Neutrino Flavor and Spin Evolution *in Astrophysical Environments*

Santa Fe Summer Workshop

Implications of Neutrino Flavor Oscillations

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Simulations of core collapse supernovae are very sophisticated:
multi-dimensional radiation hydrodynamics;
Boltzmann neutrino transport, and *detailed microphysics/EOS* . . .

Our understanding of the effects of nonzero neutrino mass (flavor oscillations; spin flip), though numerically sophisticated, is crude, and difficult to incorporate into the SN simulations.

There are *unsettled issues* in the story of supernova and early universe neutrinos.

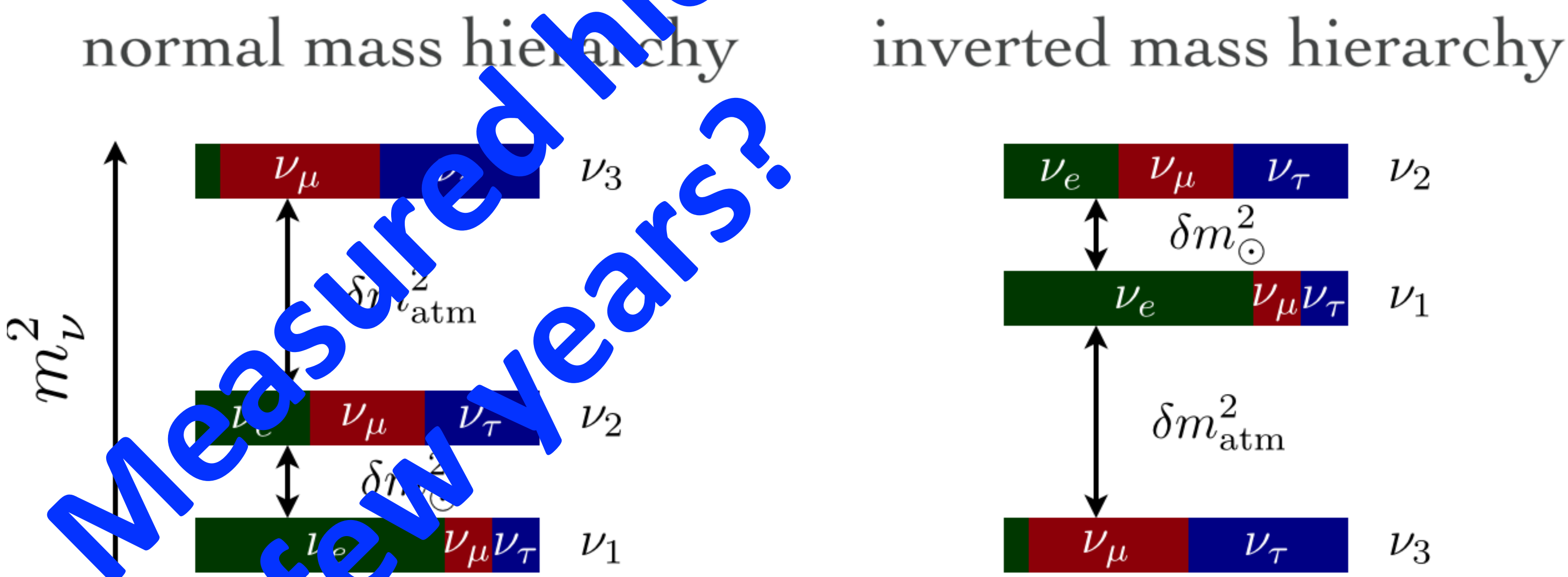
Neutrino Mass: what we know and don't know

We know the *mass-squared* differences:

$$\delta m^2_{\odot} \approx 7.6 \times 10^{-5}\,\mathrm{eV}^2$$
$$\delta m^2_{\mathrm{atm}} \approx 2.4 \times 10^{-3}\,\mathrm{eV}^2$$

$$e.g., \quad \delta m^2_{21} \equiv m^2_2 - m^2_1$$

We *do not* know the *absolute masses* or the *mass hierarchy*:



Measured hierarchy within a few years?

