Self-Consistent Treatment of Neutrino Physics in Cosmology

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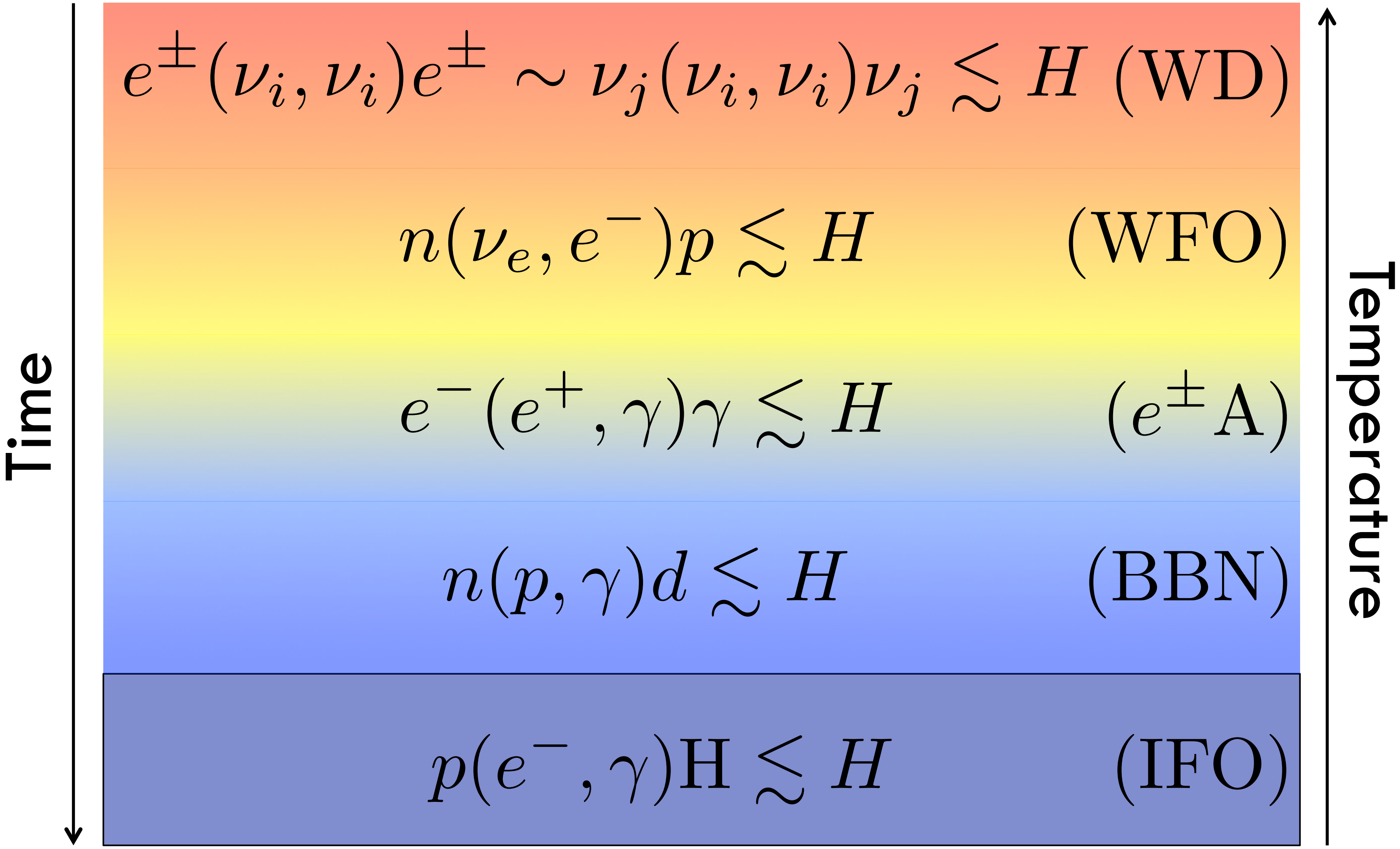
Outline

- ➢Overview of the early universe
- ➢An alternative to N_{eff}
- ➢The "Neutrino-Mass/Recombination Effect" (νMR)
- ➢Neutrino Transport during Weak Decoupling and BBN

Epochs of Interest



$e^{\pm}(\nu_i,\nu_i)e^{\pm} \sim \nu_j(\nu_i,\nu_i)\nu_j \lesssim H$ (WD)

$n(\nu_e,e^-)p \lesssim H$ (WFO)

$e^-(e^+,\gamma)\gamma \lesssim H$ ($e^{\pm}$A)

$n(p,\gamma)d \lesssim H$ (BBN)

$p(e^-,\gamma)\mathrm{H} \lesssim H$ (IFO)

Code

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BBN

Predict primordial nuclear abundances

UNITARY

Preserve unitarity in nuclear reaction network

Quantify errors

RECOMBINATION

Treat recombination with three-level atom similar to recfast [16]

Isolate neutrino signatures in cosmological power spectra

SELF-CONSISTENT

Maintain self-consistency over large range of epochs

TRANSPORT

Follow evolution of neutrino spectra

Neutrino Energy Density

$$\rho_{\mathrm{rad}} = \left(2 + \frac{7}{4}\left(\frac{4}{11}\right)^{4/3} N_{\mathrm{eff}}\right) \frac{\pi^2}{30} T_\gamma^4$$

$$\implies N_{\mathrm{eff}} = 3$$

Finite temperature radiative correction
Dicus et al. (1982) [6]
Lopez & Turner (1999)[7]

$\Delta N_{\mathrm{eff}} \sim 0.011$

Weak decoupling is not a sharp event
Dolgov, Hansen, & Semikoz (1997)[8]

