The proton radius puzzle and neutrino cross sections

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based on 1505.01489, with Gabriel Lee, John Arrington

INFO workshop, Santa Fe

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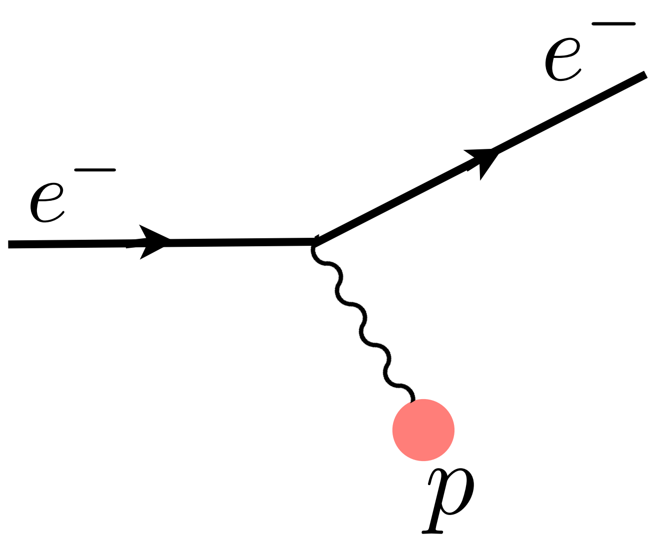
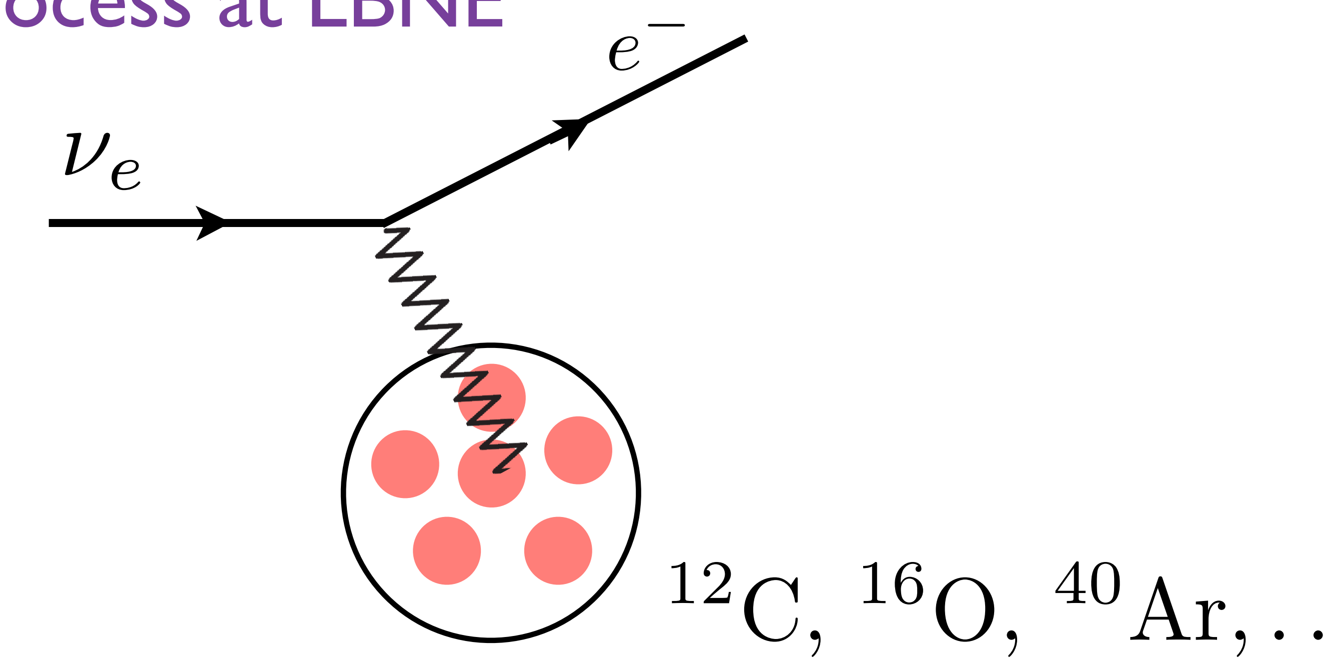
Regardless of the existence of the "proton radius puzzle":

- serious issues to confront in the precision era of lepton-nucleon scattering data
- addressing these issues will be critical to discovery potential of the accelerator neutrino program

e-p scattering



signal process at LBNE



Solving the simpler e-p problem prerequisite to more challenging neutrino processes

The applications, the problems, and the theoretical tools are central to HEP

Some facts about the Rydberg constant puzzle (a.k.a. proton radius puzzle)

1) It has generated a lot of attention and controversy

2) The *most mundane* resolution necessitates:

- 5σ shift in fundamental Rydberg constant
- discarding or revising decades of results in e-p scattering and hydrogen spectroscopy

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The New York Times

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"The good news is that it's not my problem"

This *is* HEP's problem:

3) Systematic effects in electron-proton scattering impact neutrino-nucleus scattering, *at a level large compared to precision requirements for oscillation measurements*

To give an idea of numerics, recall

$|z| \ll 1$ ⇒ Form factors ~linear in appropriate variable

⇒ need normalization and slope

	normalization	slope
electric charge form factor:	Q	r_E^2
axial form factor:	g_A	r_A^2

Determinations of r_E differ by as much as 8%.
"World average" r_A quoted with uncertainty ≲2%

Talk by A. Meyer tomorrow: model-independent analysis of deuterium, lattice QCD

Recall hydrogen spectrum:

$$E_n \sim \frac{R_\infty}{n^2} + \frac{r_E^2}{n^3}$$

$$hcR_\infty = \frac{m_e c^2 \alpha^2}{2} \approx 13.6\,\mathrm{eV}$$

proton charge radius

Disentangle 2 unknowns, R_∞ and r_E, using well-measured 1S-2S hydrogen transition *and*

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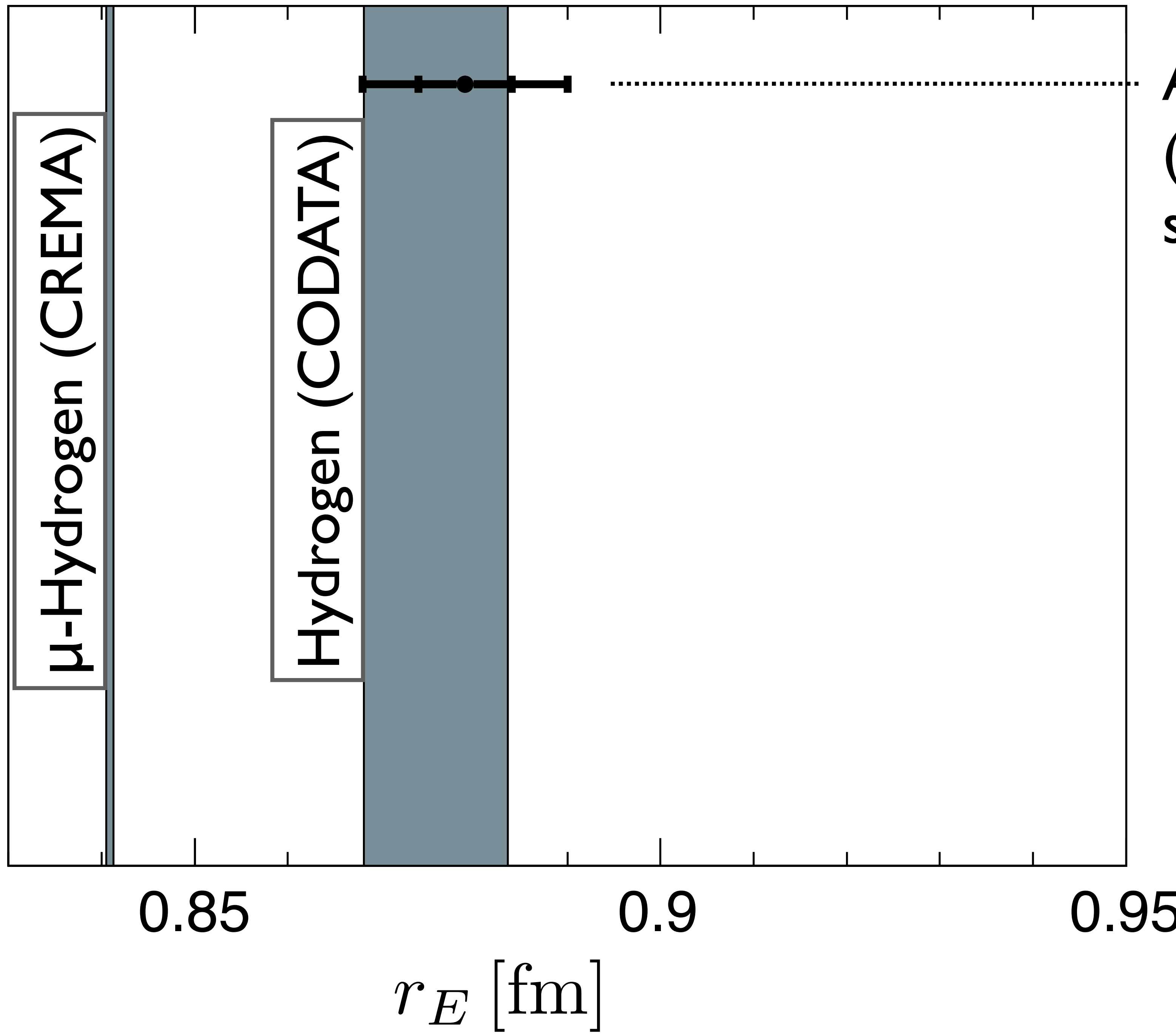
5σ discrepancy in Rydberg constant from (1+2) versus (3)



A1 analysis of Mainz data (default: 8 parameter cubic spline fit)

PRC 90, 015206

<u>this talk</u>: new extraction of proton charge and magnetic radii from electron scattering data