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = U_m \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix}$$

Pontecorvo-Maki-Nakagawa-Sakata matrix

$$U_m = U_{23}\, U_{13}\, U_{12}$$

If Neutrinos are Majorana . . .

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{-i\alpha_1/2} & 0 \\ 0 & 0 & e^{-i\alpha_2/2} \end{bmatrix}$$

$$U_{23} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix}$$

unknown:
Majorana phases α_1, α_2

Hints in $0\nu\beta\beta$ or supernovae?

$$U_{13} \equiv \begin{pmatrix} \cos\theta_{13} & 0 & e^{i\delta}\sin\theta_{13} \\ 0 & 1 & 0 \\ -e^{-i\delta}\sin\theta_{13} & 0 & \cos\theta_{13} \end{pmatrix}$$

4 parameters

$$\theta_{12}, \theta_{23}, \theta_{13}, \delta$$

$$U_{12} \equiv \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\theta_{12} \approx 0.59^{+0.02}_{-0.015}$$

$$\theta_{23} \approx 0.785^{+0.124}_{-0.124} \approx \frac{\pi}{4}$$

$$\theta_{13} \approx 0.154^{+0.065}_{-0.065}$$

$$\delta = CP \text{ violating phase} = ?$$

in VACUUM

Neutrino Rest Mass brings up many issues
which can impact cosmology and compact object physics.
Moreover, the origin of neutrino mass is likely different from the
the way other particles acquire mass via the Higgs

Outstanding Questions:

Are neutrinos Majorana or Dirac?

If Majorana , what are the Majorana Phases? (may impact $0\nu 2\beta$ decay; SN ν signal)

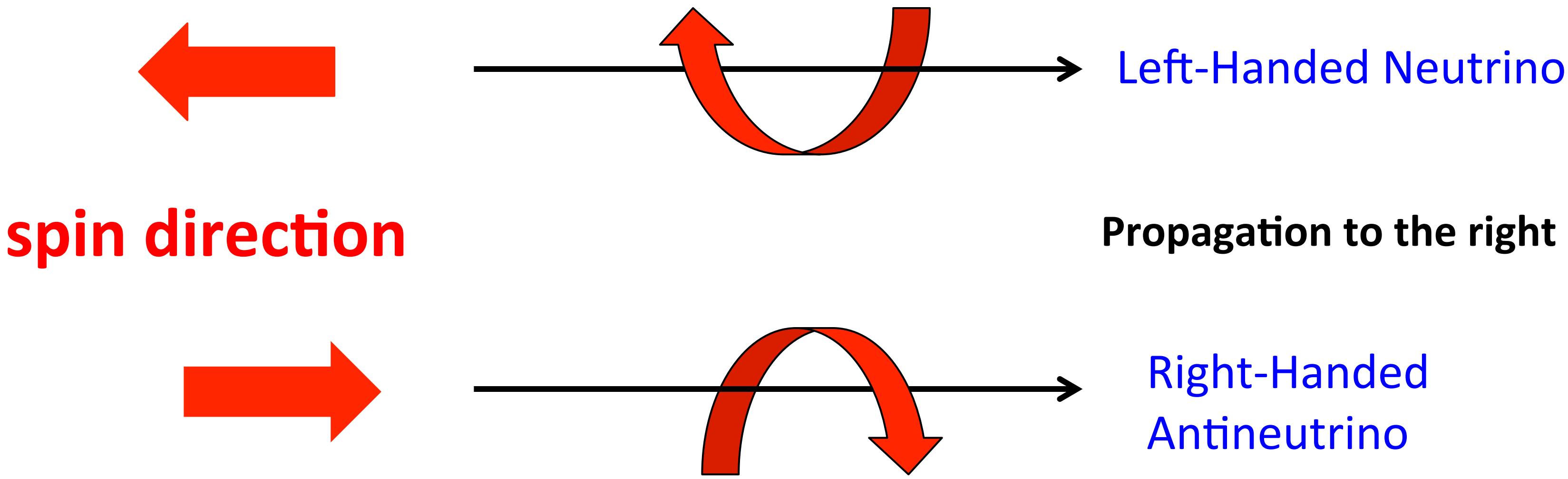
What is the CP-violating phase? (may impact lepto-genesis models; *NOT* SN)

Why are the neutrinos so light?