$\Delta N_{\mathrm{eff}} \sim 0.034$

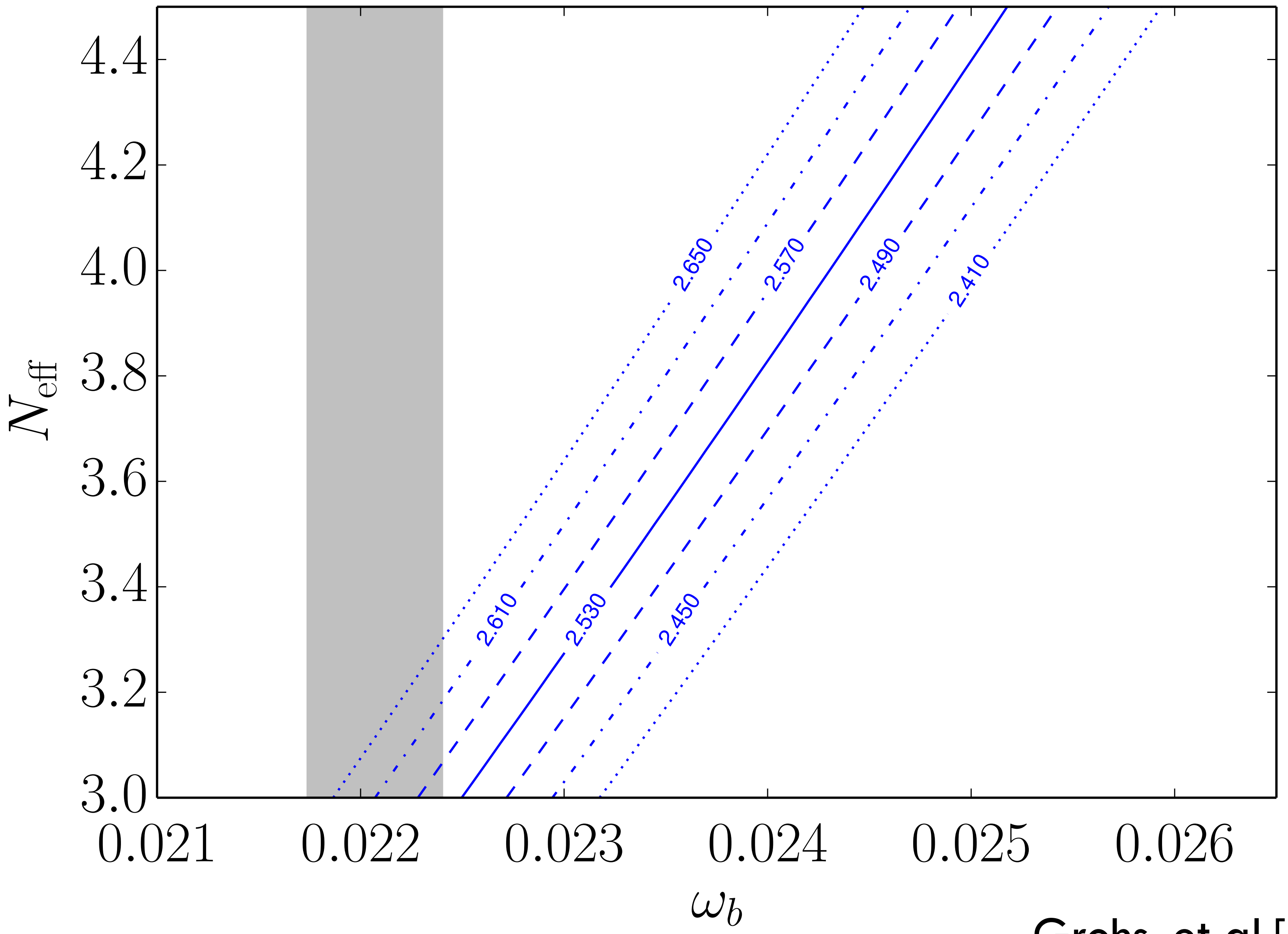
Mangano et al. (2005)[9]:

$$N_{\mathrm{eff}} = 3.046 \text{ in SM}$$

Planck XIII (2015)[10]:

$$N_{\mathrm{eff}} = 3.15 \pm 0.23$$

Contours of constant D/H



Grohs, et al.[13]

Issues with N_{eff}

- N_{eff} not measured immediately after e± annihilation or during BBN
- ρ_{rad} is not a CMB observable; how does CMB determine ρ_{rad}?
- Neutrinos have non-zero rest mass.
- What happens to N_{eff} when including Beyond-Standard-Model physics?

$$N_{\text{eff}}^{(\text{th})} \ \text{vs.} \ \tilde{N}_{\text{eff}} : \begin{cases} \rho_\gamma + \rho_\nu = \left(1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\text{eff}}^{(\text{th})}\right)\rho_\gamma \\ \tilde{N}_{\text{eff}} = N_{\text{eff}}^{(\text{th})}[r_s/r_d = (r_s/r_d)^{(\text{inp})}] \end{cases}$$

Strategy

Sound Horizon:

$$r_s = \int_0^{a_{\gamma d}} \frac{da}{a^2 H \sqrt{3(1+R)}} \qquad R \equiv \frac{3\rho_b}{4\rho_\gamma}$$

Damping Diffusion Wave Number:

$$k_d^{-2} = \int_0^{a_{\gamma d}} \frac{da}{n_e \sigma_T H a^3 6(1+R)} \left(\frac{R^2}{1+R} + \frac{16}{15} \right), \; r_d = \pi / k_d$$

$$H = \left(\frac{8\pi}{3 m_{\mathrm{pl}}^2} \rho \right)^{1/2} \to \left(\frac{8\pi}{3 m_{\mathrm{pl}}^2} \left(\rho_m + \rho_\Lambda + \rho_{\mathrm{rad}} \right) \right)^{1/2}$$

Ratio: $r_s / r_d = d_s / d_d = \theta_s / \theta_d, \; \theta \equiv \dfrac{d}{D_A}$

Free-Electron Fraction

Typical reaction of Interest:

$$e^- + p^+ \rightleftharpoons \mathrm{H} + \gamma$$

Covariant form of Boltzmann Eqn.:

$$p^\sigma \frac{\partial f}{\partial x^\sigma} - \Gamma^\sigma_{\mu\nu} p^\mu p^\nu \frac{\partial f}{\partial p^\sigma} = \hat{C}[f]$$

Evolution of X_e:

$$\frac{dX_e}{dt} = (1 - X_e)\beta - X_e^2 n_b \alpha^{(2)}$$

β : Ionization coefficient

n_b : Baryon number density

$\alpha^{(2)}$: Recombination coefficient

Boltzmann Eqn. Correction

Correction to Boltzmann Equation due to Peebles (1968):

