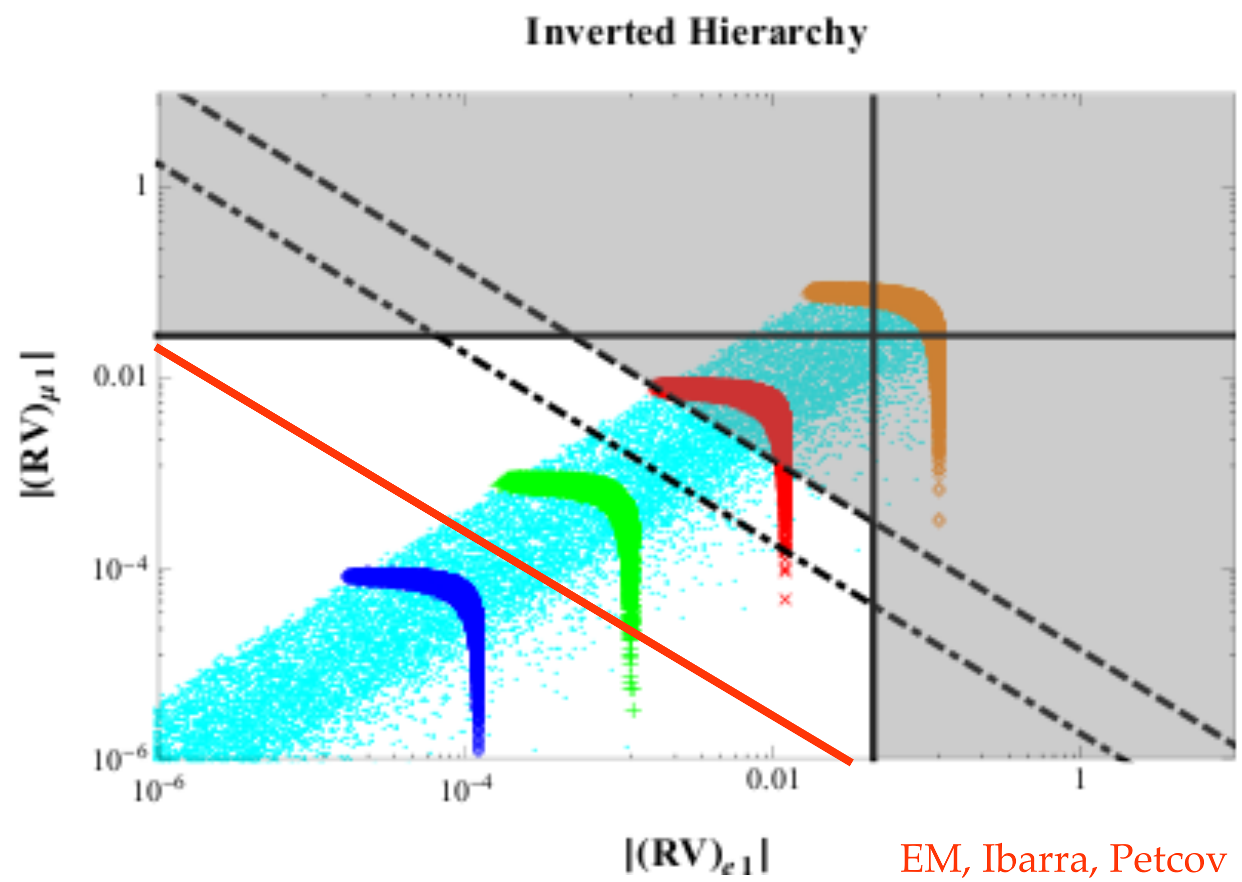
flavour structure fixed by neutrino oscillation parameters and $(RV)_{\ell k} \propto y\,v/M_k$