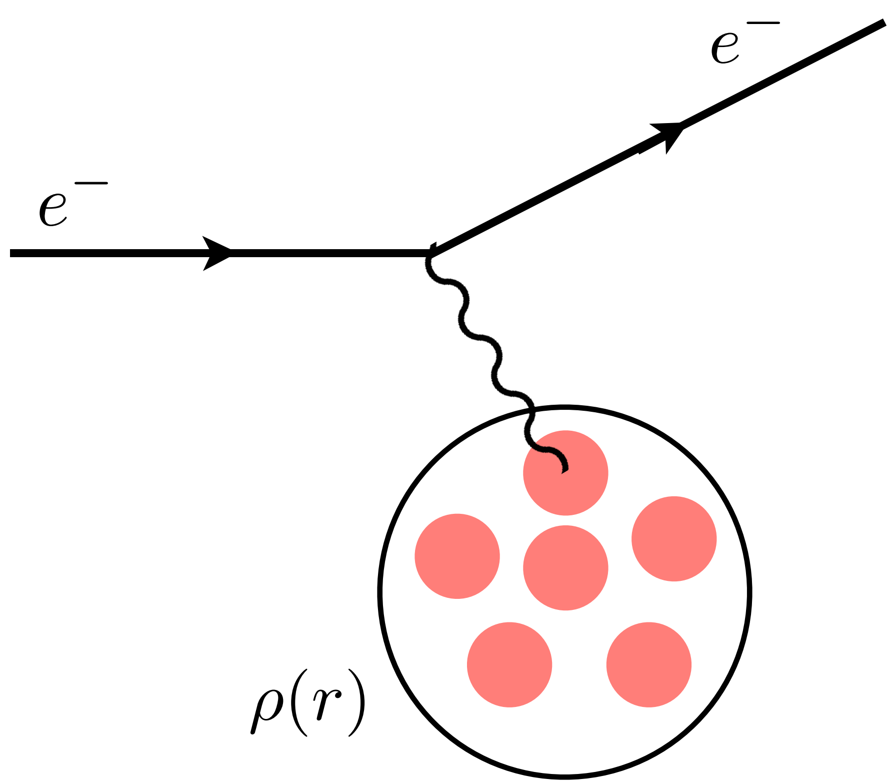
preliminaries

What is the proton charge radius?

recall scattering from extended classical charge distribution:

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{pointlike}} |F(q^2)|^2$$

$$F(q^2) = \int d^3r\, e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \rho(\boldsymbol{r})$$
$$= \int d^3r \left[1 + i\boldsymbol{q}\cdot\boldsymbol{r} - \frac{1}{2}(\boldsymbol{q}\cdot\boldsymbol{r})^2 + \dots\right]\rho(r)$$
$$= 1 - \frac{1}{6}\langle r^2\rangle \boldsymbol{q}^2 + \dots$$



for the relativistic, QM, case, *define* radius as slope of form factor

$$r_E^2 \equiv 6 \frac{d}{dq^2} G_E(q^2)\Big|_{q^2=0}$$

$$\langle J^\mu \rangle = \gamma^\mu F_1 + \frac{i}{2m_p}\sigma^{\mu\nu} q_\nu F_2$$

$$G_E = F_1 + \frac{q^2}{4m_p^2} F_2 \qquad G_M = F_1 + F_2$$

similarly for r_M from G_M

Consider separately two datasets

- “Mainz”: high statistics 2010 Mainz A1 collaboration data (1422 datapoints)

- “world”: global cross section and polarization data excluding Mainz (406 datapoints below Q^2=1GeV^2)

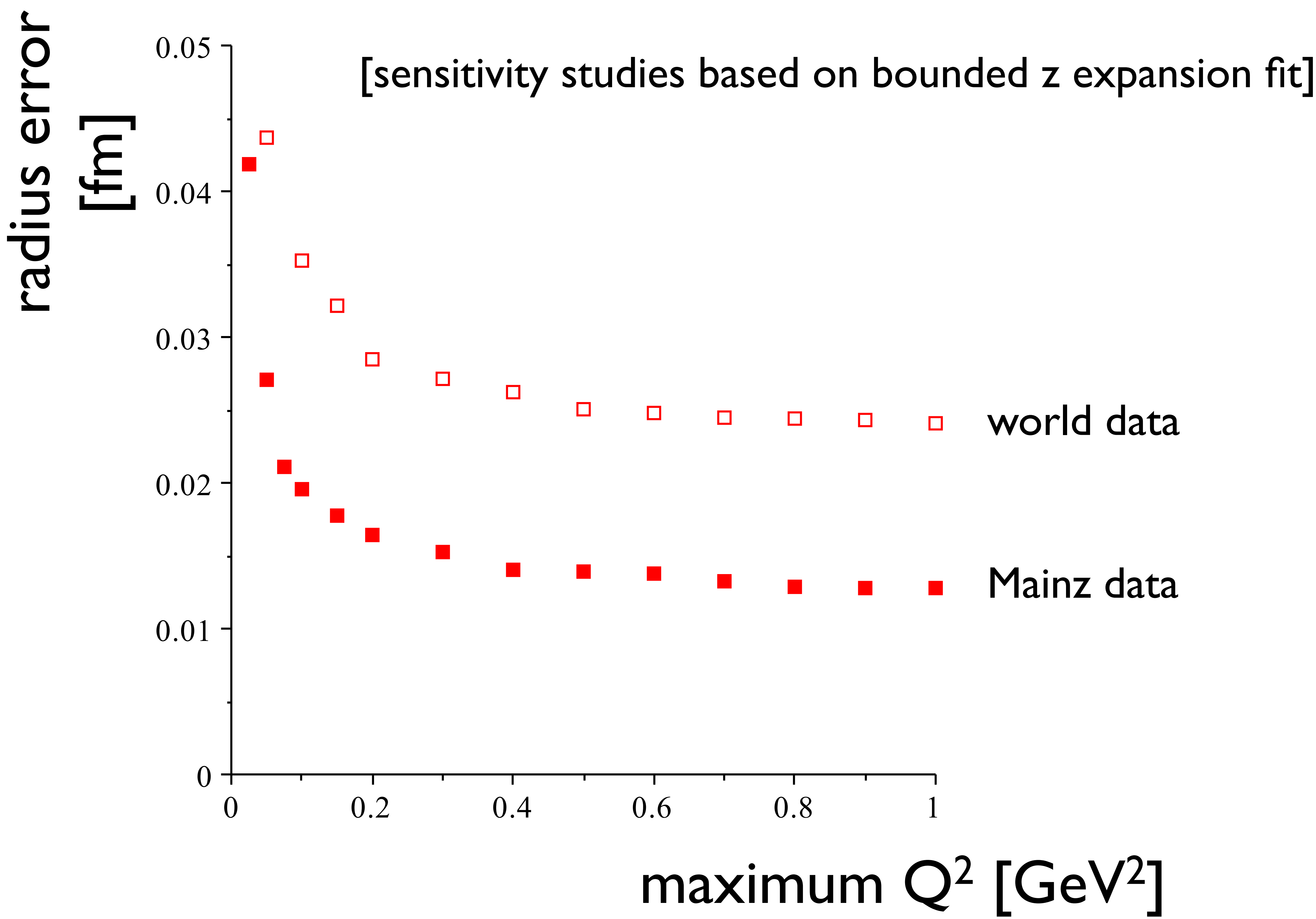
Focus first on r_E and the Mainz dataset, addressing in succession:

- Form factor shape
- Radiative corrections
- Uncorrelated systematic errors
- Correlated systematic errors

After fixing procedures, present final results for r_E and r_M, for Mainz and world datasets

form factor shape

Radius defined as slope. Requires data over finite Q^2 range



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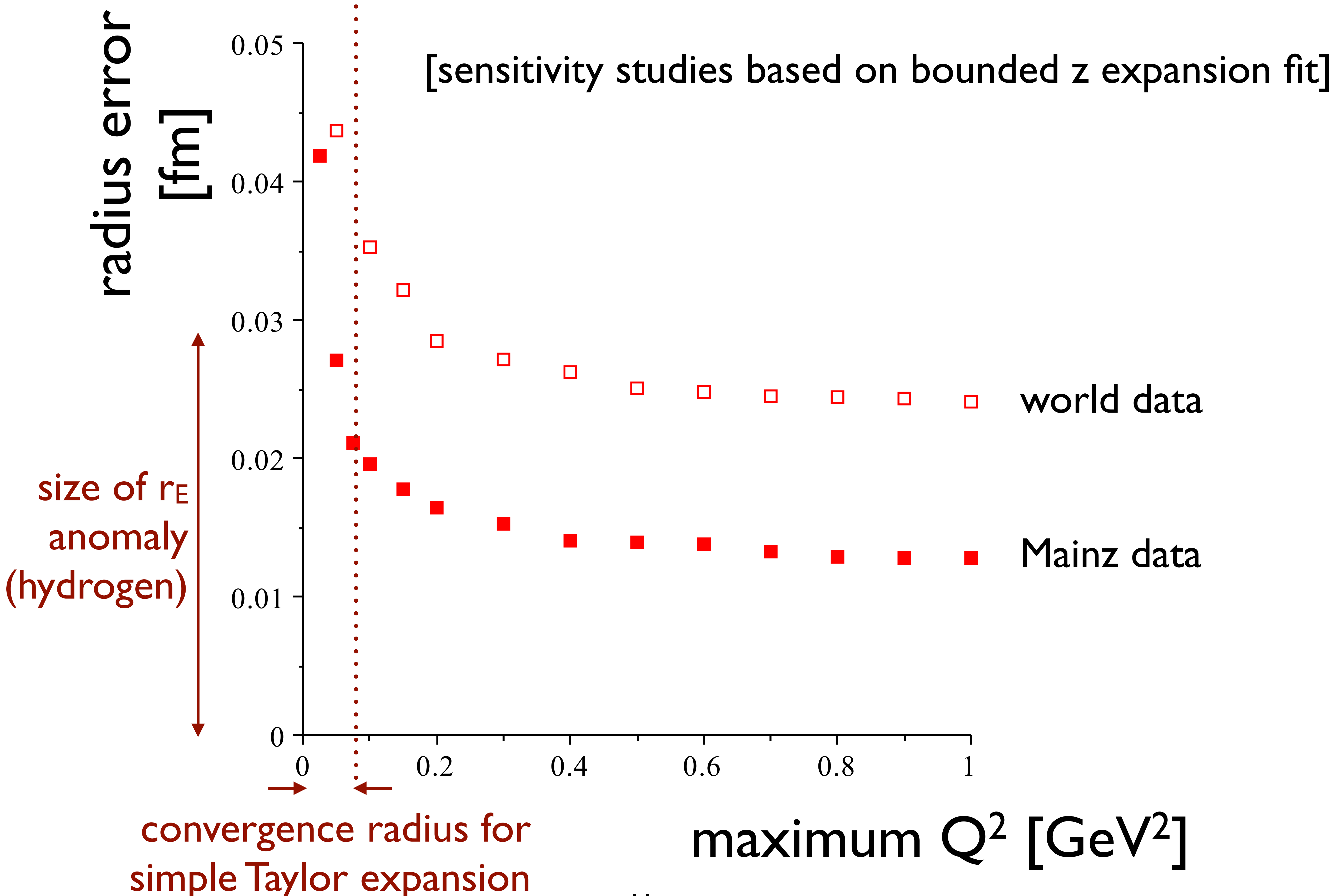
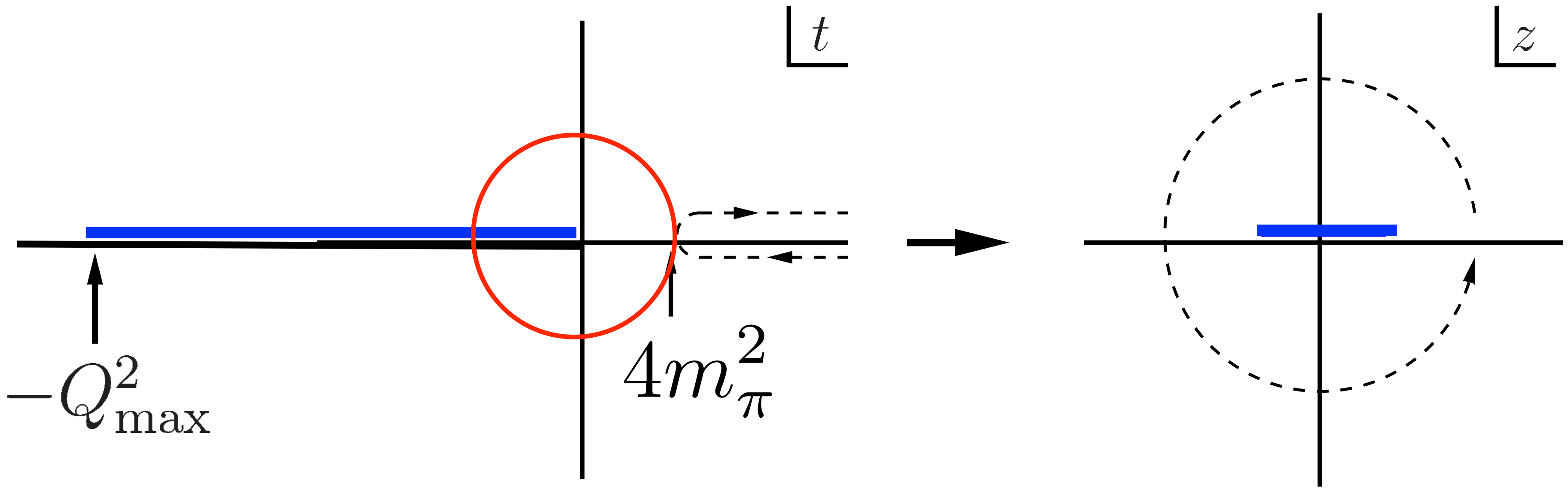