If See-Saw schemes are the answer, what are the mass scales of sterile neutrinos?

Are there flavor-changing neutral currents?
(feature of many supersymmetric models ; impact on CC SN)

Majorana Neutrino Spin Flip



neutrino spin flip, *i.e.*, converting a left-handed neutrino to a right-handed antineutrino is always $\propto m_\nu / E_\nu$

The surprise is that it can be done coherently and possibly collectively in environments with anisotropic neutrino distributions

Bruno Pontecorvo

recognized that the handedness of the weak interaction meant that non-zero neutrino rest mass could enable neutrino spin flip from active, left-handed states, to **sterile**, right-handed states.

Soviet Physics – JETP **26**, 984 (1968)

A take-away message from the experiments is that neutrinos have *non-zero rest masses*

This fact begs the question: Are there sterile neutrino states?

$$|\nu_e\rangle = \cos\theta\,|\nu_1\rangle \,+\, \sin\theta\,|\nu_2\rangle$$

$$|\nu_s\rangle = -\sin\theta\,|\nu_1\rangle \,+\, \cos\theta\,|\nu_2\rangle$$

If sterile neutrinos mix with active neutrinos in vacuum like this, then they are not really ***sterile*** !!

$$\text{active neutrino cross section} \quad \sigma \sim G_{\rm F}^2\, E_\nu^2$$

$$\text{“sterile” neutrino cross section} \; \sigma \sim (G_{\rm F}^2\, \sin^2\theta)\, E_\nu^2$$

in medium it's a different story . . .

neutrinos can scatter on *any* particles that carry weak charge, including *other neutrinos*, and this generates potentials that can make the neutrinos change flavors

like photons acquire an index of refraction when traveling through glass

But, unlike for photons . . .

Potentials that govern how a neutrino changes its flavor depend on the flavor states of neutrino: **NONLINEAR**

As we saw, each Neutrino is a Quantum System

In quantum mechanics a system can be in two or more seemingly mutually exclusive states at the same time!
(*e.g.,* Schroedinger's Cat is both alive *and* dead)

As it propagates along a neutrino can be in a superposition of different flavors, and the *medium* around it can influence the relative mix of these flavors.

But (some of) this medium the neutrino moves through consists of *other neutrinos*.

The upshot is that how neutrinos *change their flavor*
depends on the *flavor states* of the neutrinos in the "medium".

NONLINEAR !!!

Physicists are good at using physical reasoning to get at how nature works.

But the real world is *nonlinear* and the human mind (*at least mine*!) seems less than adept at grappling with nonlinearity.

Linear: double some parameter in a model and, *e.g.*, some other quantity doubles.

Nonlinear: double some parameter, other quantities shoot up by, *e.g.*, huge amounts, and perhaps a whole new, **unexpected** phenomenon presents itself.

I will describe just such a surprising result of nonlinearity discovered in supercomputer calculations by UCSD and LANL scientists (H. Duan; G. M. Fuller UCSD; J. Carlson LANL; Y-Z. Qian U. Minn)

Medium-Enhanced/Suppressed Neutrino Flavor Transformation

Photons acquire an index of refraction from forward scattering on the atoms (electrons) in, for example, glass or air. The propagation speed of light in materials like these is reduced from the vacuum propagation speed. It is as if the photons acquire an "*effective mass*" in medium.

Likewise, a neutrino can acquire an index of refraction by forward scattering on any particles that carry weak charge (e.g., electrons, positrons, quarks, or other neutrinos). Again, we can think of this as an "effective mass" acquired by the neutrino in medium. Flavor mixing properties altered as well.