Lepton flavour violation

$\mathbf{M_1} = \mathbf{100}\ \mathrm{GeV}$



$\mathbf{M_1} = \mathbf{1000}\ \mathrm{GeV}$



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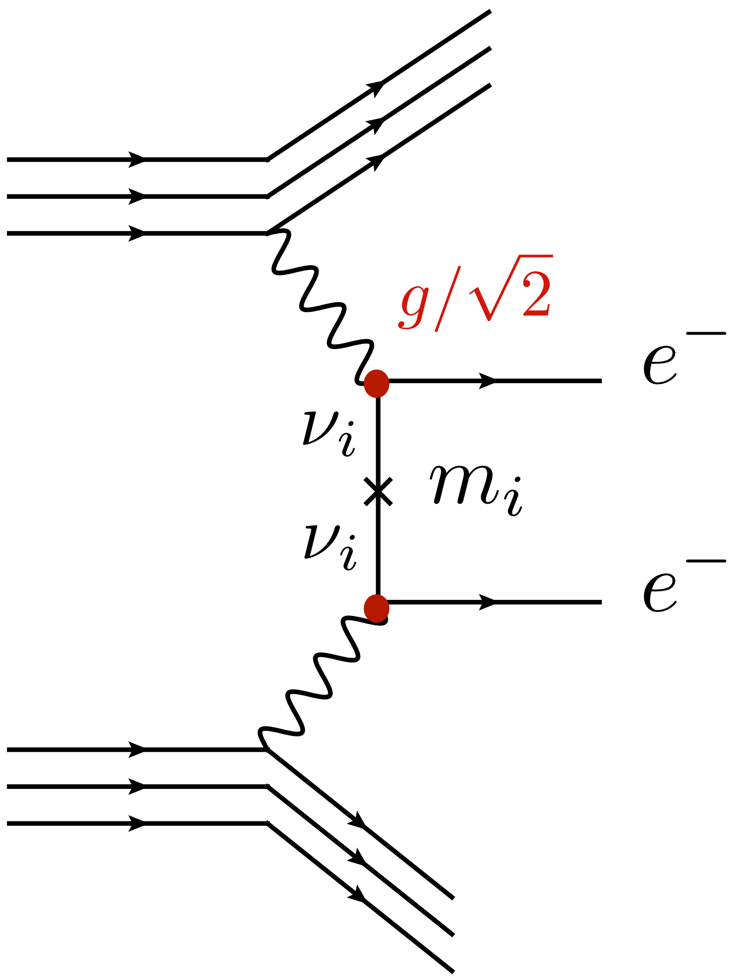
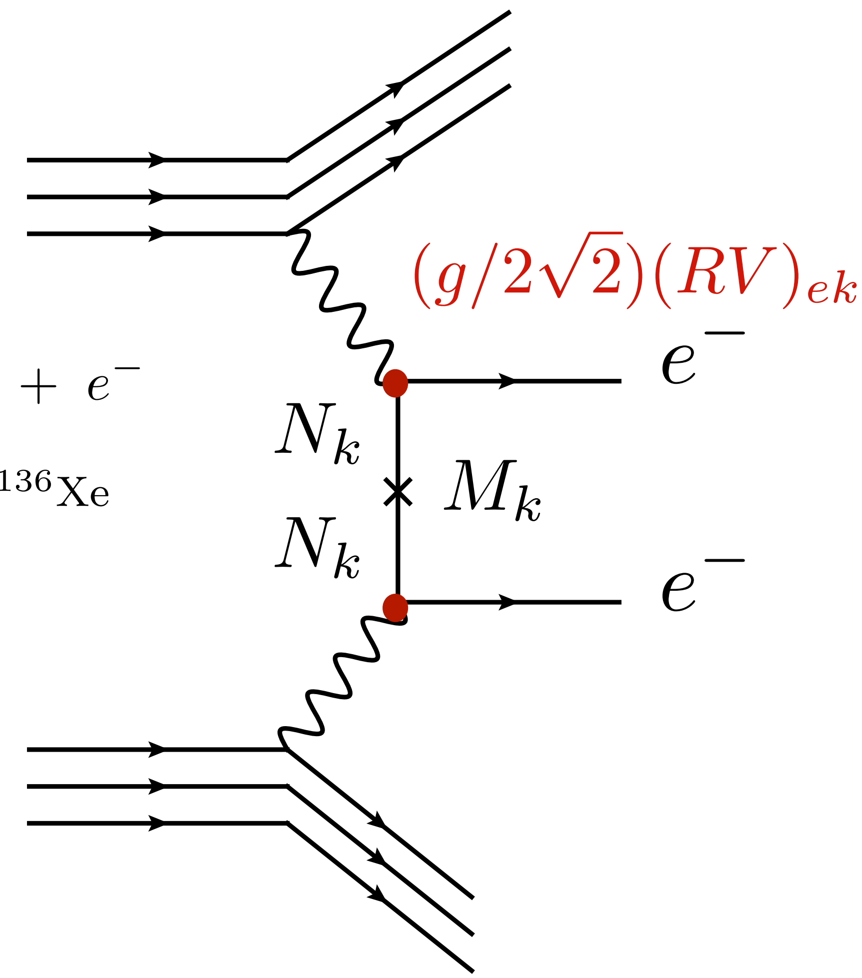
Neutrinoless double beta decay



$$(A,Z) \Longrightarrow (A,Z+2) + e^- + e^-$$

$^{48}\mathrm{Ca}$, $^{76}\mathrm{Ge}$, $^{82}\mathrm{Se}$, $^{124}\mathrm{St}$, $^{130}\mathrm{Te}$, $^{136}\mathrm{Xe}$