Unfortunately, for the proton form factors, a simple Taylor expansion has finite (small) radius of convergence



Fortunately, the analytic structure of amplitudes allows us to "resum" by change of variables into expansion covering the entire physical region

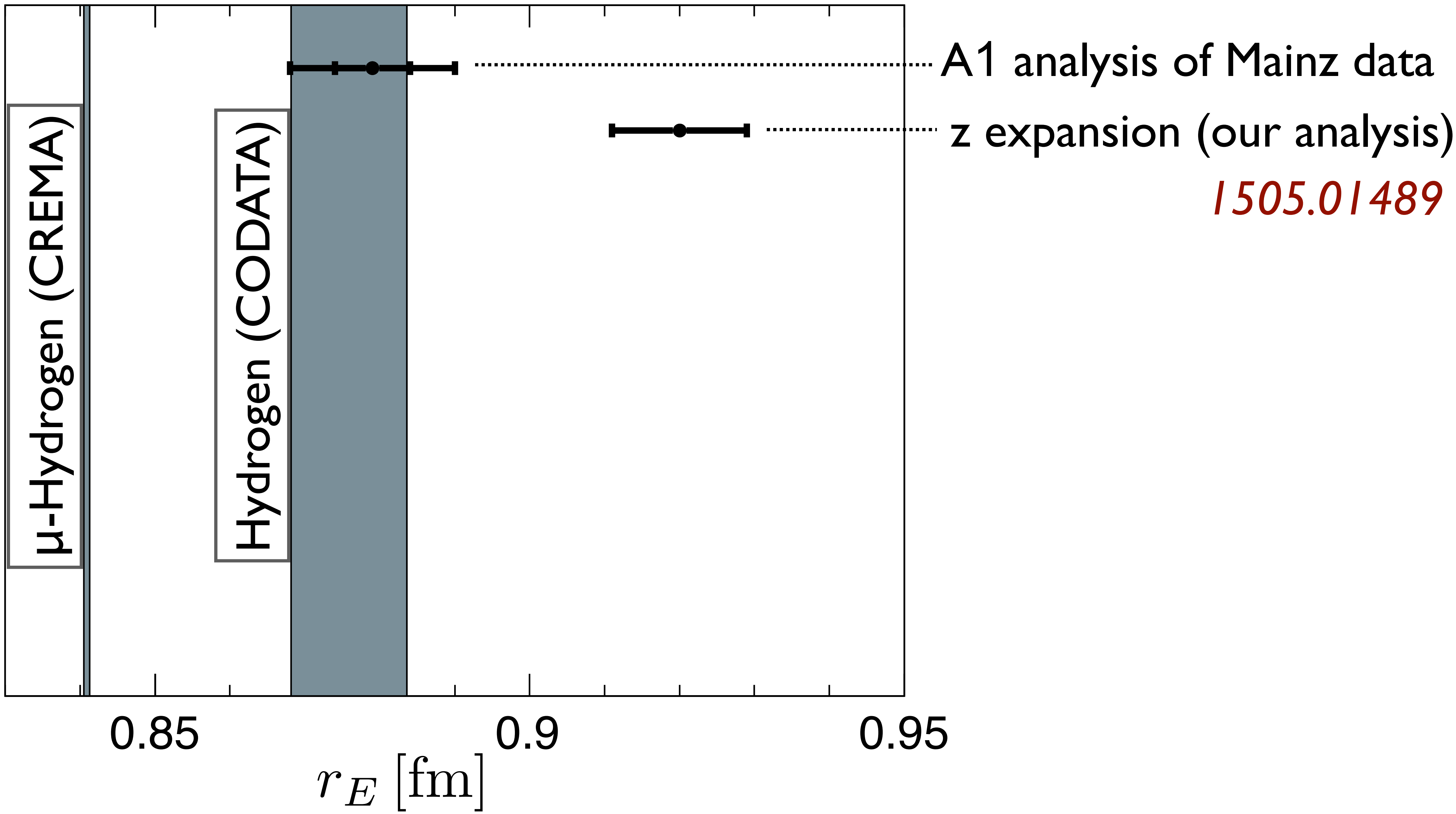
$$z(t, t_{\rm cut}, t_0) = \frac{\sqrt{t_{\rm cut} - t} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - t} + \sqrt{t_{\rm cut} - t_0}}$$

$4m_\pi^2$ (isoscalar channel)

point mapping to z=0
(scheme choice)

$$G_E(q^2) = \sum_k a_k [z(q^2)]^k$$

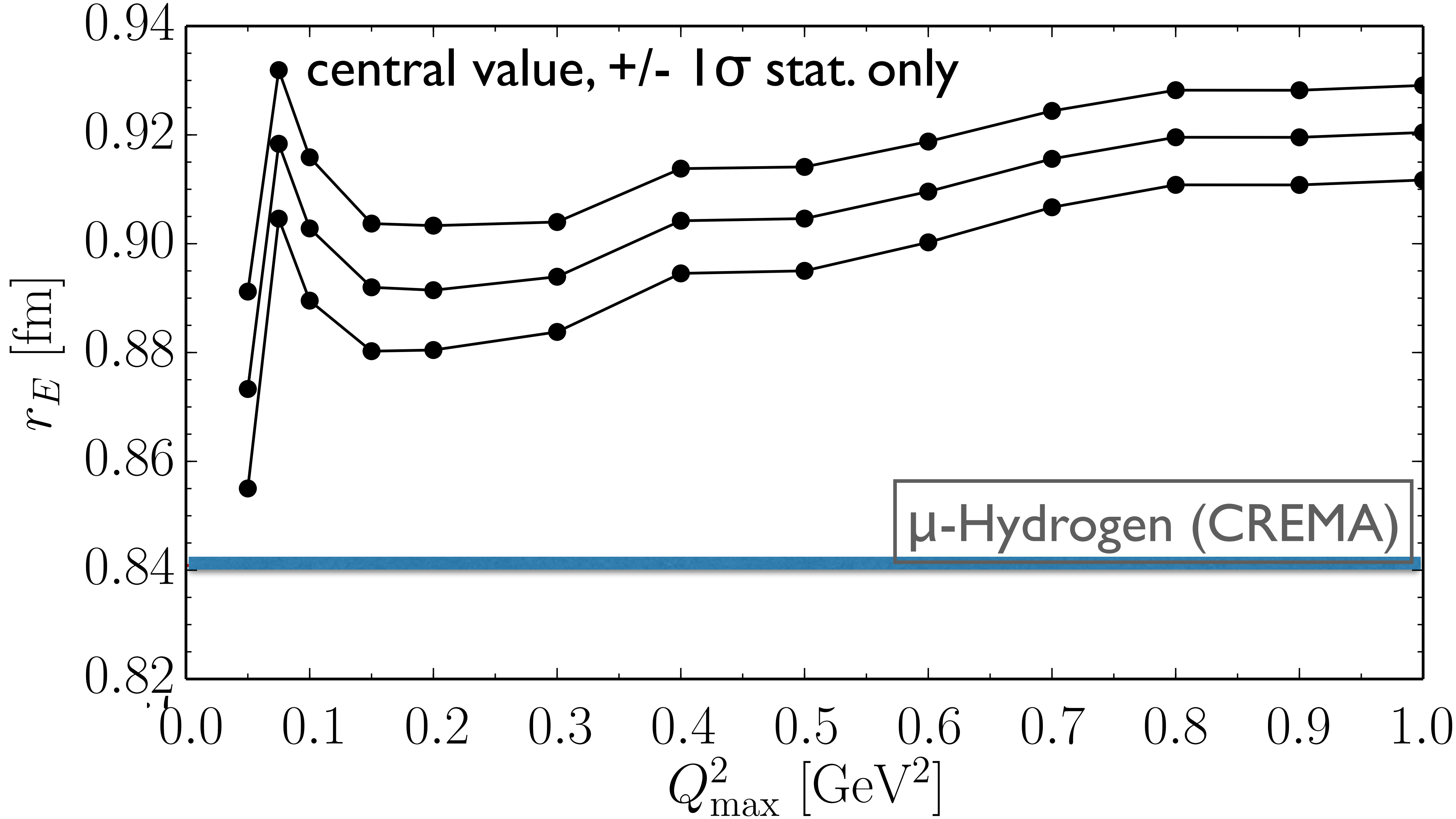
fit for undetermined order unity coefficients a_k



Require form factors to lie within QCD-constrained class of curves: *larger (7σ) discrepancy with* μ-*Hydrogen !*

Besides 7σ discrepancy with μH, now 3σ tension with H, 3σ with A1 analysis of same dataset.