$$\frac{dX_e}{dt} = \left[(1 - X_e)\beta - X_e^2 n_b \alpha^{(2)}\right] C$$

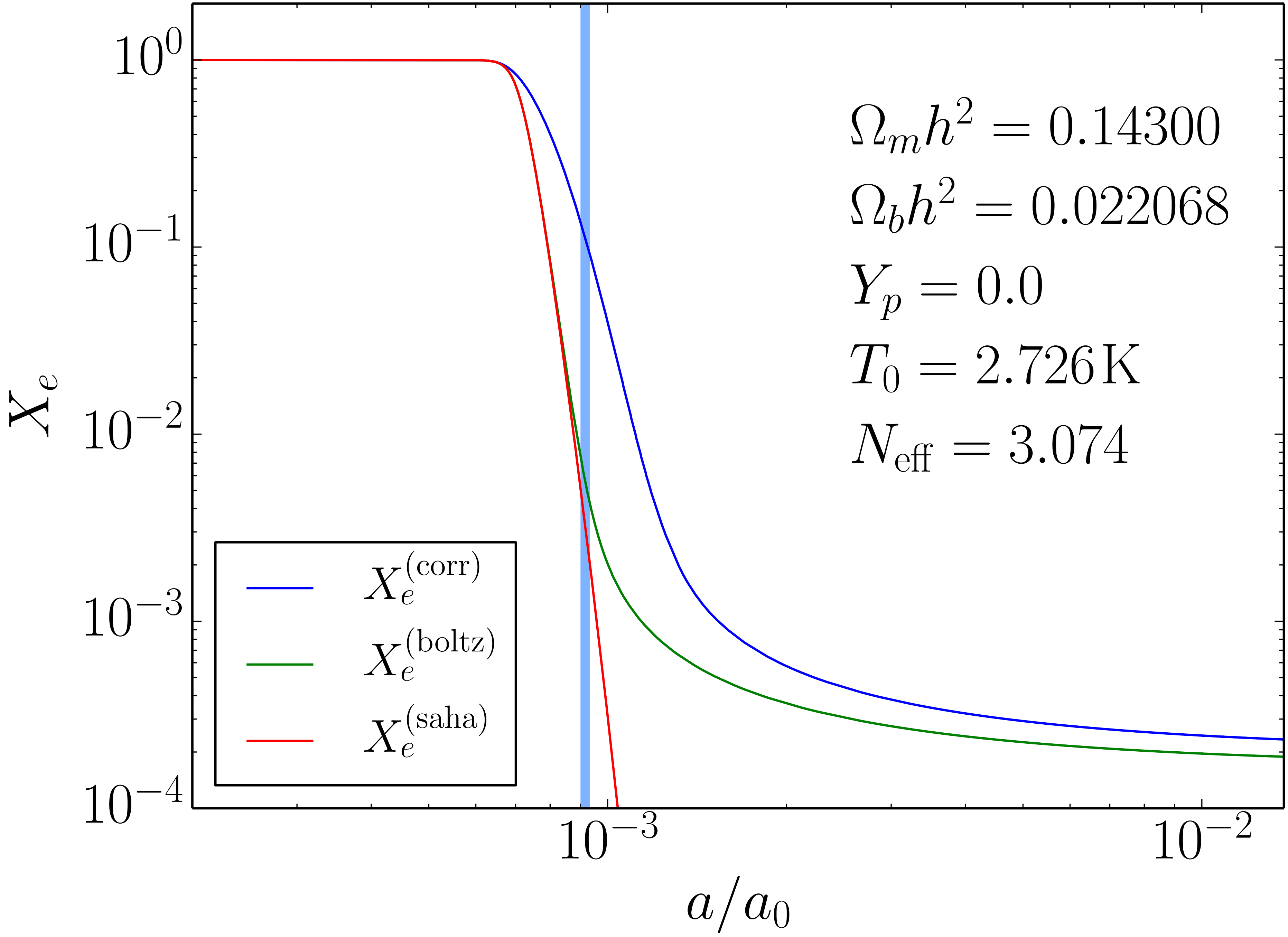
Correction Factor for 3-level atom:

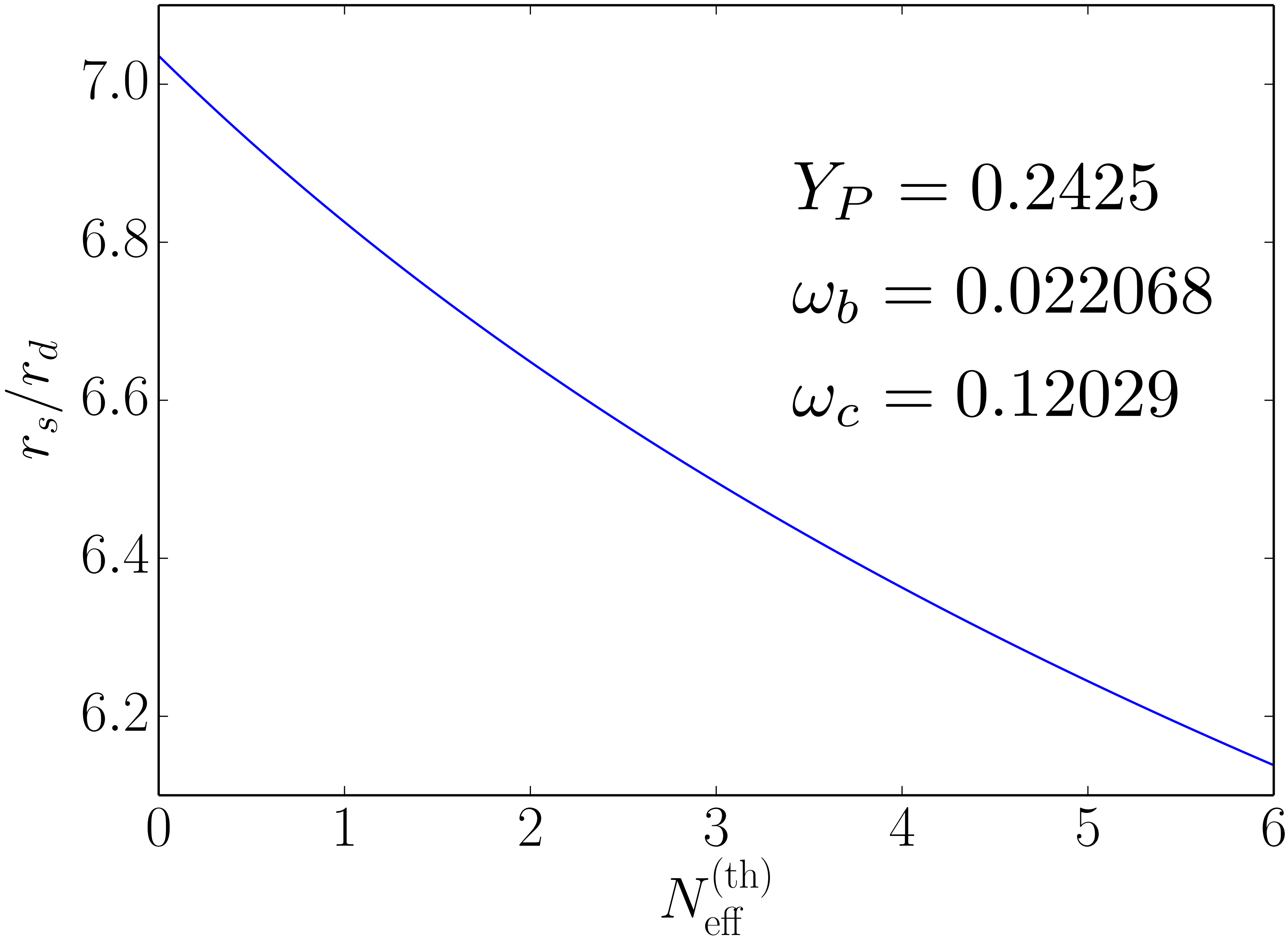
$$C = \frac{\Lambda_\alpha + \Lambda_{2\gamma}}{\Lambda_\alpha + \Lambda_{2\gamma} + \beta^{(2)}}$$

Λ_α : Lyman-alpha photon redshift rate

$\Lambda_{2\gamma}$: 2-photon decay rate

$\beta^{(2)}$: Lyman-alpha production rate


X_e
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-3}
10^{-2}
a/a_0
$\Omega_m h^2 = 0.14300$
$\Omega_b h^2 = 0.022068$
$Y_p = 0.0$
$T_0 = 2.726\,\mathrm{K}$
$N_{\mathrm{eff}} = 3.074$
$X_e^{(\mathrm{corr})}$
$X_e^{(\mathrm{boltz})}$
$X_e^{(\mathrm{saha})}$


r_s/r_d
7.0
6.8
6.6
6.4
6.2
$Y_P = 0.2425$
$\omega_b = 0.022068$
$\omega_c = 0.12029$
0
1
2
3
4
5
6
$N_{\rm eff}^{\rm (th)}$