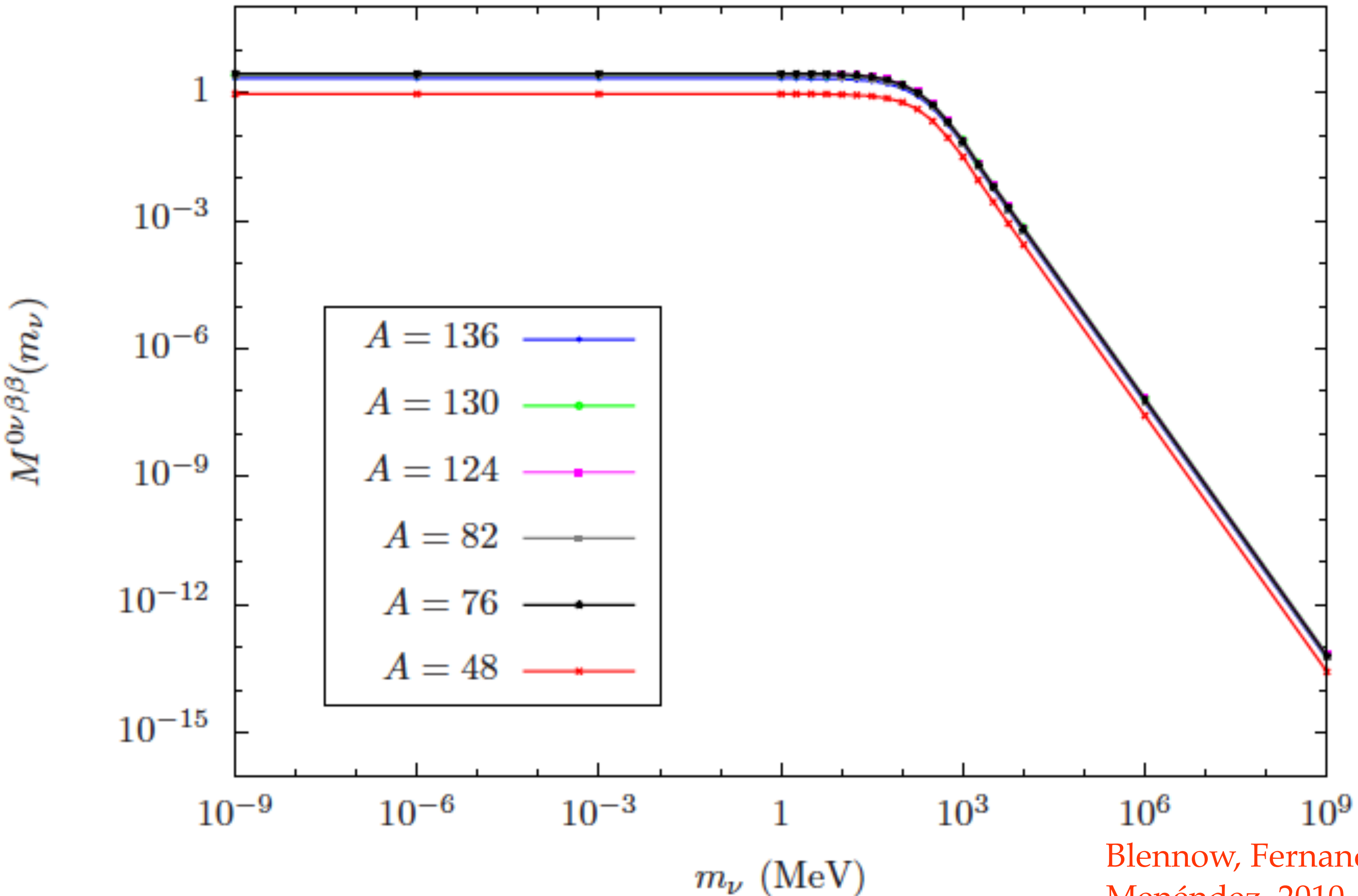


$$\frac{\Gamma_{0\nu\beta\beta}}{\ln 2} = G_{01} \left| \sum_j U_{ej}^2 \frac{\hat{m}_j}{m_e} \mathcal{M}^{0\nu\beta\beta}(\hat{m}_j) \right|^2$$

Kinematic Factor:
from leptonic degrees of freedom

Nuclear Matrix Elements:
dependence on the mass of the virtual neutrino propagating

Nuclear Matrix Elements



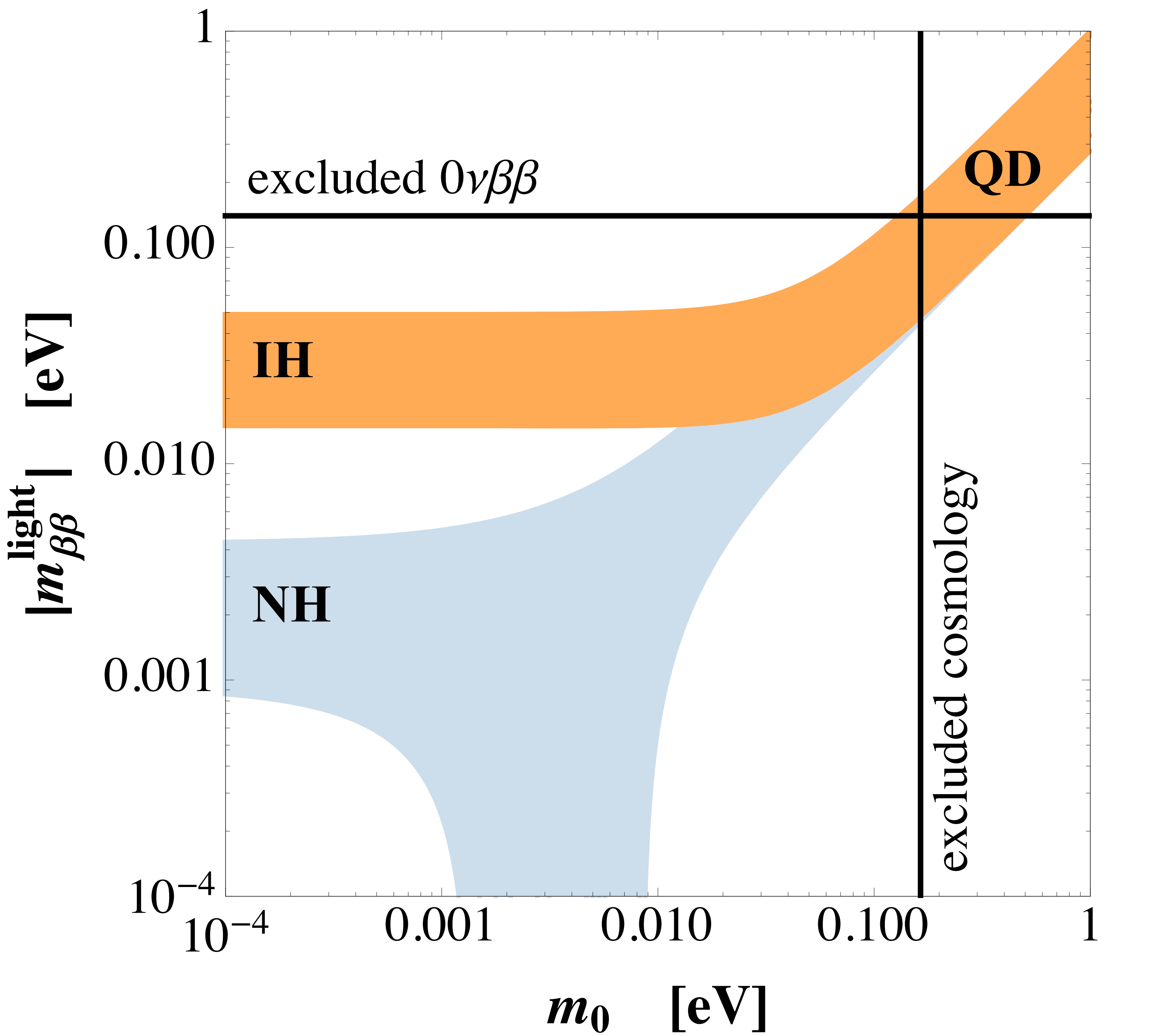
Blennow, Fernandez-Martinez, Lopez-Pavon, Menéndez, 2010

$$\Gamma_{0\mu\beta\beta} \propto |m_{\beta\beta}| \equiv \left| m_{\beta\beta}^{\text{light}} + m_{\beta\beta}^{\text{heavy}} \right|$$

for $M_k \gtrsim 100$ MeV

$$m_{\beta\beta}^{\text{heavy}} \approx -\sum_k (RV)^2_{ek}\, f(A)\, \frac{M_a^2}{M_k}$$