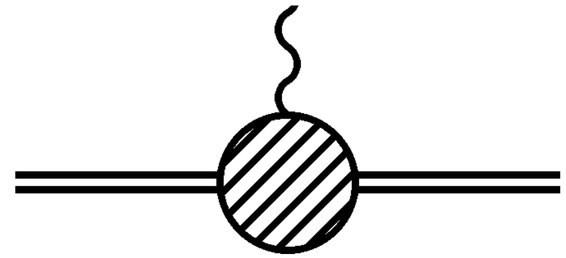
Also: tension between fit to entire dataset and fit to data subsets



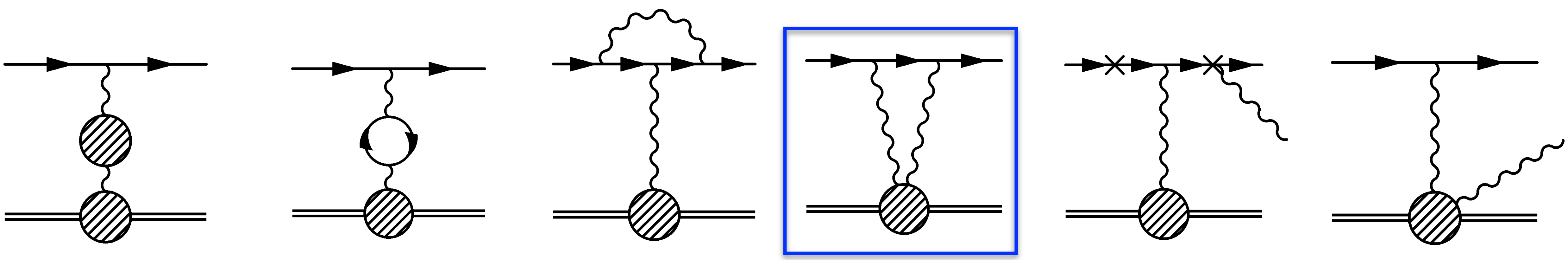
⇒ Revisit theoretical and experimental systematics

systematics:
radiative corrections

In order to isolate the proton vertex defining form factors and radius

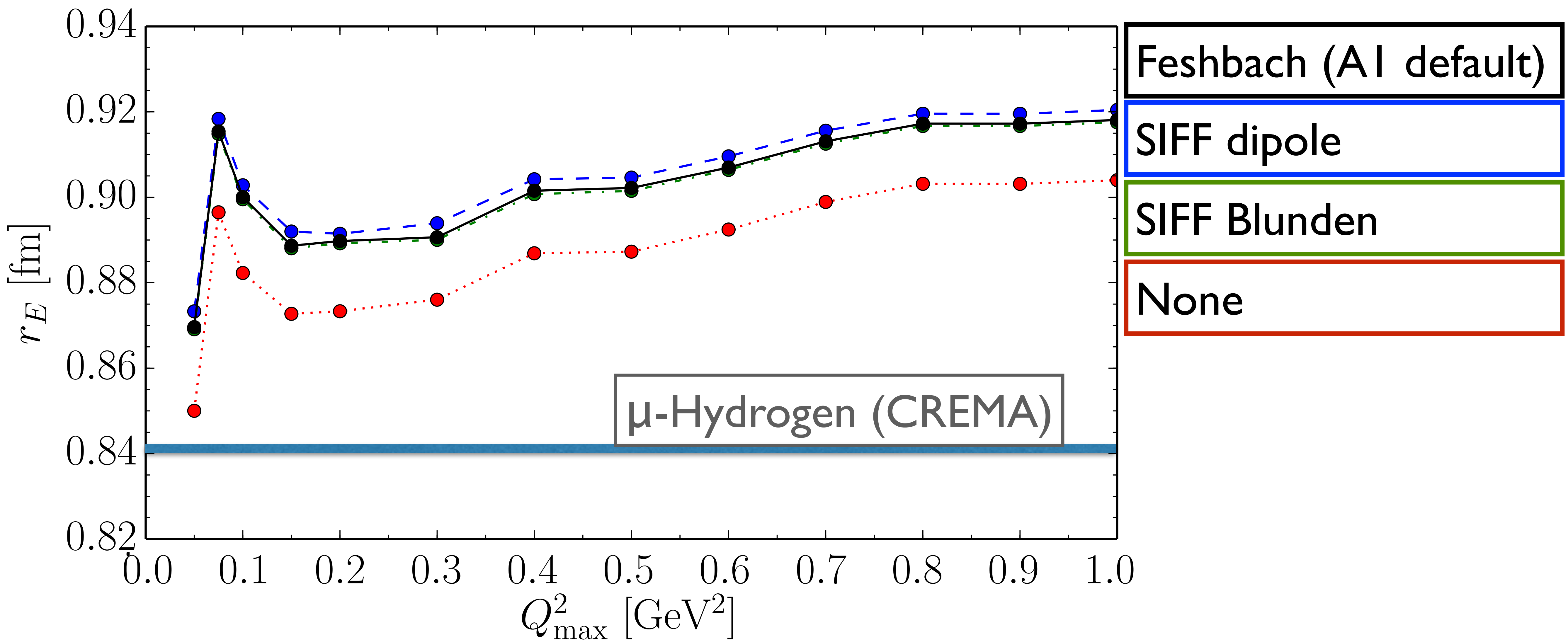


must subtract off radiative corrections that are part of the experimental measurement:



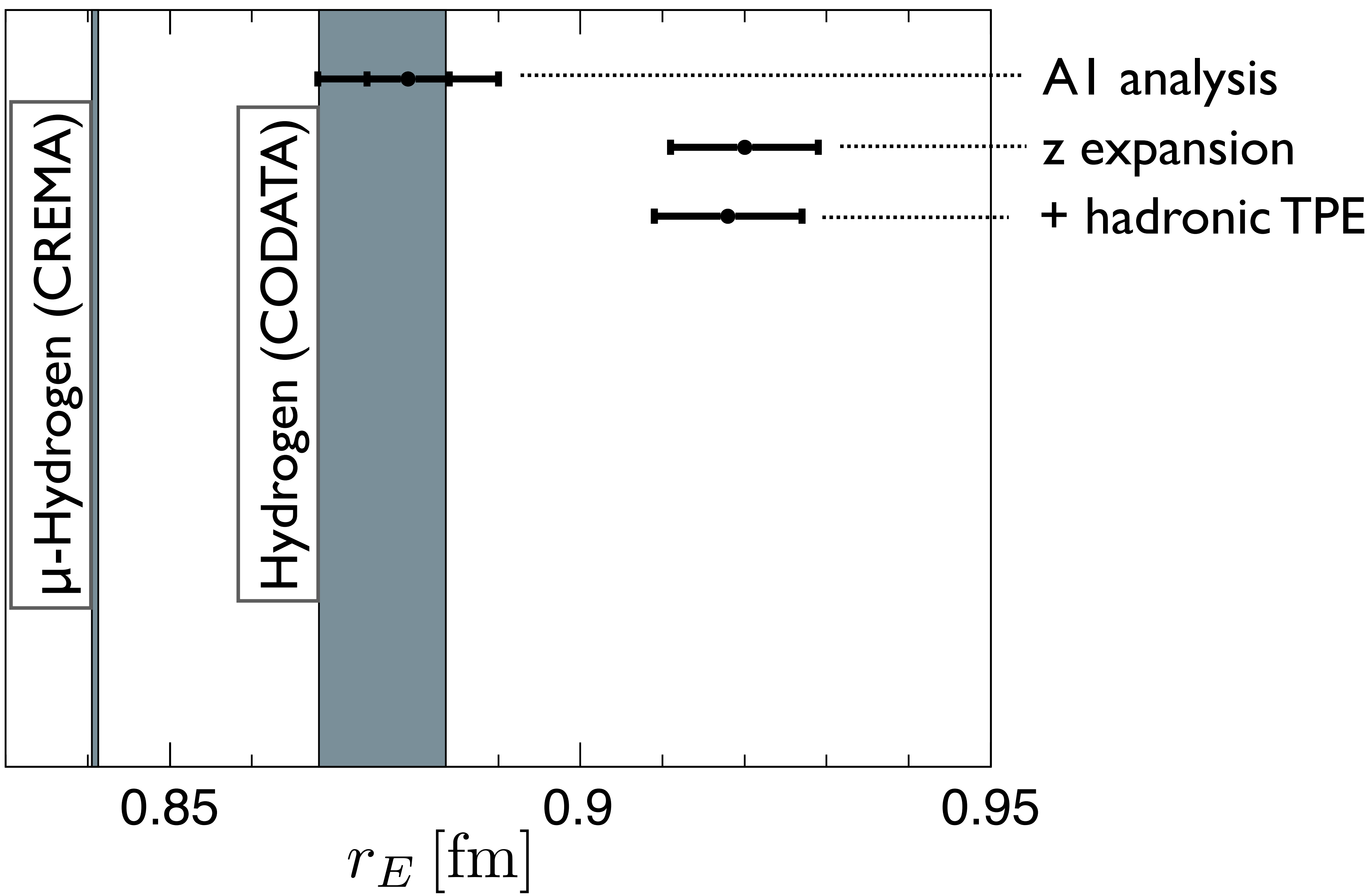
Through one-loop order, only essential difficulty is with Two-Photon Exchange: beyond present technology to compute from first principles, insufficient data to fully constrain