In optically birefringent media like calcite crystals, the index of refraction depends on the polarization state of the photon.

All media of interest in the cosmos are "optically birefringent" for neutrinos. That is, the effective mass acquired by a neutrino depends on its flavor state.

How Quantum Mechanical Systems Evolve – The Rules

- when you make a measurement you have to get an eigenvalue and system is "*collapsed*" into the corresponding eigenstate

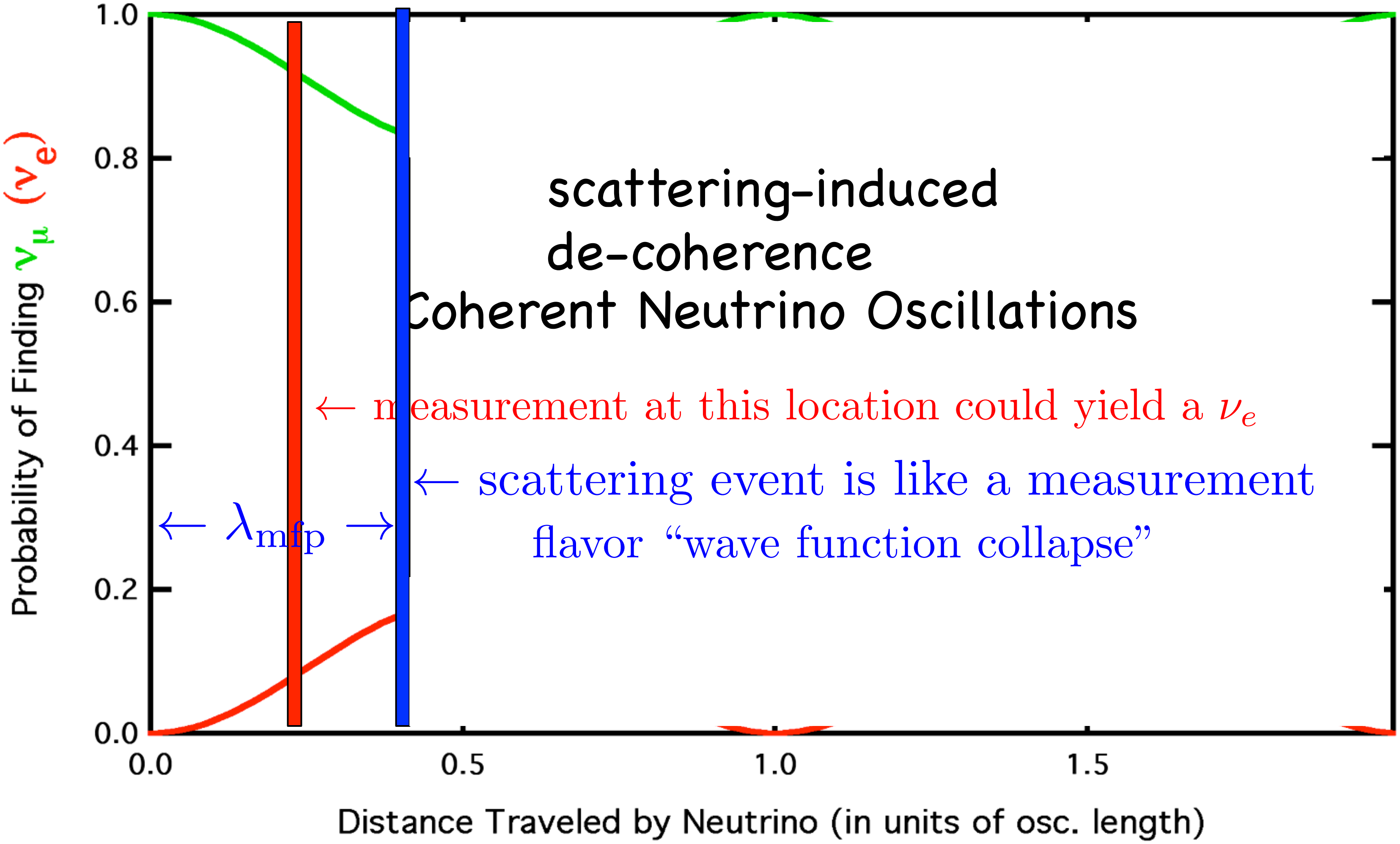
- two ways system can evolve in time:

  Schroedinger-like evolution SMOOTH/Continuous

  state reduction ("wave function collapse") because of a "measurement" ABRUPT

Simple Example: two-by-two vacuum neutrino oscillations

$$|\nu_e\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle$$
$$|\nu_\mu\rangle = -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle$$



$$|\Psi\,(t=0)\rangle = |\nu_\mu\rangle = -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle$$

$$|\Psi\,(t)\rangle = -\sin\theta\, e^{-iE_1t}\,|\nu_1\rangle + \cos\theta\, e^{-iE_2t}\,|\nu_2\rangle$$

So what is unique about the early universe, BBN, and the cosmic neutrino background (CνB) as a lab for studying neutrinos?

In a nutshell:

There are a ***huge*** number of these neutrinos.
They make themselves felt in Big Bang Nucleosynthesis,
at the γ-decoupling epoch, and by their gravitational effects.

Moreover, the relic density of these neutrinos
and their energy spectra could give unique insights
into the physics of the very early universe

Neutrinos and Lepton Number Violation in Astrophysics

Leptogenesis/Baryogenesis:

Baryon Number: $\eta = (n_{\rm b} - n_{\bar{\rm b}})/n_\gamma \approx 6.1 \times 10^{-10}$ (measured in *CMB*)

Lepton numbers: $L_{\nu_e,\nu_\mu,\nu_\tau} \leq 0.1$ (He or D plus neutrino mixing parameters)

Compact Object Physics is ***exquisitely sensitive*** to *lepton number violating processes* and neutrino flavor/spin physics:

Nucleosynthesis (e.g., the **r-process**) is sensitive to neutrino flavor/spin

- role of neutrinos in core collapse SN and neutron stars mergers?

Cosmology/BSM neutrino sector physics/Dark Matter

Will approach a ***nearly over-determined*** situation with advent of

- Next generation CMB polarization: N_{eff}, He, baryon-number, Σm_ν
- 30m-class telescopes: Primordial Deuterium to 2%
- X-ray (Astro-H) and gamma-ray observatories (Fermi)

E. Grohs, GMF, C.T. Kishimoto, M. W. Paris, ***JCAP*** **05**, 017 (2014). arXiv:1502.02718

radiation energy density at the photon decoupling epoch

$N_{\rm eff}$ defined through

$$\rho_{\rm rad} = \left[2 + \tfrac{7}{4}\left(\tfrac{4}{11}\right)^{\frac{4}{3}} N_{\rm eff}\right] \frac{\pi^2}{30}\, T^4$$

$$N_{\rm eff}^{\rm theory} = 3.046$$

Polarization release Planck XIII (2015)

$N_{\rm eff} = 3.15 \pm 0.23$ (Planck TT+ low P + BAO)