νMR effect

Neutrinos free-stream after weak decoupling:

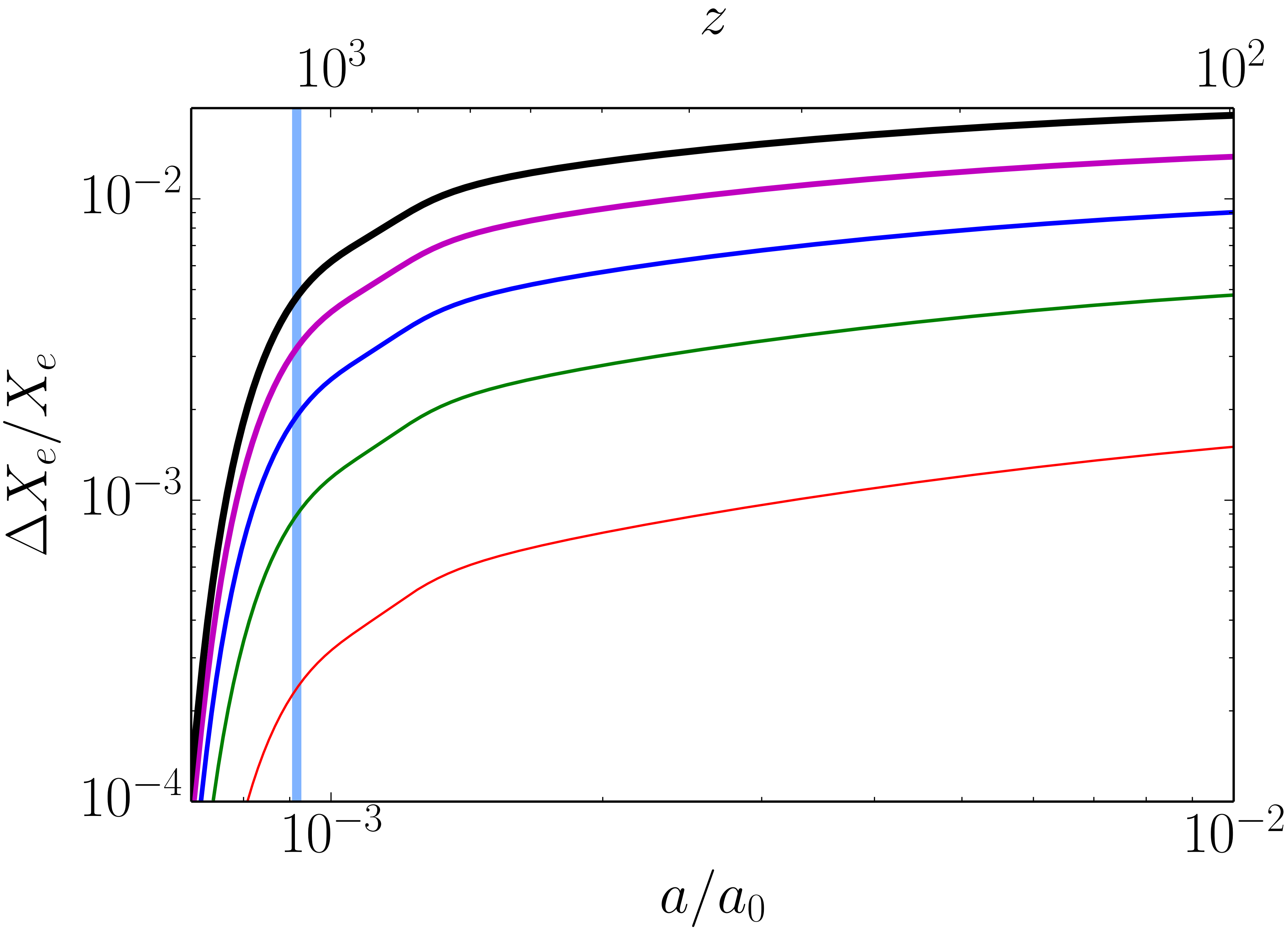
$$\rho_\nu(m \neq 0) = \int_0^\infty \frac{p^2\sqrt{p^2+m^2}dp}{e^{p/T}+1} > \int_0^\infty \frac{p^3 dp}{e^{p/T}+1} = \rho_\nu(m=0)$$

Naïve Scaling Argument:

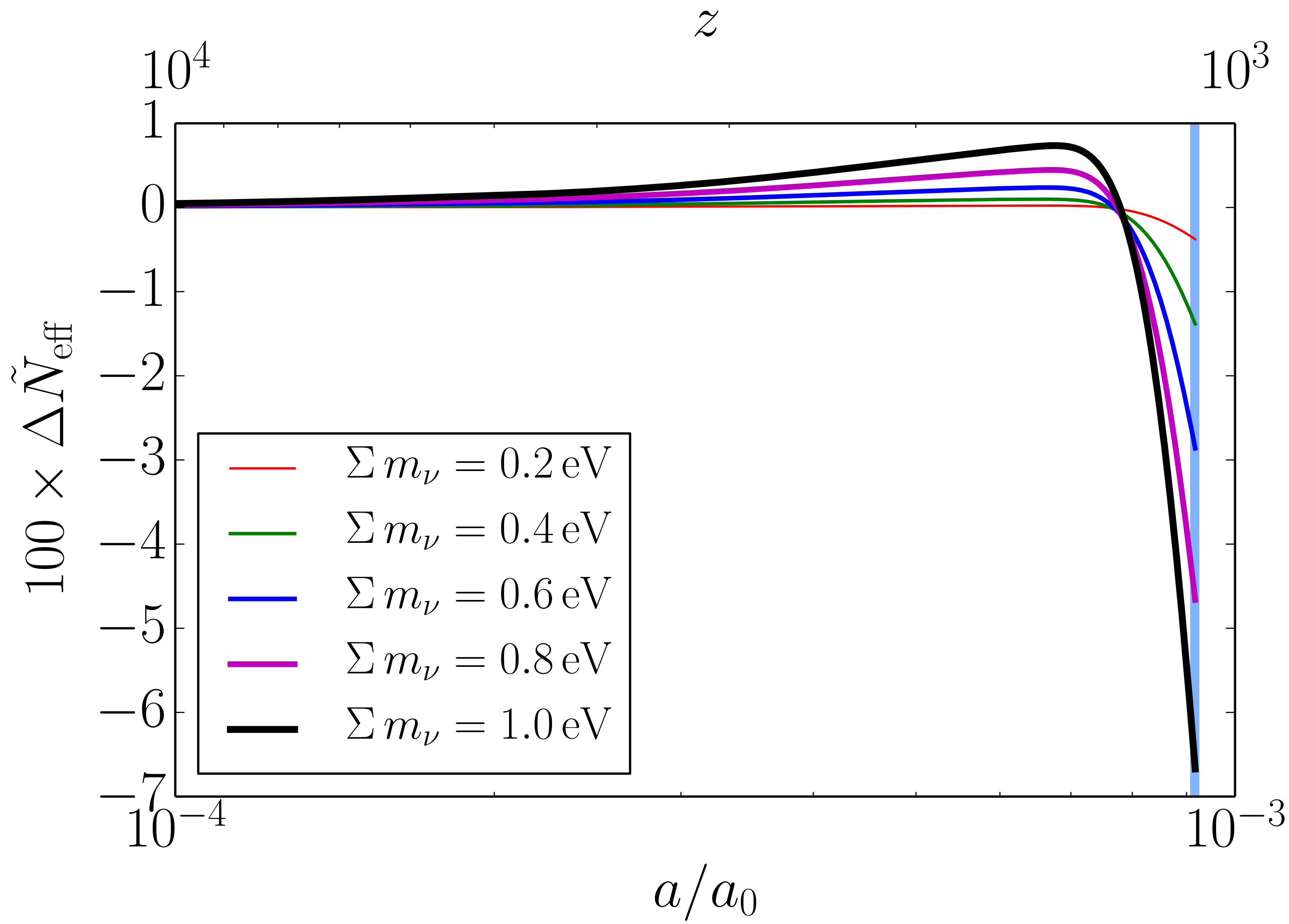
$$r_s \propto \frac{1}{H},\, r_d \propto \frac{1}{\sqrt{H}} \implies \frac{r_s}{r_d} \propto \frac{1}{\sqrt{H}} \sim \frac{1}{\rho^{1/4}}$$