Consider a range of one-loop Two-Photon Exchange (TPE) corrections



Model dependence in TPE, but appears small for r_E

Take Blunden et al. hadronic model as default *PRC 72, 034612*

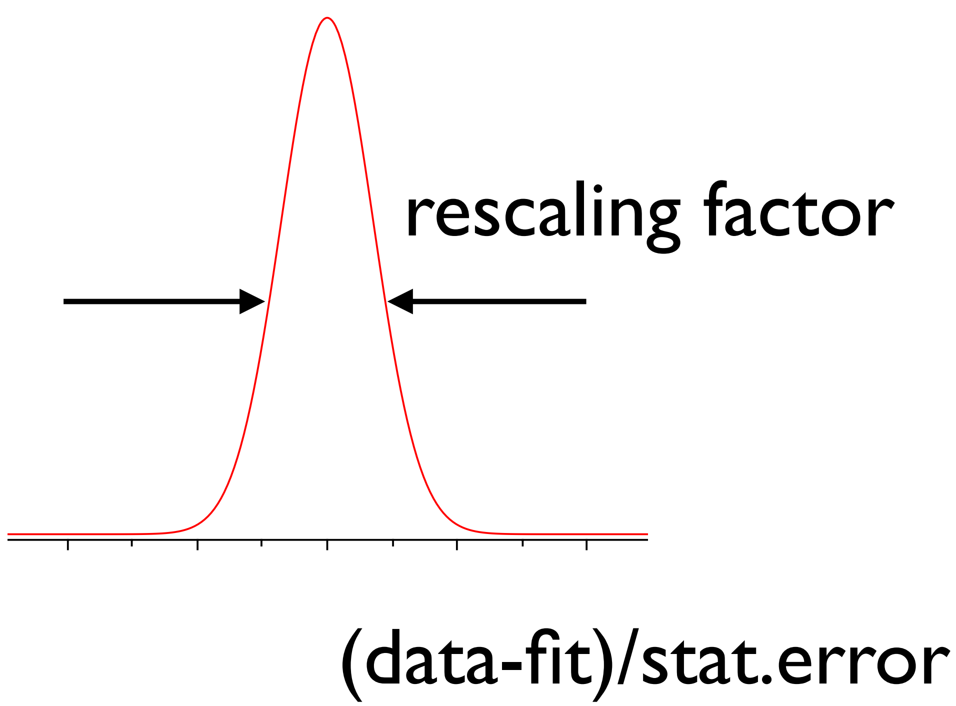


Return later to log-enhanced higher-order effects

systematics: uncorrelated errors

In the A1 dataset, kinematically uncorrelated systematic errors are deduced by examining subset fluctuations around initial fit

- perform initial fit to entire dataset
- for each beam/spectrometer data subset, rescale statistical errors to account for systematics



Potential concerns:

- inferred systematic can be extremely small (as low as 0.05%)

- repeated measurements at identical kinematics drive systematic uncertainties to zero

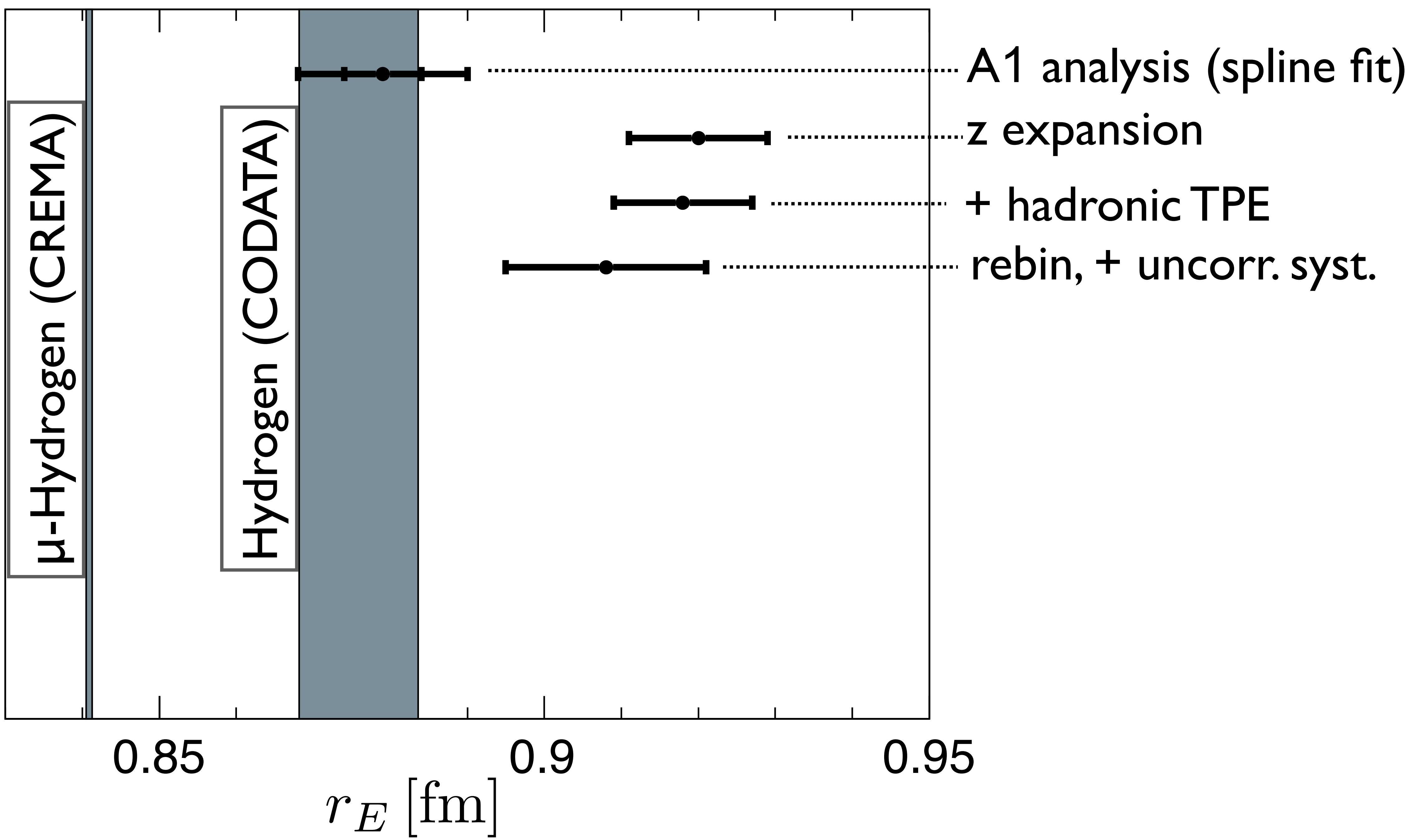
Address these concerns:

- combine ("rebin") data taken at identical kinematics

- include constant systematic error independent of statistics (0.3-0.4% based on confidence level analysis)

details: backup slide

Same fit to rebinned dataset:



systematics: correlated errors

In the A1 dataset, correlated systematic errors are estimated by considering modifications to each data subset:

$$d\sigma \to (1+\delta)d\sigma$$

where δ depends on kinematics. e.g.:

$$\delta \propto \frac{\theta - \theta_{\rm min}}{\theta_{\rm max} - \theta_{\rm min}}$$

We performed a more general analysis with a variety of functional forms and different subset groupings.

details: backup slide

Observations:

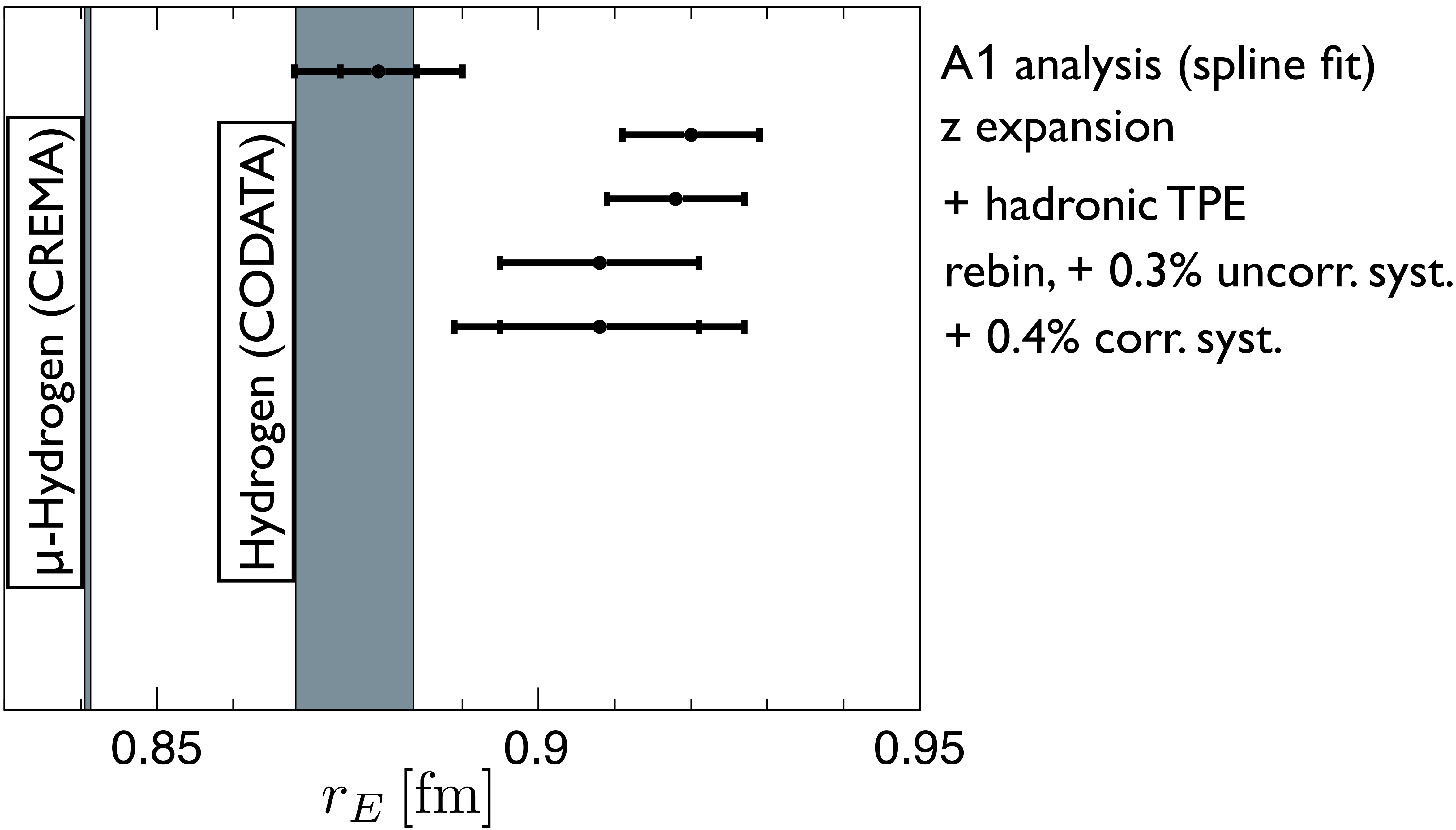
- especially for r_M, significant cancellation between effects of corrections applied to different spectrometers

$$\delta r_M = 0.016\,({\rm spec.A}) - 0.008\,({\rm spec.B}) + 0.002\,({\rm specC}) = 0.010\,{\rm fm}$$

$$\delta r_M = 0.016\,({\rm spec.A}) + 0.008\,({\rm spec.B}) + 0.002\,({\rm specC}) = 0.026\,{\rm fm}$$