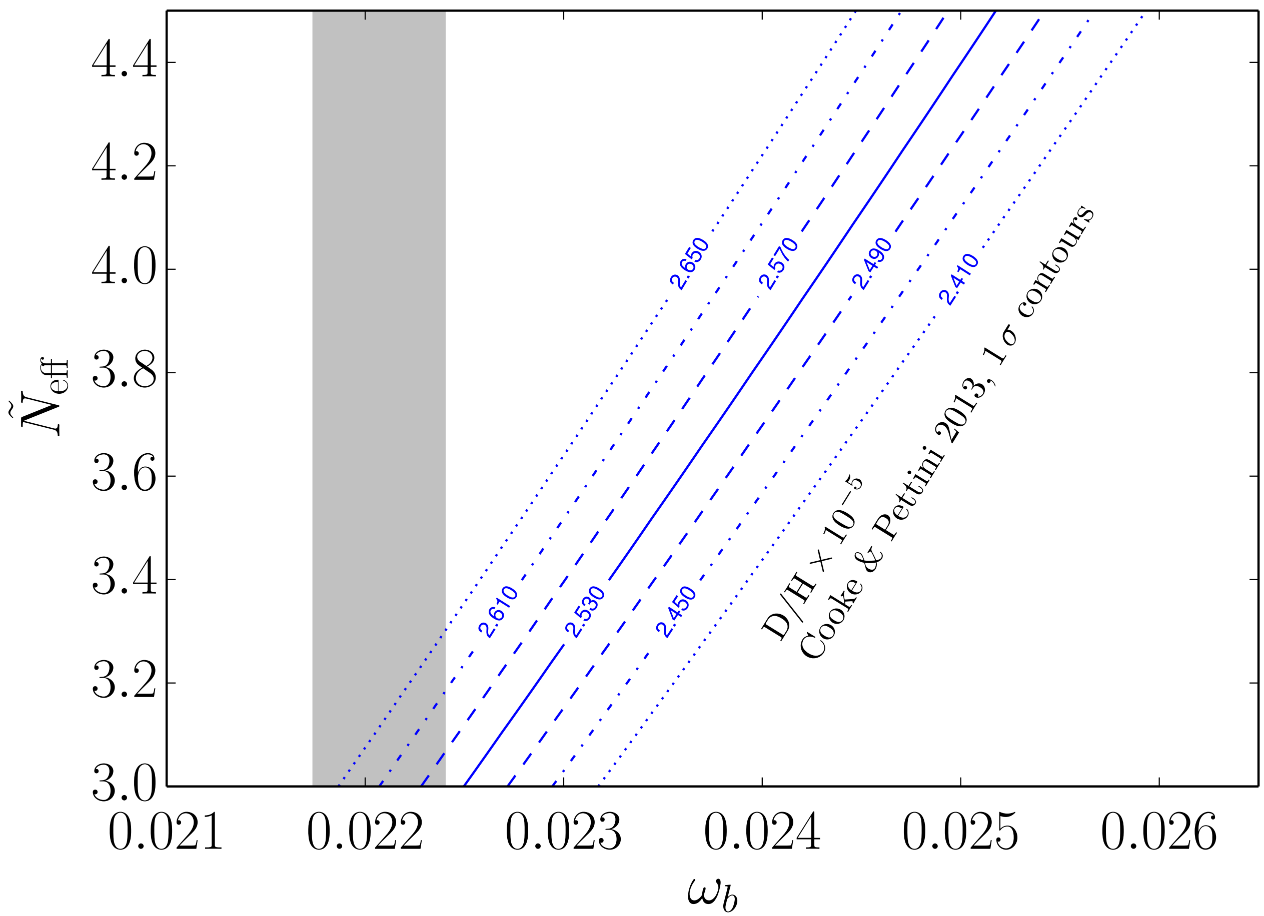
$\sum m_\nu < 0.23\,{\rm eV}$ (Planck TT+ low P + lensing + BAO + JLA + H_0))

Claimed eventual 2% precision from CMB polarization data
– but don't actually measure $N_{\rm eff}$

But cosmological neutrinos **cannot** be described by $N_{\rm eft}$ and $\sum m_\nu$ alone - see E. Grohs *et al.* (2014, 2015)

Tension between measured $\tilde{N}_{\rm eff}$ and baryon closure fraction $\omega_{\rm b}$

E. Grohs, GMF, C. T. Kishimoto, M. W. Paris arXiv:1502.02718; arXiv:1412.6875



Importance of *Thirty-Meter*-class telescopes
for deuterium abundance D/H and, hence, neutrino physics and BSM physics

Astrophysical Probes of Neutrino Rest Mass

(Abazajian et al., arXiv:1103.5083)