Expectation: Larger Hubble rate implies Larger $\tilde{N}_{\mathrm{eff}}$

However, calculation yields: $\tilde{N}_{\mathrm{eff}} = 2.995,\ \Sigma m_\nu = 0.23\ \mathrm{eV}$

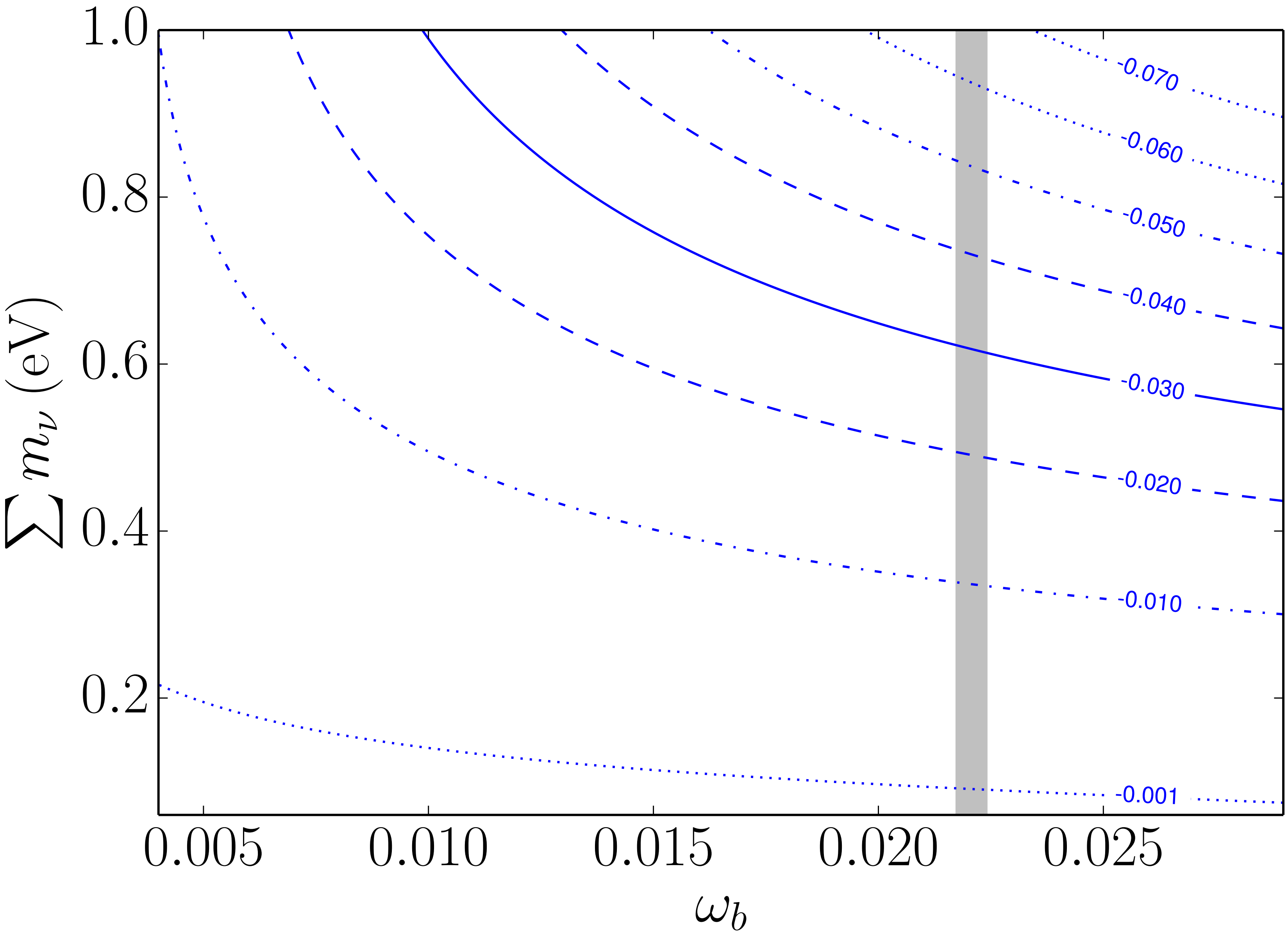


Grohs, et al.[12]



Grohs, et al.[12]