- take 0.4% angular correction (vs. A1's 0.2%) applied uniformly to beam/spectrometer groupings as consistent with known uncertainties

Same fit, including correlated systematic error:



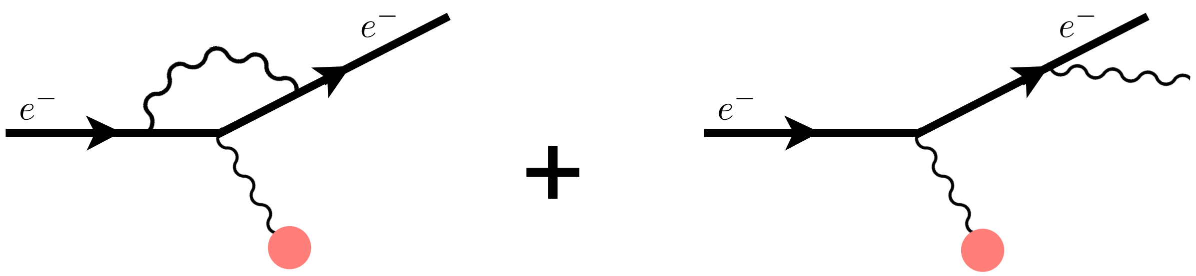
larger systematic shift would require:

- greater than 0.4% variation over subsets
- more extreme functional form
- conspiracy between shifts applied to different subsets

What could such a shift look like?

Large logarithms spoil QED perturbation theory when $Q^2 \sim \mathrm{GeV}^2$

$$|F(q^2)|^2 \to |F(q^2)|^2 \left(1 - \underbrace{\frac{\alpha}{\pi} \log \frac{Q^2}{m_e^2} \log \frac{E^2}{(\Delta E)^2}}_{\approx 0.5} + \dots \right)$$



(electrons *really* like to radiate)

A standard ansatz sums leading logarithms by exponentiating 1st order:

$$|F(q^2)|^2 \left(1 - \frac{\alpha}{\pi} \log \frac{Q^2}{m_e^2} \log \frac{E^2}{(\Delta E)^2} + \dots \right) \to |F(q^2)|^2 \exp \left[- \frac{\alpha}{\pi} \log \frac{Q^2}{m_e^2} \log \frac{E^2}{(\Delta E)^2} \right]$$

Yennie, Frautschi, Suura, 1961

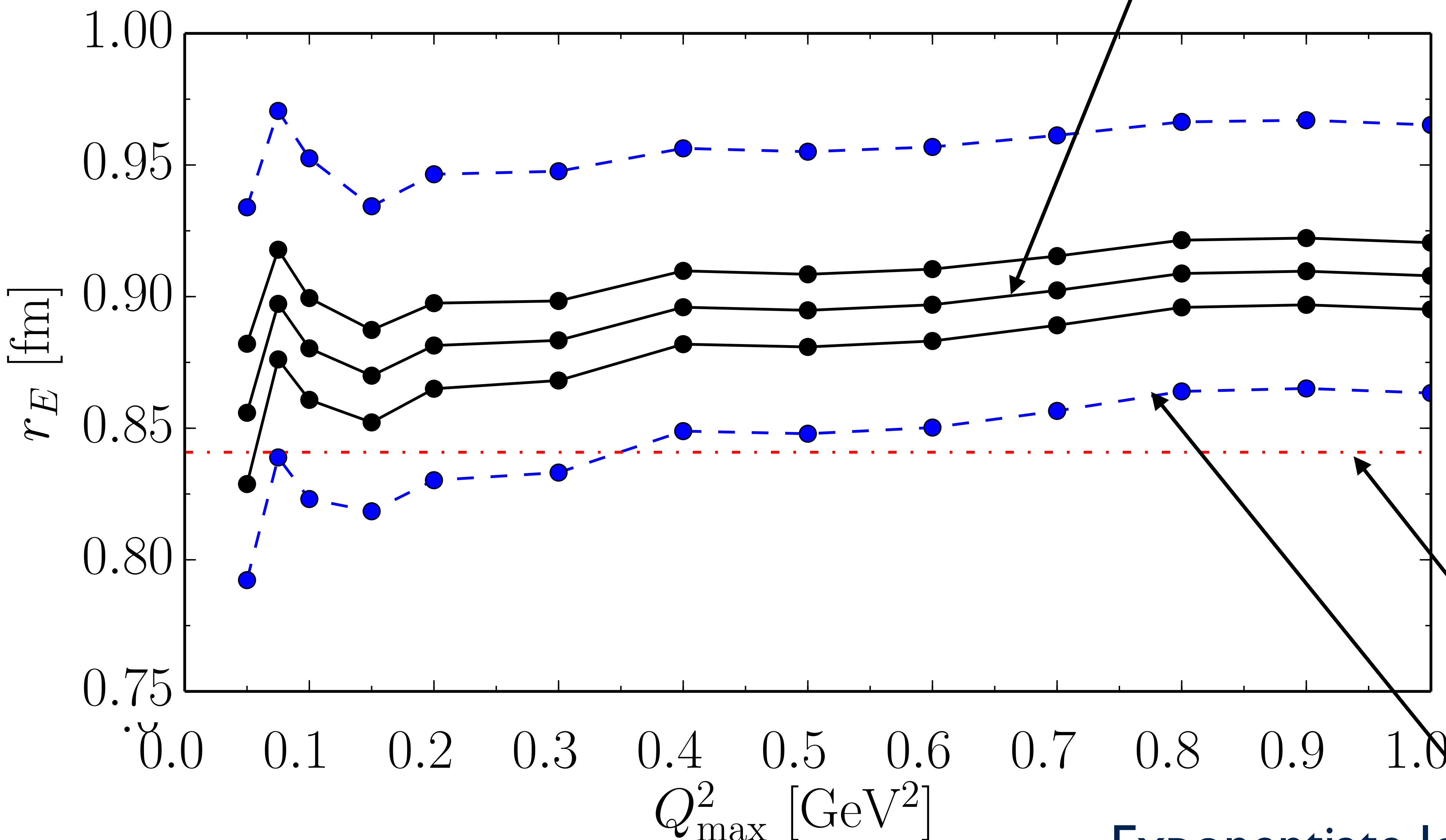
Captures leading logarithms when

$$Q \sim E \,, \quad \Delta E \sim m_e$$

As consistency check, should find the same result for resumming:

$$\log^2 \frac{Q^2}{m_e^2} \qquad \text{vs.} \quad \log \frac{Q^2}{m_e^2} \log \frac{E^2}{(\Delta E)^2}$$