Effective Majorana neutrino mass



@ tree-level

$$m_{\beta\beta}^{\text{light}} = \sum_{i=1}^{3} (U_{\text{PMNS}})_{ei}^2 \, m_i = -\sum_k (R\,V)_{ek}^2 \, M_k$$

Effective Majorana neutrino mass

$$m_{\beta\beta}^{\rm light} = \sum_{i=1}^{3} (U_{\rm PMNS})^2_{ei}\, m_i = -\sum_k (R\,V)^2_{ek}\, M_k$$

$$m_{\beta\beta}^{\rm heavy} \approx -\sum_k (RV)^2_{ek}\, f(A)\, \frac{M_a^2}{M_k}$$

sizeable (heavy) sterile neutrino contribution $m_{\beta\beta}^{\rm heavy}$ can be achieved if

1. $M_1 \in$ [1 eV, 100 MeV], $M_k >$ 100 MeV

   strongly constrained by cosmology, N_{eff}

Blennow, Fernandez-Martinez, Lopez-Pavon, Menéndez, 2010

Hernandez, Kekic, Lopez-Pavon, 2013; 2014

2. $M_k \in$ [100 MeV, 1000 GeV]

   fine-tuning in the seesaw parameter space

Blennow, Fernandez-Martinez, Lopez-Pavon, Menéndez, 2010
Ibarra, EM, Petcov, 2010; 2011

Mitra, Senjanovic, Vissani, 2012
Lopez-Pavon, Pascoli, Wong 2012

The lepton number violation introduced through the RH neutrino Majorana mass term, *required to obtain a sizable effect in the 0νββ decay rate*, also appears at one-loop level in the light neutrino sector

Radiative corrections to neutrino masses

$$\mathcal{L}_\nu \;=\; -\overline{\nu_{\ell L}}\,(m_D)_{\ell s}\,\nu_{sR} - \frac{1}{2}\,\overline{\nu^c_{sL}}\,(M_R)_{st}\,\nu_{tR} \;+\; \text{h.c.}$$

$$\mathcal{M} \;=\; \begin{pmatrix} m_\nu^{1-\text{loop}} & m_D \\ m_D^T & M_R \end{pmatrix} = U^*\,\text{diag}\,(m_i, M_k)\,U^\dagger$$

At one-loop the dominant contribution comes from the ν self-energy: Pilaftsis, 1992

$$m_\nu^{1-\text{loop}} \;=\; \frac{1}{(4\,\pi\,v)^2}\,m_D\,\left(M_R^{-1}\,F(M_R M_R^\dagger) + F(M_R^\dagger M_R)\,M_R^{-1}\right)\,m_D^T$$

$$F(x) \equiv \frac{x}{2}\,\left(3\log(x/M_Z^2)\,(x/M_Z^2-1)^{-1} + \log(x/M_H^2)\,(x/M_H^2-1)^{-1}\right)$$

Radiative corrections to ν mass matrix can be relevant in seesaw scenarios with sizeable neutrino Yukawa couplings

$$m_\nu = m_\nu^{\text{tree}} + m_\nu^{1-\text{loop}} \;=\; U^*_{\text{PMNS}}\,\text{diag}(m_1, m_2, m_3)\,U^\dagger_{\text{PMNS}}$$

Radiative corrections to neutrino masses

$$(m_\nu)_{\ell\ell'} = -(m_D V)_{\ell k} \left[M_k^{-1} - \frac{1}{(4\pi v)^2} M_k \left(\frac{3 \log(M_k^2/M_Z^2)}{M_k^2/M_Z^2 - 1} + \frac{\log(M_k^2/M_H^2)}{M_k^2/M_H^2 - 1} \right) \right] (V^T m_D^T)_{k\ell'}$$

$$\equiv -(m_D V)_{\ell k} \Delta_k^{-1} (V^T m_D^T)_{k\ell'} = (U^*_{\rm PMNS} \, {\rm diag}(m_1, m_2, m_3) \, U^\dagger_{\rm PMNS})_{\ell\ell'}$$

One-loop generalization of the Casas-Ibarra parametrization: Lopez-Pavon, EM, Petcov, 2015

$$\left(\pm i \, \hat{m}^{-1/2} U^\dagger_{\rm PMNS} R V \Delta^{1/2} \right) \left(\pm i \, \hat{m}^{-1/2} U^\dagger_{\rm PMNS} R V \Delta^{1/2} \right)^T \equiv O \, O^T = 1$$

$$RV = \mp i \, U_{\rm PMNS} \, \hat{m}^{1/2} \, O \, \Delta^{-1/2}$$

Contribution to the effective Majorana neutrino mass from light ν's exchange:

$$m_{\beta\beta}^{\rm light} = m_{\beta\beta}^{\rm tree} + m_{\beta\beta}^{1-{\rm loop}} = \sum_{i=1}^{3} (U_{PMNS})^2_{ei} m_i$$

Two realizations of type I seesaw scenario

Let's consider another parametrization of the Dirac and Majorana mass matrices:

$$\mathcal{M} \equiv \begin{pmatrix} \mathbf{O} & m_D \\ m_D^T & M_R \end{pmatrix} = \begin{pmatrix} \mathbf{O} & \mathbf{Y}_1\, v/\sqrt{2} & \epsilon\, \mathbf{Y}_2\, v/\sqrt{2} \\ \mathbf{Y}_1^T v/\sqrt{2} & \mu' & \Lambda \\ \epsilon\, \mathbf{Y}_2^T\, v/\sqrt{2} & \Lambda & \mu \end{pmatrix}$$