Contours of constant $\Delta\tilde{N}_{\text{eff}}$



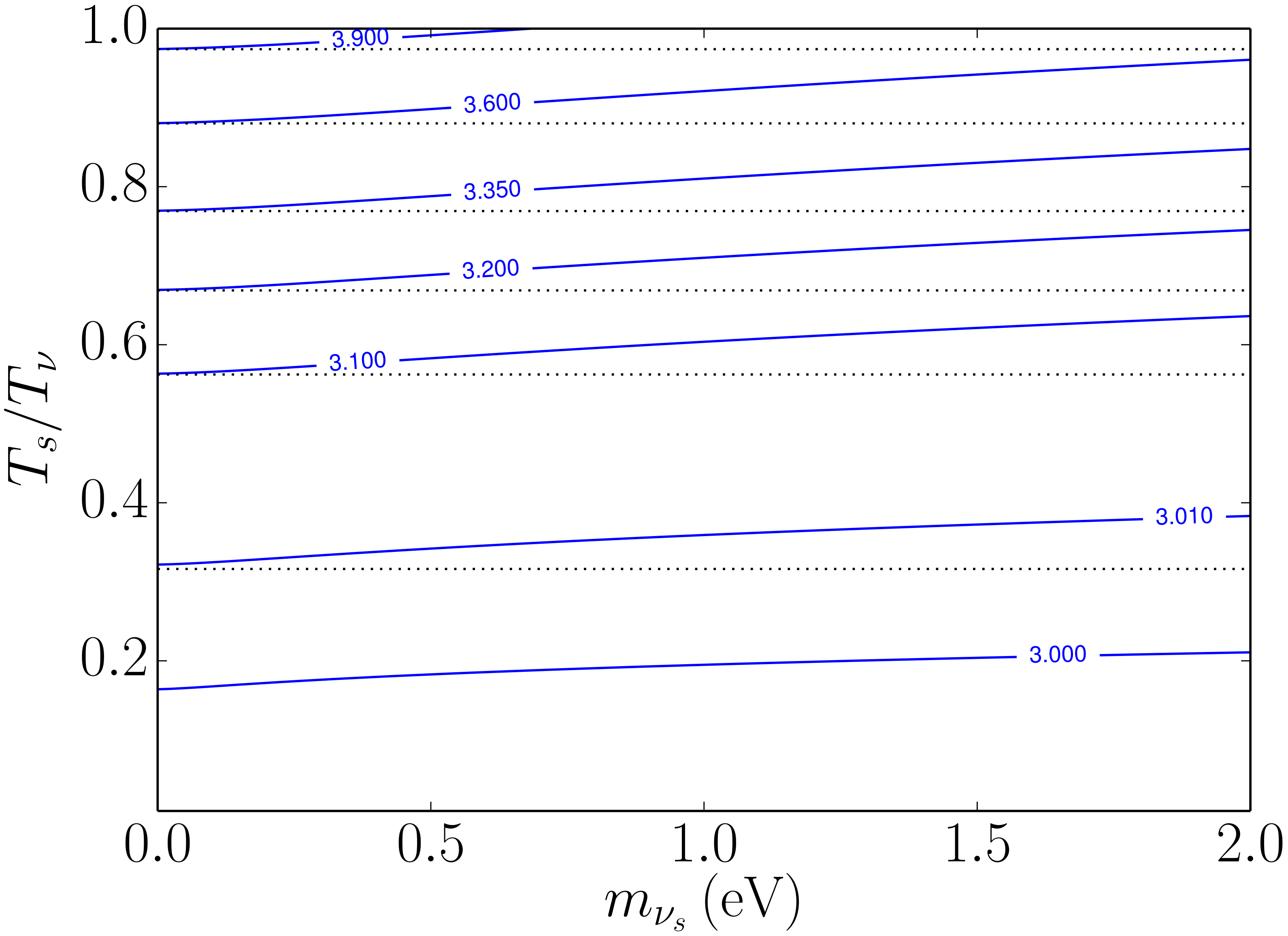
Grohs, et al.[13]

Hints for light sterile neutrinos?

- mini-BooNE
  - neutrino oscillation experiment $\nu_e \to \nu_s \to \nu_\mu$
  - appearance with $\delta m^2 \sim 1\ \mathrm{eV}^2$
  - result inconsistent with flavor oscillation alone
- Neutrino reactor anomaly
  - 3σ deficit neutrinos detected in short-baseline (<100m) reactor vexperiments

    $\bar{\nu}_e$ deficit from $\bar{\nu}_e \to \bar{\nu}_s$ (???) – a disappearance experiment
  - A. Hayes et al. (2013)[15] find “large corrections”
- Extra radiation at photon-decoupling (Neff) ??
  - CMB observations (PolarBear, ACT, SPT, Planck, CMBPol,…)
  - ‘extra’ RED could reconcile H_0 and σ_8 inferred from CMB and astronomical observation

Contours of constant $\tilde{N}_{\mathrm{eff}}$



Grohs, et al.[12]

Weak Interactions

Channels:

$$\nu_i + \nu_j \leftrightarrow \nu_i + \nu_j$$

$$\nu_i + \bar{\nu}_j \leftrightarrow \nu_i + \bar{\nu}_j$$

$$\nu_i + \bar{\nu}_i \leftrightarrow \nu_j + \bar{\nu}_j$$

$$\nu_i + e^{\pm} \leftrightarrow \nu_i + e^{\pm}$$

$$\bar{\nu}_i + e^{\pm} \leftrightarrow \bar{\nu}_i + e^{\pm}$$

$$\nu_i + \bar{\nu}_i \leftrightarrow e^- + e^+$$

Summed-Squared Amplitude examples:

$$\nu_e(1) + \nu_e(2) \leftrightarrow \nu_e(3) + \nu_e(4)$$

$$\langle |\mathcal{M}|^2 \rangle = 2^7 G_F^2 (P_1 \cdot P_2)^2$$

$$\nu_e(1) + e^-(2) \leftrightarrow e^-(3) + \nu_e(4)$$

$$\begin{aligned}\langle |\mathcal{M}|^2 \rangle = 2^5 G_F^2 [&(1 + 2\sin^2\theta_W)^2 (P_1 \cdot Q_2)(Q_3 \cdot P_4) \\ &+ 4\sin^4\theta_W (P_1 \cdot Q_3)(Q_2 \cdot P_4) \\ &- 2\sin^2\theta_W (1 + 2\sin^2\theta_W) m_e^2 (P_1 \cdot P_4)]\end{aligned}$$

Boltzmann Equation Revisited

Covariant form of Boltzmann Eqn.:

$$p^{\sigma}\frac{\partial f}{\partial x^{\sigma}} - \Gamma^{\sigma}_{\mu\nu}p^{\mu}p^{\nu}\frac{\partial f}{\partial p^{\sigma}} = \hat{C}[f]$$

In comoving coordinates: $f(\vec{x},\vec{p},t) \to f(p,t) \to f(\epsilon,t)$

With collision operator:

$$\frac{Df_1}{Dt} = \int \frac{s}{2E_1}\frac{d^3q_2}{(2\pi)^3 2E_2}\frac{d^3q_3}{(2\pi)^3 2E_3}\frac{d^3p_4}{(2\pi)^3 2E_4} \\ \times \langle|\mathcal{M}|^2\rangle(2\pi)^4\delta^4(P_1+Q_2-Q_3-P_4) \\ \times [f_3f_4(1-f_1)(1-f_2)-f_1f_2(1-f_3)(1-f_4)]$$

Collision Term Reduction

1. Nine-dimensional integral over phase space of particles 2, 3, and 4
2. Conservation of four-momentum – Five-dimensional integral
3. Isotropy – Three-dimensional integral
4. Integration Limits Trick – Two-dimensional integral