Default fit: exponentiate complete one loop radiative corrections



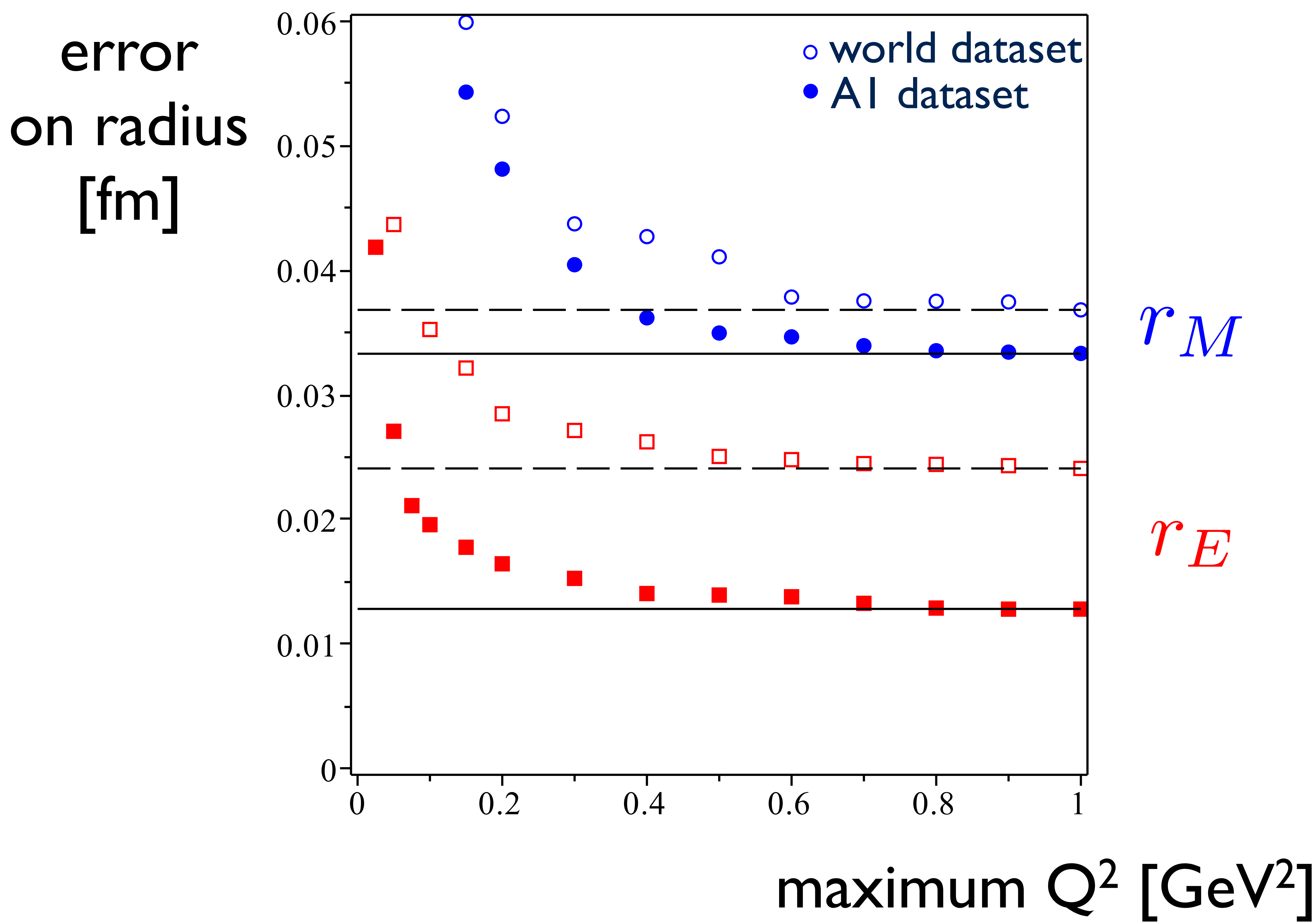
muonic hydrogen

Exponentiate $\log^2(Q^2/m_e^2)$

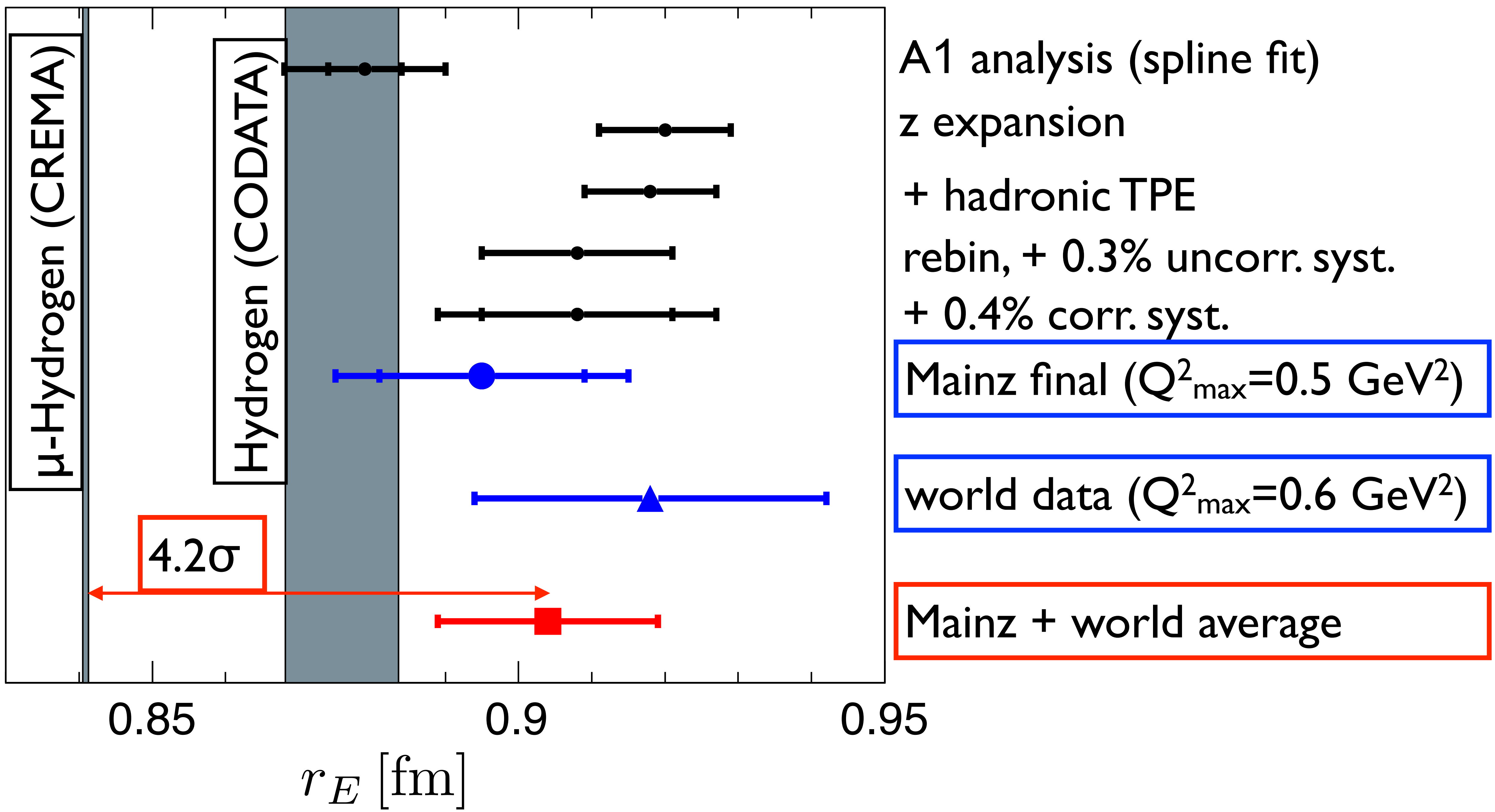
More detailed analysis of subleading radiative corrections required and in progress. Will present results using standard radiative correction models.

final results

Maximize radius sensitivity, minimize possible high-Q^2 systematics:



Proton charge radius

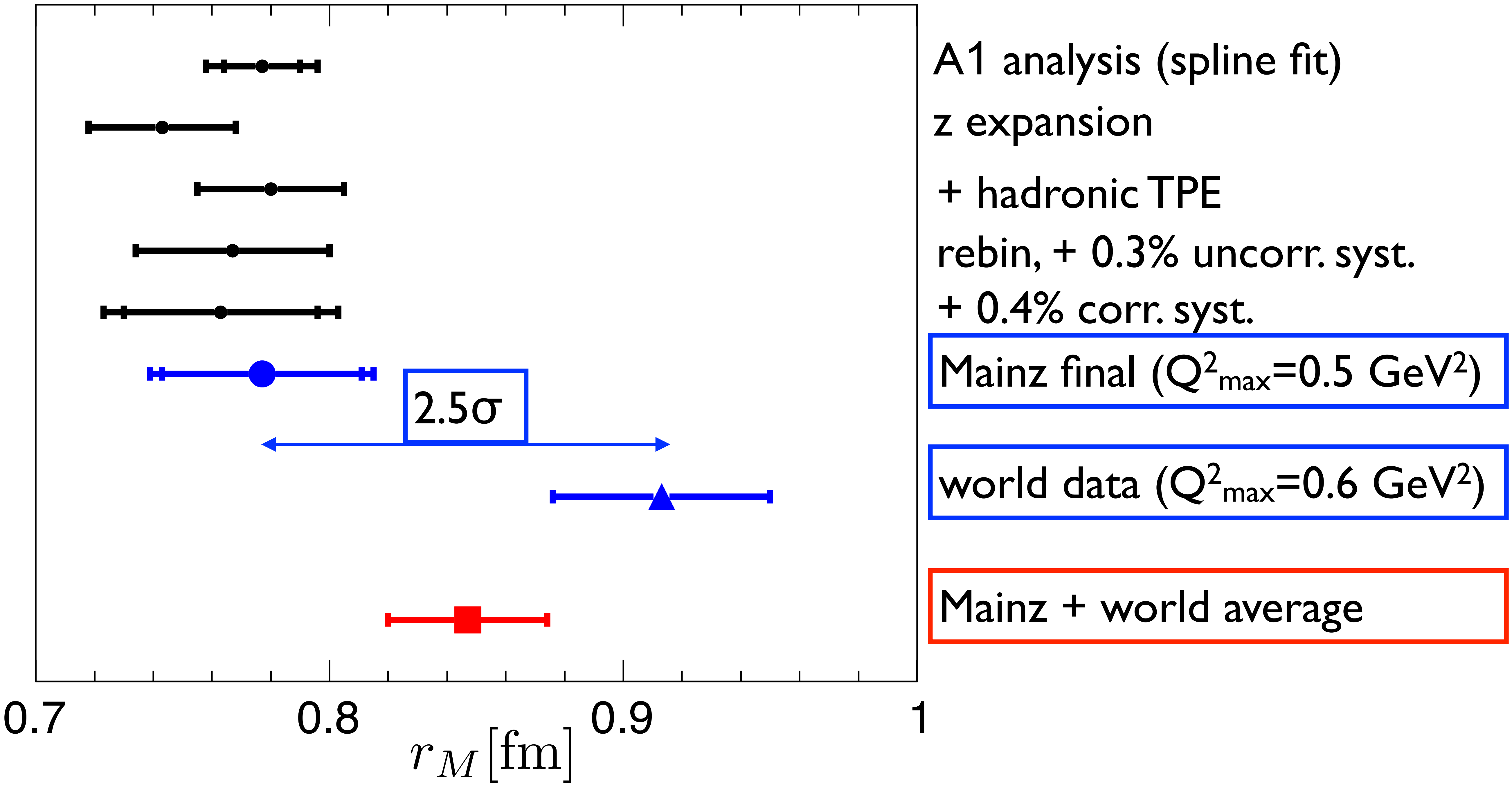


$r_E^{Mainz} = 0.895(14)(14)$

$r_E^{world} = 0.918(24)$

simple average: $r_E^{avg.} = 0.904(15)$

Proton magnetic radius



r_M^{Mainz} = 0.777(34)(17)

r_M^{world} = 0.913(37)

simple average: $r_M^{avg.}$ = 0.847(27)

summary

Performed the most comprehensive analysis of global electron-proton scattering data

r_E summary

Employing standard models for radiative corrections, and reasonable experimental systematics: Mainz and world values consistent. Combination is 4σ from muonic hydrogen

r_M summary

Mainz and world values differ by 2.5σ.

Implications:

most mundane resolution involves 5σ shift in Rydberg, and discarding/revising large body of results in both electron scattering and hydrogen spectroscopy.

Tension in low- and high-Q^2 data may point to underestimated systematic. Identified naively subheading radiative corrections as a concern.

The same issues facing electron-proton scattering are critical for the HEP accelerator neutrino program.

thanks for your attention (!)

back up

Mainz data rebinning

- one set of points (E_{beam}=315 MeV, θ=30.01°) inconsistent with statistical scatter. Excluded.