Tree-level ν mass matrix:

$$m_\nu^{\rm tree} = \frac{v^2}{2\,(\Lambda^2 - \mu'\mu)} \left(\mu\, \mathbf{Y}_1\, \mathbf{Y}_1^T + \epsilon^2\, \mu'\, \mathbf{Y}_2\, \mathbf{Y}_2^T - \Lambda\, \epsilon\, (\mathbf{Y}_2\, \mathbf{Y}_1^T + \mathbf{Y}_1\, \mathbf{Y}_2^T)\right)$$

μ, μ' and ϵ interpreted as lepton number breaking parameters

$$L' = L_e + L_\mu + L_\tau + L_1 - L_2$$

Two realizations of type I seesaw scenario

$$m_\nu^{\rm tree} = \frac{v^2}{2\,(\Lambda^2 - \mu'\mu)}\left(\mu\,\mathbf{Y}_1\,\mathbf{Y}_1^T + \epsilon^2\,\mu'\,\mathbf{Y}_2\,\mathbf{Y}_2^T - \Lambda\,\epsilon\,(\mathbf{Y}_2\,\mathbf{Y}_1^T + \mathbf{Y}_1\,\mathbf{Y}_2^T)\right)$$

$$\mu' \gg \Lambda,\; y_{1\alpha}\,v \gg \mu,\; \epsilon\,y_{2\alpha}\,v \;(\textbf{extended seesaw limit})$$ Kang, Kim, 2007

$$M_1 \approx (\Lambda^2/\mu' - \mu)\,, \qquad (RV)_{\ell 1} \approx i\frac{v}{\sqrt{2}\,M_1}\left[y_{1\ell}\,\frac{\Lambda}{\mu'-\mu} - \epsilon\,y_{2\ell}\left(1 - \frac{\Lambda^2}{2(\mu'-\mu)^2}\right)\right]$$

$$M_2 \approx \mu' + \Lambda^2/\mu'\,, \qquad (RV)_{\ell 2} \approx \frac{v}{\sqrt{2}\,M_2}\left[y_{1\ell}\left(1 - \frac{\Lambda^2}{2(\mu'-\mu)^2}\right) + \epsilon\,y_{2\ell}\,\frac{\Lambda}{\mu'-\mu}\right]$$

$$\frac{(RV)_{\ell 1}}{(RV)_{\ell 2}} \approx i\sqrt{\frac{M_2}{M_1}}$$

Two realizations of type I seesaw scenario

$$m_\nu^{\rm tree} = \frac{v^2}{2\,(\Lambda^2 - \mu'\mu)}\left(\mu\,\mathbf{Y}_1\,\mathbf{Y}_1^T + \epsilon^2\,\mu'\,\mathbf{Y}_2\,\mathbf{Y}_2^T - \Lambda\,\epsilon\,(\mathbf{Y}_2\,\mathbf{Y}_1^T + \mathbf{Y}_1\,\mathbf{Y}_2^T)\right)$$

$$\mu' \gg \Lambda,\ y_{1\alpha}\,v \gg \mu,\ \epsilon\,y_{2\alpha}\,v\ \ (\textbf{extended seesaw limit})$$ Kang, Kim, 2007

tree-level contributions to the effective Majorana neutrino mass:

$$m_{\beta\beta}^{\rm light} \approx \frac{v^2}{2\,(\Lambda^2/\mu' - \mu)}\left(\frac{\mu}{\mu'}\,y_{1e}^2 - 2\,\epsilon\,\frac{\Lambda}{\mu'}\,y_{1e}\,y_{2e}\right) \overset{\Lambda^2 \gg \mu\mu'}{\approx} \frac{v^2}{2\,\Lambda^2}\left(\mu\,y_{1e}^2 - 2\,\epsilon\,\Lambda\,y_{1e}\,y_{2e}\right)$$

$$m_{\beta\beta}^{\rm heavy} \approx f(A)\,\frac{v^2\,M_a^2}{2\,(\Lambda^2/\mu' - \mu)^3}\left(\frac{\Lambda^2}{\mu'^2}\,y_{1e}^2 - 2\,\epsilon\,\frac{\Lambda}{\mu'}\,y_{1e}\,y_{2e}\right) \overset{\Lambda^2 \gg \mu\mu'}{\approx} f(A)\,\frac{\mu'\,v^2\,M_a^2}{2\,\Lambda^4}\left(y_{1e}^2 - 2\,\epsilon\,\frac{\mu'}{\Lambda}\,y_{1e}\,y_{2e}\right)$$

one-loop contribution to the effective Majorana neutrino mass:

$$m_{\beta\beta}^{1-\rm loop} \approx \frac{\mu'}{2}\,\frac{y_{1e}^2}{(4\,\pi)^2}\left(\frac{3\ln\left(\mu'^2/M_Z^2\right)}{\mu'^2/M_Z^2 - 1} + \frac{\ln\left(\mu'^2/M_H^2\right)}{\mu'^2/M_H^2 - 1}\right)$$

$$\overset{\mu' \gg M_H, M_Z}{\approx} \frac{y_{1e}^2}{(4\,\pi)^2}\left(\frac{3\,M_Z^2}{2\,\mu'}\ln\left(\mu'^2/M_Z^2\right) + \frac{M_H^2}{2\,\mu'}\ln\left(\mu'^2/M_H^2\right)\right)$$