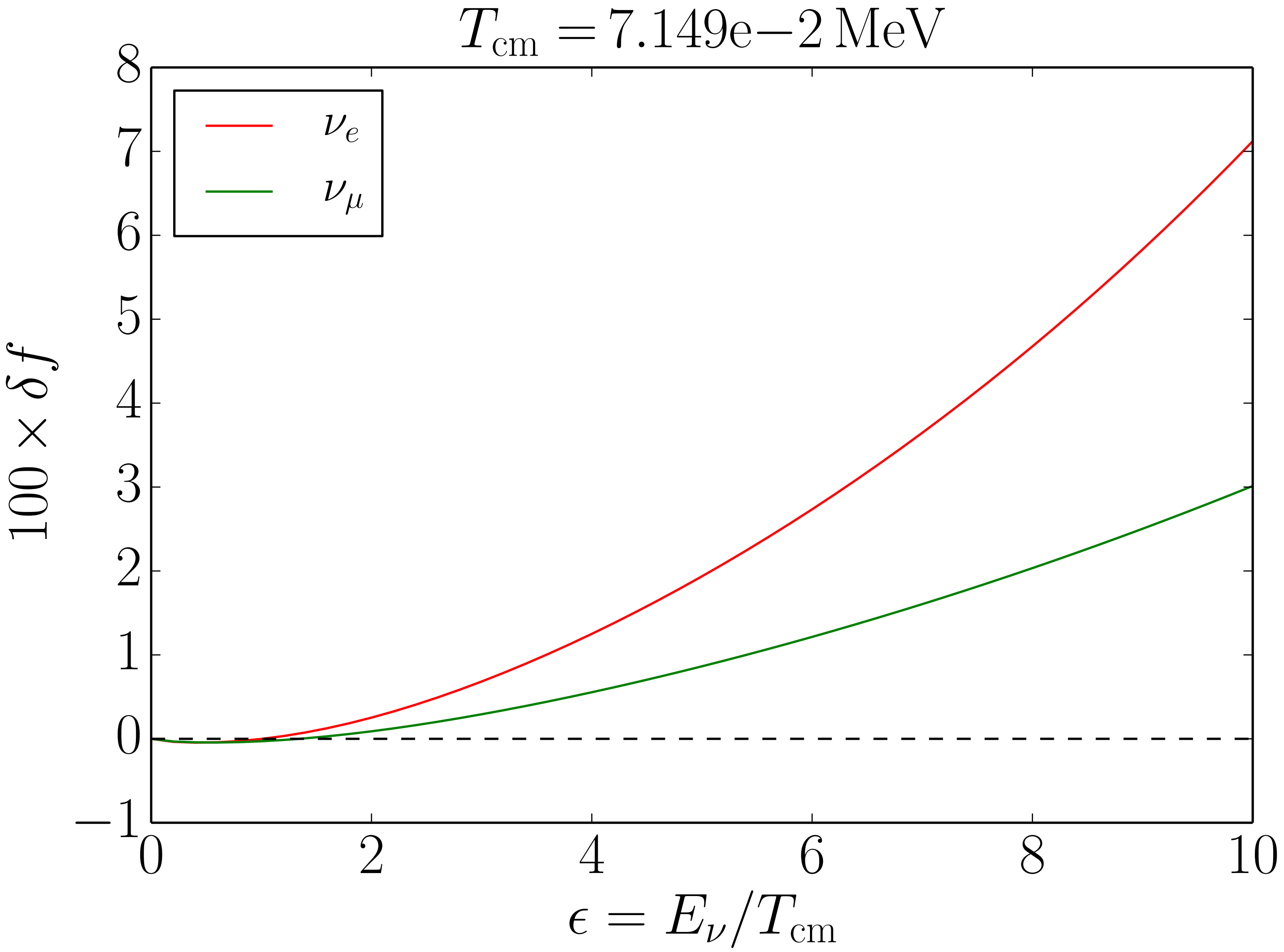
$$\frac{Df_1}{Dt} = \frac{\kappa}{32(2\pi)^3} \int_0^\infty dp_2\, p_1 p_2^3 \int_0^{p_1+p_2} dp_3 W(p_1, p_2, p_3) F(p_1, p_2, p_3, p_1 + p_2 - p_3)$$

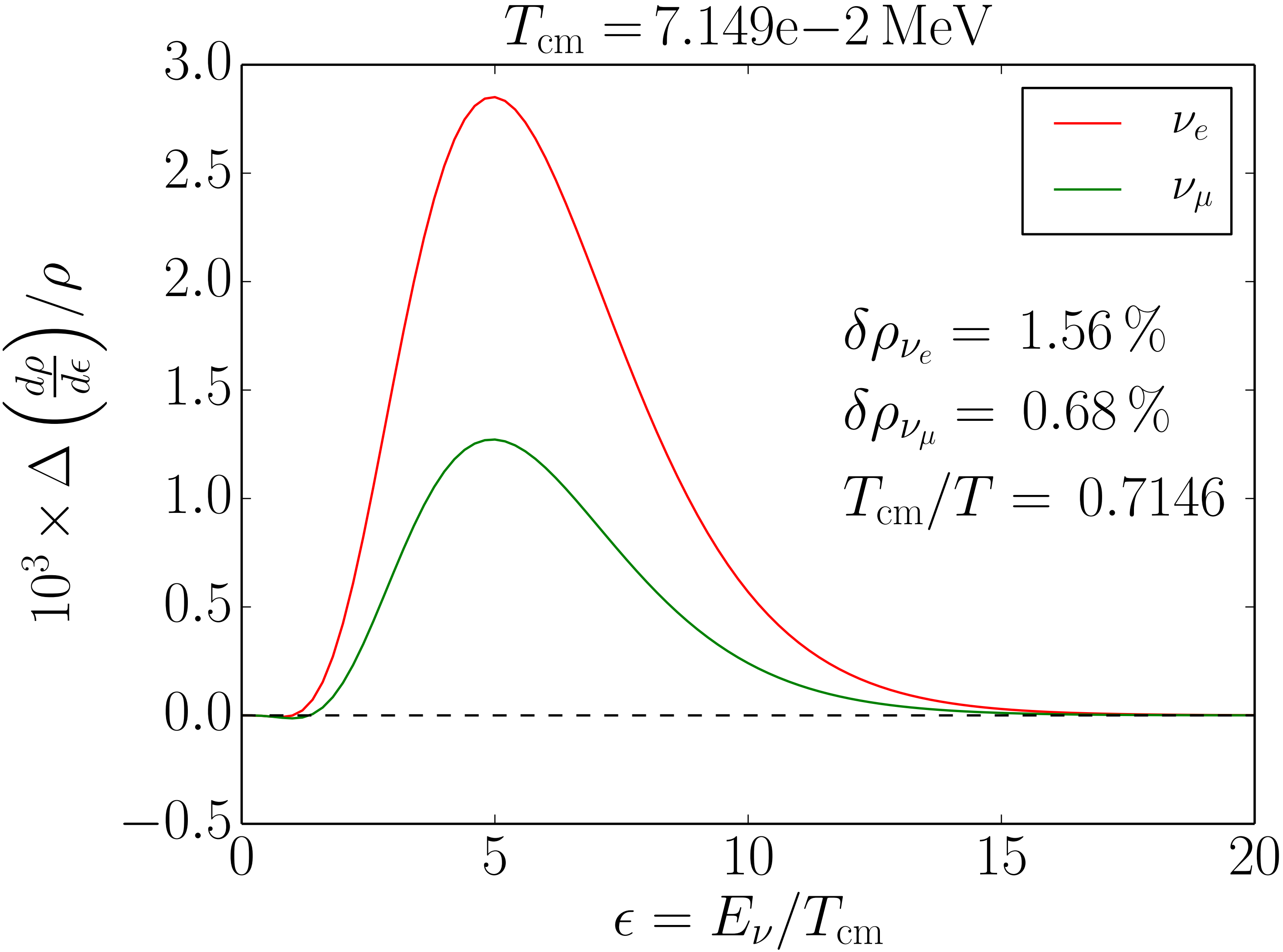
$$W(p_1, p_2, p_3) = \int_{x_0}^{1} dx \frac{(1-x)^2}{\sqrt{p_1^2 + p_2^2 + 2p_1 p_2 x}}$$