- 657 independent cross section measurements (from original 1422)

spec.	beam	N_σ	χ^2_{red}	CL (%)	χ^2_{red}	CL (%)
A	180	29	0.59	96.1	0.46	99.4
	315	23	0.54	96.4	0.44	99.1
	450	25	1.52	4.8	1.00	46.7
	585	28	1.54	3.4	1.03	42.8
	720	29	1.05	39.9	0.87	66.4
	855	21	0.92	56.8	0.77	76.0
B	180	61	0.85	79.8	0.65	98.3
	315	46	1.05	38.5	0.76	88.5
	450	68	0.90	71.7	0.67	98.2
	585	60	0.61	99.2	0.50	99.96
	720	57	1.29	6.9	0.97	53.7
	855	66	1.88	0.002	1.15	19.6
C	180	24	0.88	63.3	0.68	88.0
	315	24	1.16	27.2	0.78	76.8
	450	25	1.53	4.3	1.08	35.9
	585	18	0.83	66.3	0.65	86.4
	720	32	1.11	30.2	0.90	62.3
	855	21	0.79	73.7	0.62	90.5

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	720	32	1.11	30.2	0.90	62.3
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Constant 0.25% uncorrelated systematic

Outlier

Mainz data rebinning

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	855	21	0.79	73.7	0.62	90.5

Constant 0.3% uncorrelated systematic

Constant 0.4% uncorrelated systematic

Mainz correlated systematics

In the A1 analysis, correlated systematic errors are estimated by considering modifications to each data subset:

$$d\sigma \to (1+\delta)d\sigma$$

where δ depends on kinematics

Since the normalizations of individual data subsets are free parameters, only variations in δ over subsets relevant. Simple ansatz:

$$1+\delta_{\rm corr} = 1 + a\frac{x - x_{\rm min}}{x_{\rm max} - x_{\rm min}}$$

A1 analysis:

- $x=\theta$
- $a\approx 0.2\%$, equal in sign and magnitude for all beam/ spectrometer subsets

We performed a more general analysis with different functional forms and different subset groupings,

- $x=\theta$, $1/\theta$, Q^2, $1/Q^2$, E', $1/E'$, ε, $\sin^4(\theta/2)$

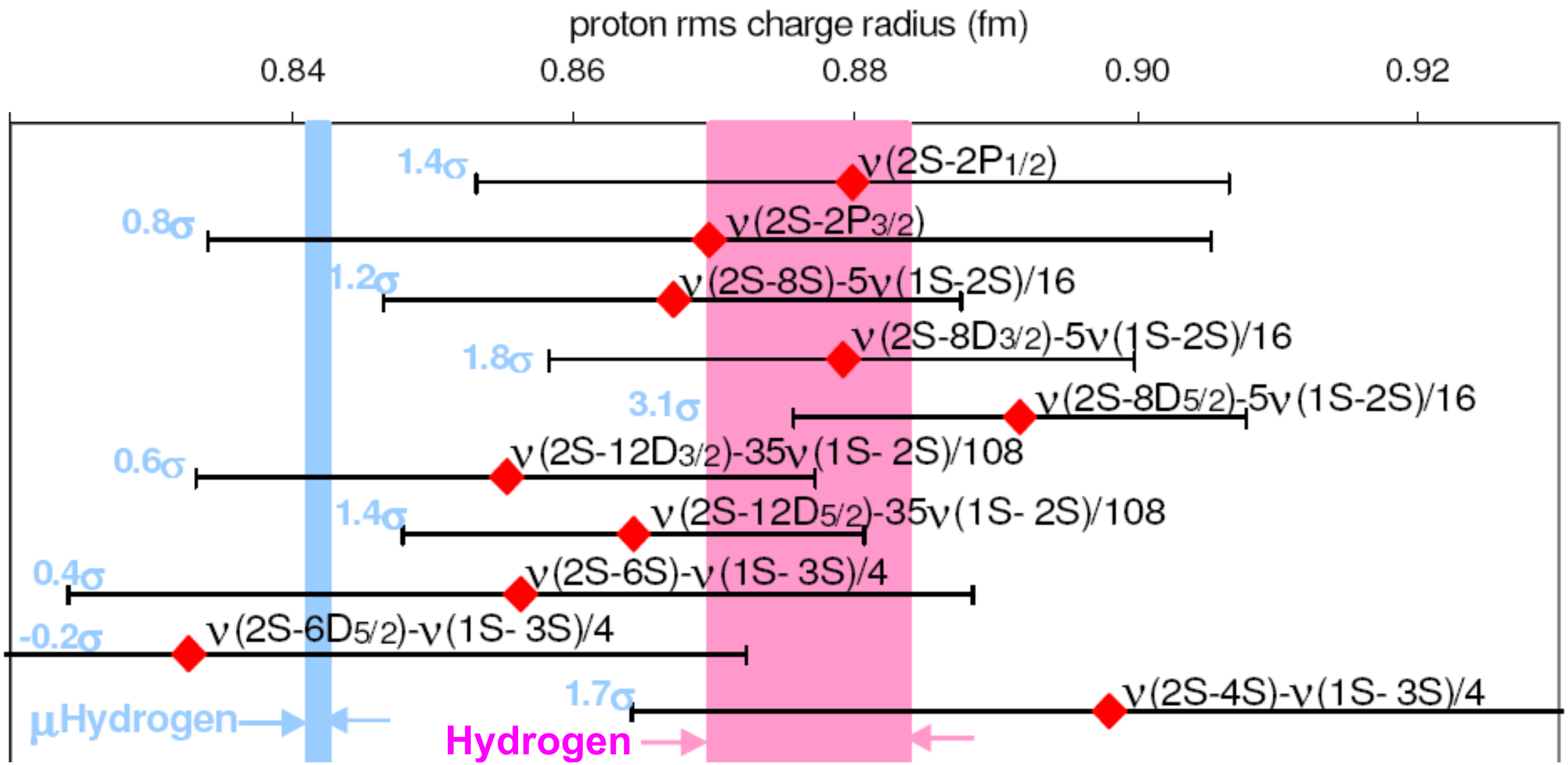
- data groupings: beam/spectrometer (18 subsets) spectrometer (3 subsets); normalization (34 subsets)

Observations:

- especially for r_M, significant cancellation between corrections applied to three spectrometers when a=constant

- take results for $x=\theta$, a=0.4%, applied to beam/spectrometer groupings as "minimum" consistent with known uncertainties

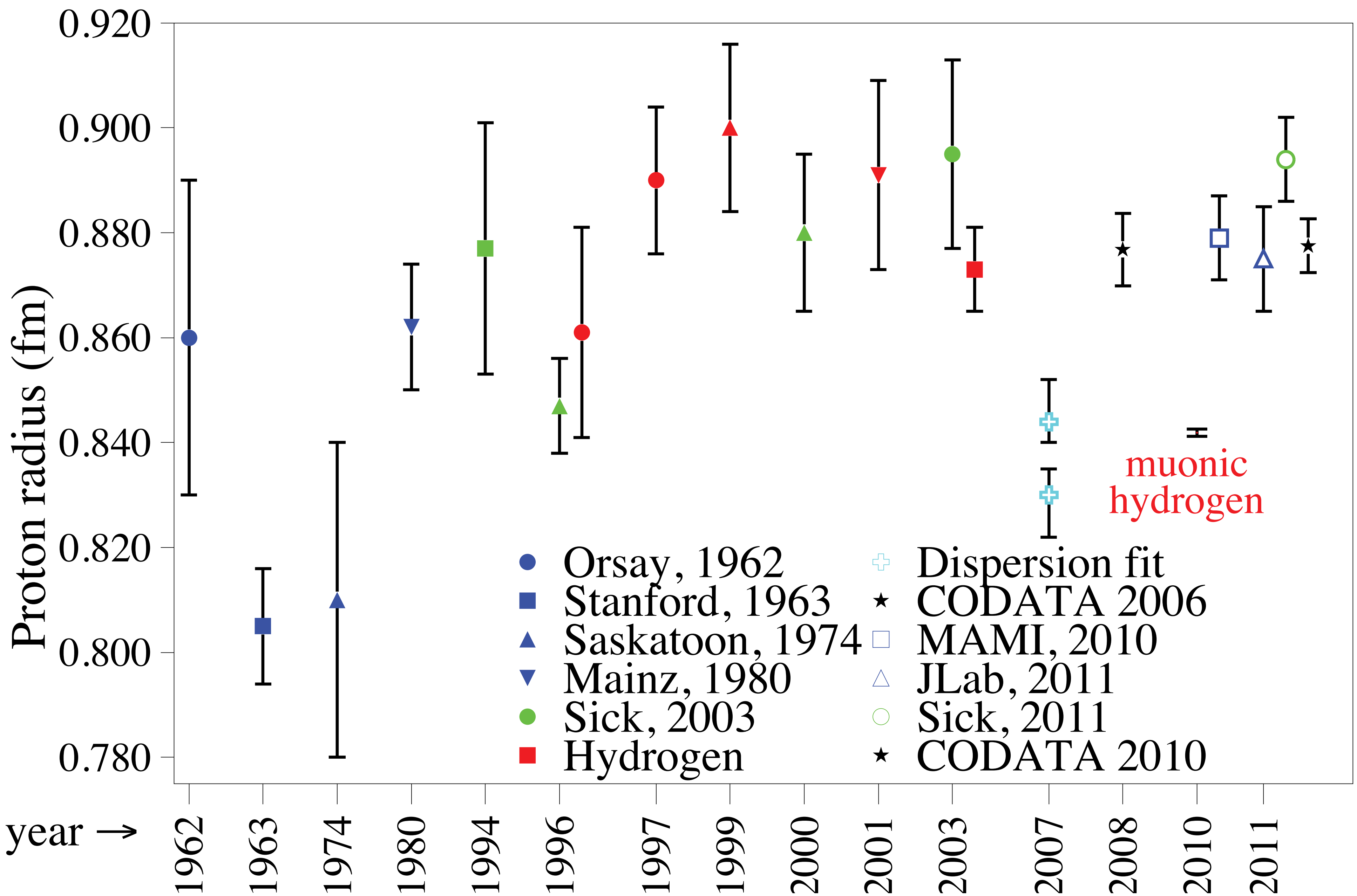
Experimental landscape: hydrogen



plot courtesy E. Hessels, proton radius workshop 2014

- no straightforward systematic explanation identified, but ~5σ deviation results from summing many ~2σ effects

Experimental landscape: historical e-p extractions



From Pohl et al., Ann.Rev.Nucl.Part.Sci. 63, 175