Two realisations of type I seesaw scenario

$$m_\nu^{\rm tree} = \frac{v^2}{2\,(\Lambda^2 - \mu'\,\mu)}\left(\mu\,\mathbf{Y}_1\,\mathbf{Y}_1^T + \epsilon^2\,\mu'\,\mathbf{Y}_2\,\mathbf{Y}_2^T - \Lambda\,\epsilon\,(\mathbf{Y}_2\,\mathbf{Y}_1^T + \mathbf{Y}_1\,\mathbf{Y}_2^T)\right)$$

$$\Lambda \gg y_{1\alpha}\,v \gg \mu',\,\mu,\;\epsilon\,y_{2\alpha}\,v\;(\textbf{inverse seesaw limit})$$

Mohapatra, Valle, 1986
Branco, Grimus, Lavoura, 1989

$$M_1 \approx \Lambda - \frac{\mu+\mu'}{2}\,,\qquad (RV)_{\ell 1} \approx i\,\frac{v}{2M_1}\left[y_{1\ell}\left(1+\frac{\mu-\mu'}{4\Lambda}\right) - \epsilon\,y_{2\ell}\left(1-\frac{\mu-\mu'}{4\Lambda}\right)\right]$$

$$M_2 \approx \Lambda + \frac{\mu+\mu'}{2}\,,\qquad (RV)_{\ell 2} \approx \frac{v}{2M_2}\left[y_{1\ell}\left(1-\frac{\mu-\mu'}{4\Lambda}\right) + \epsilon\,y_{2\ell}\left(1+\frac{\mu-\mu'}{4\Lambda}\right)\right]$$

$$\frac{(RV)_{\ell 1}}{(RV)_{\ell 2}} \approx i\sqrt{\frac{M_2}{M_1}}$$

Two realizations of type I seesaw scenario

$$m_\nu^{\rm tree} = \frac{v^2}{2\,(\Lambda^2 - \mu'\mu)}\left(\mu\,\mathbf{Y}_1\,\mathbf{Y}_1^T + \epsilon^2\,\mu'\,\mathbf{Y}_2\,\mathbf{Y}_2^T - \Lambda\,\epsilon\,(\mathbf{Y}_2\,\mathbf{Y}_1^T + \mathbf{Y}_1\,\mathbf{Y}_2^T)\right)$$

$$\Lambda \gg y_{1\alpha}\,v \gg \mu',\,\mu,\,\epsilon\,y_{2\alpha}\,v\;(\textbf{inverse seesaw limit})$$

Mohapatra, Valle, 1986
Branco, Grimus, Lavoura, 1989

tree-level contributions to the effective Majorana neutrino mass:

$$m_{\beta\beta}^{\rm light} \approx \frac{v^2}{2\,\Lambda^2}\left(\mu\,y_{1e}^2 - 2\,\epsilon\,\Lambda\,y_{1e}\,y_{2e}\right)$$

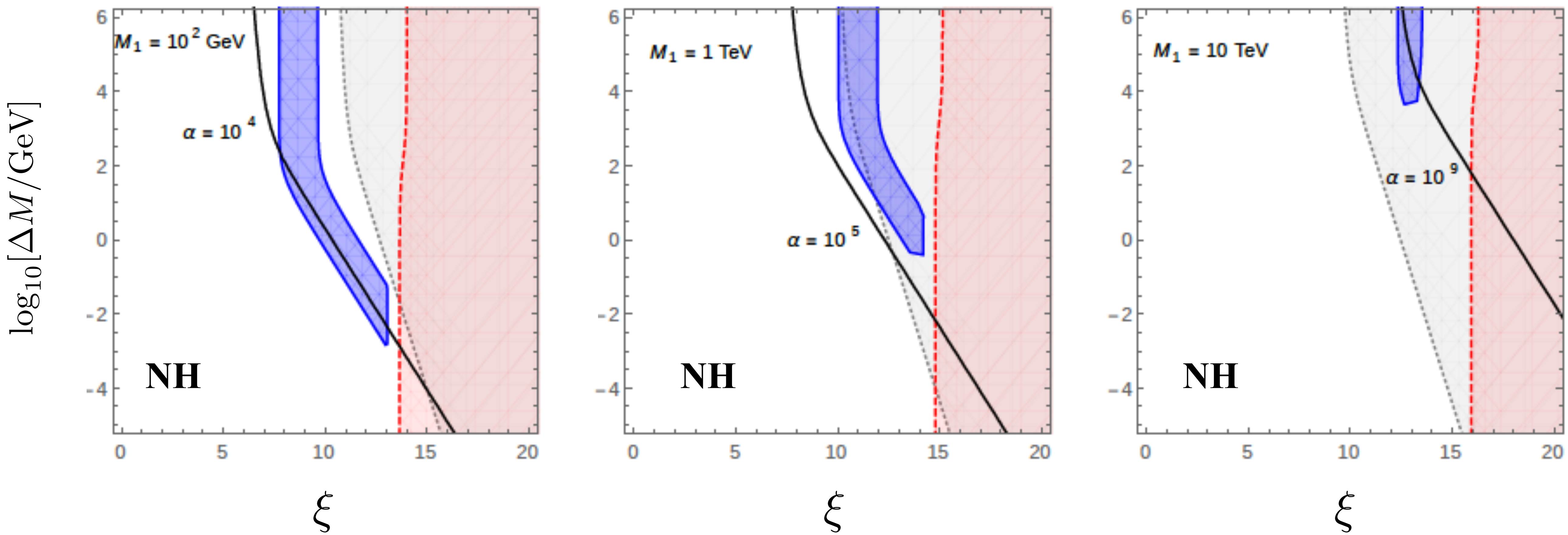
$$m_{\beta\beta}^{\rm heavy} \approx f(A)\,\frac{v^2\,M_a^2}{2\,\Lambda^4}\left((2\,\mu + \mu')\,y_{1e}^2 - 2\,\epsilon\,\Lambda\,y_{1e}\,y_{2e}\right)$$

one-loop contribution to the effective Majorana neutrino mass:

$$m_{\beta\beta}^{1-\rm loop} \approx \frac{1}{(4\,\pi)^2}\left(\epsilon\,\Lambda\,y_{1e}\,y_{2e} - \frac{\mu}{2}\,y_{1e}^2\right)\left(\frac{3\ln\left(\Lambda^2/M_Z^2\right)}{\Lambda^2/M_Z^2 - 1} + \frac{\ln\left(\Lambda/M_H^2\right)}{\Lambda^2/M_H^2 - 1}\right)$$
$$- \frac{\mu + \mu'}{2}\,\frac{y_{1e}^2}{(4\,\pi)^2}\left(\frac{4M_H^2M_Z^2 - \Lambda^2\left(M_H^2 + 3M_Z^2\right)}{(\Lambda^2 - M_Z^2)\,(\Lambda^2 - M_H^2)} + \frac{\ln\left(\Lambda^2/M_H^2\right)}{\left(\Lambda^2/M_H^2 - 1\right)^2} + \frac{3\ln\left(\Lambda^2/M_Z^2\right)}{\left(\Lambda^2/M_Z^2 - 1\right)^2}\right)$$

Large heavy neutrino contribution to 0νββ

$$10^{-2}\ \text{eV} < |m_{\beta\beta}^{\text{light}} + m_{\beta\beta}^{\text{heavy}}| < 0.5\ \text{eV}$$



Lopez-Pavon, EM, Petcov, 2015

tuning parameter:

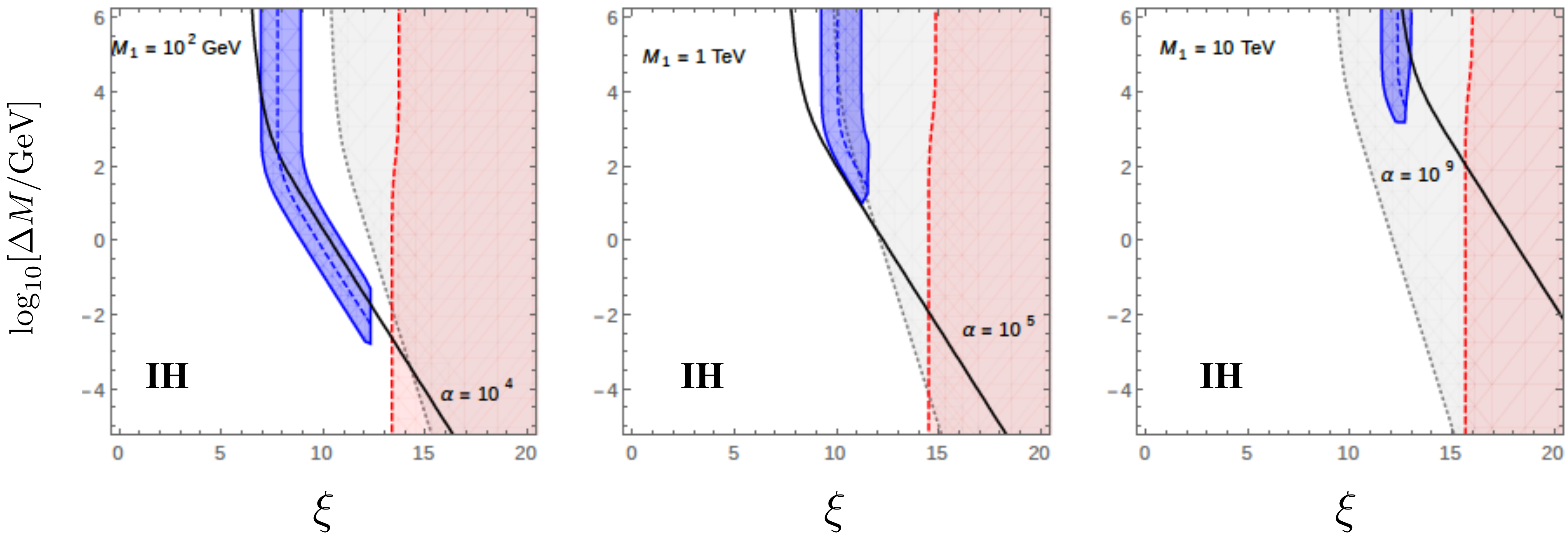
$$\alpha \equiv |m_{\beta\beta}^{1-\text{loop}}|/|m_{\beta\beta}^{\text{light}}|\,, \qquad m_{\beta\beta}^{\text{light}} \equiv m_{\beta\beta}^{\text{tree}} + m_{\beta\beta}^{1-\text{loop}} = \sum_{i=1}^{3} (U_{PMNS})_{ei}^2 m_i$$

bound from two-loop contribution:

$$m_{\beta\beta}^{2-\text{loop}} \sim \frac{y_{1e}^2}{(4\,\pi)^2}\, m_{\beta\beta}^{1-\text{loop}} \ll m_{\beta\beta}^{\text{light}}$$