$$x_0 = \max\left(-1, 1 - \frac{2p_3(p_1 + p_2 - p_3)}{p_1 p_2}\right)$$

Computational Parameters

1. All calculations done in the weak eigenbasis.
2. 100 bins, 101 abscissas linearly spaced between $0 \leq \epsilon \leq 20$.
3. Input temperature $T_{in} = 8$ MeV, final temperature $T_{cm} \sim 9$ keV.
4. Fifth-order polynomial interpolator.
5. Acceptance tolerance $(\mathrm{net}/\mathrm{frs})^{(\mathrm{tol})} = 30$.
6. Electrons and positrons always in thermal/chemical equilibrium.


$T_{\rm cm} = 7.149{\rm e}{-2}\,{\rm MeV}$
ν_e
ν_μ
$100 \times \delta f$
$\epsilon = E_\nu / T_{\rm cm}$
8
7
6
5
4
3
2
1
0
−1
0
2
4
6
8
10


$T_{\rm cm} = 7.149{\rm e}{-2}\,{\rm MeV}$
ν_e
ν_μ
$\delta\rho_{\nu_e} = 1.56\,\%$
$\delta\rho_{\nu_\mu} = 0.68\,\%$
$T_{\rm cm}/T = 0.7146$
$10^3 \times \Delta\left(\frac{d\rho}{d\epsilon}\right)/\rho$
$\epsilon = E_\nu/T_{\rm cm}$
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
0
5
10
15
20

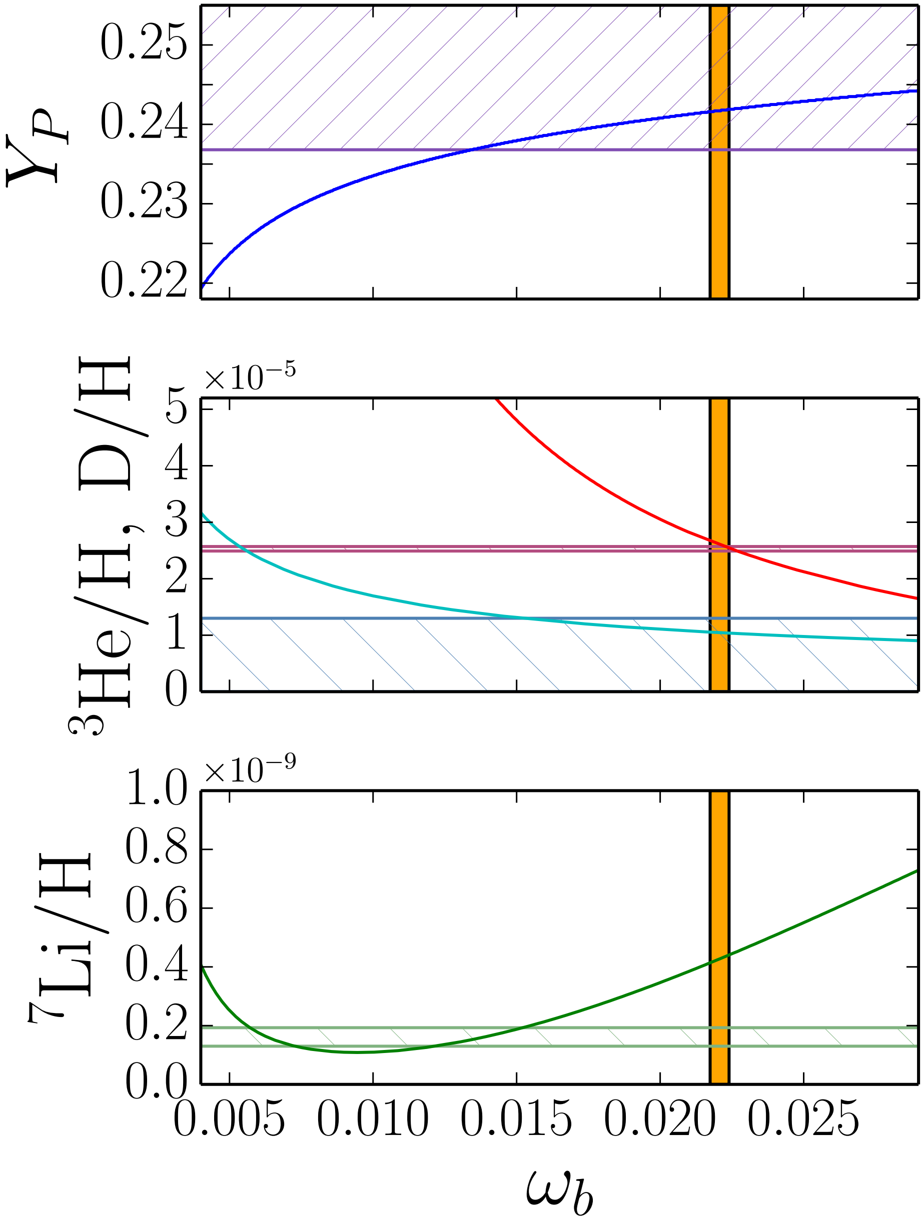
Nucleosynthesis

Without neutrino transport (sharp WD):

$$Y_P = 0.2425,$$
$$\mathrm{D/H} = 2.615 \times 10^{-5},$$

With neutrino transport:

$$Y_P = 0.2426,\ \delta(Y_P) \simeq 4 \times 10^{-4}$$
$$\mathrm{D/H} = 2.628 \times 10^{-5},$$
$$\Longrightarrow\ \delta(\mathrm{D/H}) \simeq 5 \times 10^{-3}$$



Summary and Future Work

➢BURST

➢Effect of neutrino rest mass on ionization equilibrium freeze-out.
arXiv: 1412.6875

➢Probing neutrino physics with a self-consistent treatment of the weak decoupling, nucleosynthesis, and photon decoupling epochs.
arXiv: 1502.02718

➢Beyond Standard Model Physics

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