Large heavy neutrino contribution to 0νββ

$$10^{-2}\ \text{eV} < |m_{\beta\beta}^{\text{heavy}}| < 0.5\ \text{eV}$$



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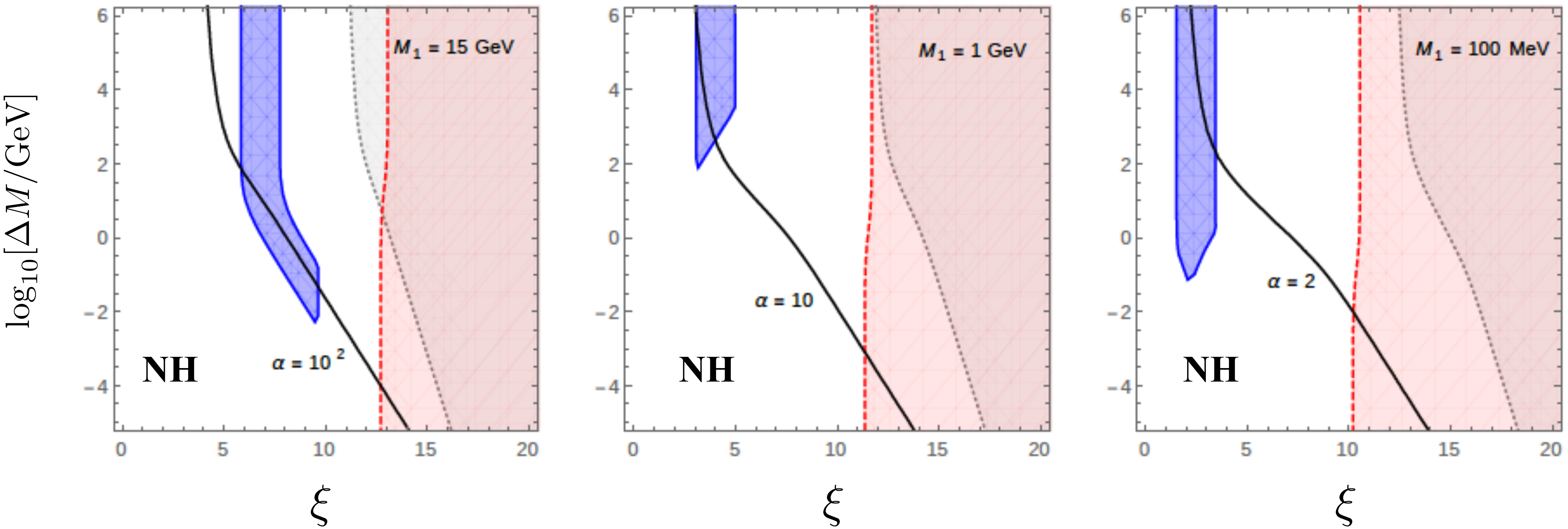

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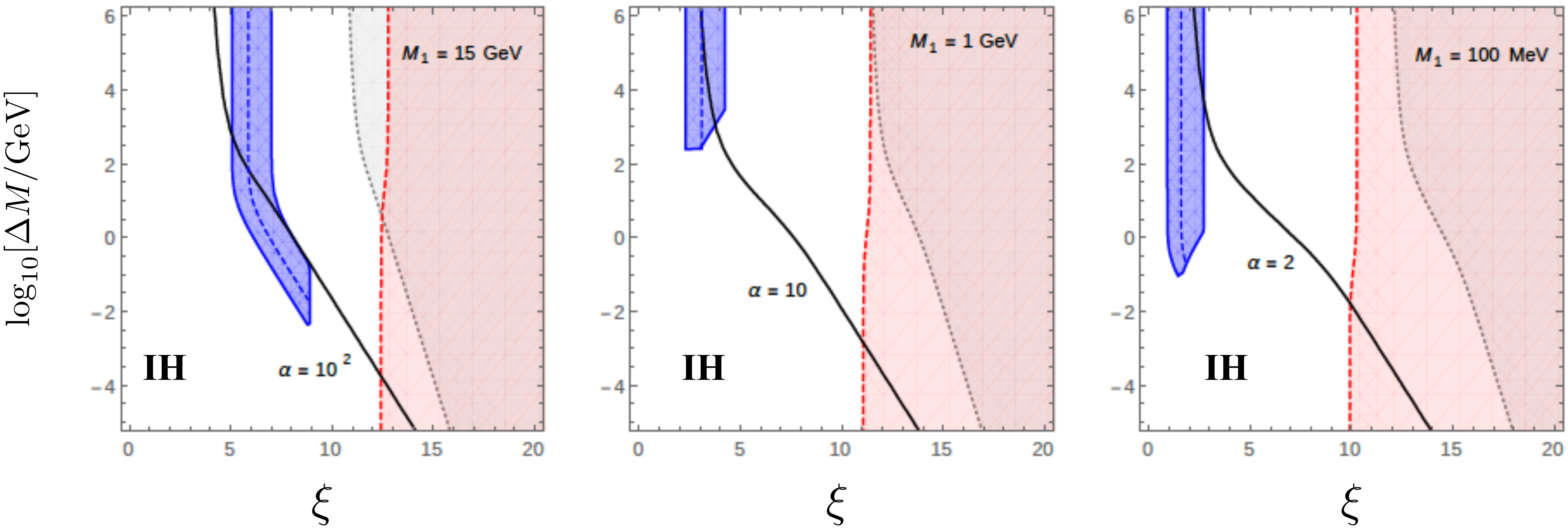

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bound from two-loop contribution:

$$m_{\beta\beta}^{2-\text{loop}} \sim \frac{y_{1e}^2}{(4\,\pi)^2}\,m_{\beta\beta}^{1-\text{loop}} \ll m_{\beta\beta}^{\text{light}}$$

Summary

A minimal extension of the Standard Model, which provides a mechanism for the generation of neutrino masses and mixing, consists of adding singlet RH neutrinos

The RH neutrino mass introduces a new scale in the theory, which can be of the same order as or smaller than the EW symmetry breaking scale

- ❖ For RH neutrino masses in the range [100 MeV, 10 TeV] an enhancement of $0\nu\beta\beta$ decay rate is always possible, due to sizeable heavy Majorana neutrino contribution
- ❖ Sizeable contribution of heavy neutrinos with masses $\gtrsim$ few GeV to $0\nu\beta\beta$ decay implies a fine-tuned cancellation between the tree-level and one-loop expressions of the light neutrino mass matrix to stabilize neutrino masses and mixing
- ❖ Such cancellation is always possible and consistent with oscillation data, low energy constraints from direct searches, charged lepton flavour violation, non-unitarity
- ❖ A fine-tuning of 1 part in 10^4 for RH mass ~100 GeV is unavoidable to obtain a dominant contribution in $0\nu\beta\beta$ decay rate. The tuning is very mild in the low mass regime and an enhancement of the rate can be easily achieved