Probe	Current/Reach $\sum m_\nu$ (eV)	Key Systematics	Current Surveys	Future Surveys
CMB Primordial	1.3/0.6	Recombination	WMAP, Planck	None
CMB Primordial w/ Distance	0.58/0.35	Distance measurements	WMAP, Planck	None
Lensing of CMB	∞/0.2-0.05	NG of Secondary anisotropies	Planck, ACT [47], SPT, PolarBear, EBEX, QUIET II [48]	CMBPol [44]
Galaxy Distribution	0.6/0.1	Nonlinearities, Bias	SDSS [9, 10], DES [43], BOSS [15]	LSST [17], WF-MOS [11], HET-DEX [12]
Lensing of Galaxies	0.6/0.07	Baryons, NL, Photo-z	CFHT-LS [42],DES [43], HyperSuprime	LSST, Euclid [57], DUNE [58]
Lyman α	0.2-?/0.1	Bias, Metals, QSO continuum	SDSS, BOSS, Keck	BigBOSS [59]
21 cm	∞/0.1-0.006	Foregrounds	Lofar [46], MWA [49], Paper, GMRT	SKA [50], FFTT [38]
Galaxy Clusters	0.3-?/0.1	Mass Function, Mass Calibration	SDSS, SPT, DES, Chandra	LSST
Core-Collapse Supernovae	NH (If $\theta_{13} > 10^{-3}$) IH (Any θ_{13})	Emergent ν spectra	SuperK, ICECube	Noble Liquids, Gadzooks

Table I: Cosmological probes of neutrino mass. "Current" denotes published (although in some cases controversial, hence the range) 95% C.L/ upper bound on $\sum m_\nu$ obtained from currently operating surveys, while "Reach" indicates the forecasted 95% sensitivity on $\sum m_\nu$ from future observations. These numbers have been derived for a minimal 7-parameter vanilla+m_ν model. The six other parameters are: the amplitude of fluctuations, the slope of the spectral index of the primordial fluctuations, the baryon density, the matter density, the epoch of reionization, and the Hubble constant.

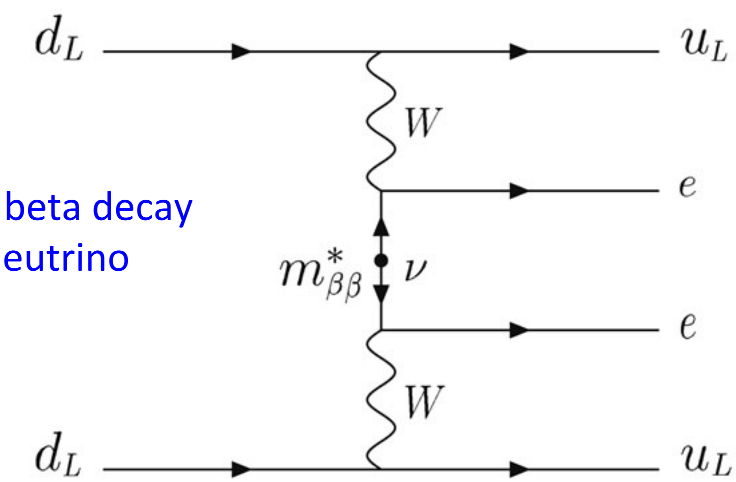
Each of these probes faces technological, observational, and theoretical challenges in its quest to extract a few percent level signal. Table I highlights the key theoretical systematics each probe will have to overcome to obtain a reliable constraint on neutrino masses.

CMB + large-scale structure observations *do not* actually measure the neutrino rest mass, but rather a convolution of this with the relic neutrino energy spectrum.

This is why $0\nu\beta\beta$-decay experiments and ^3H-decay endpoint experiments are important complementary probes of neutrino mass neutrino-less double beta decay experiments are a powerful probe of neutrino physics, complementary to cosmological and other laboratory probes. They can provide unique insights.

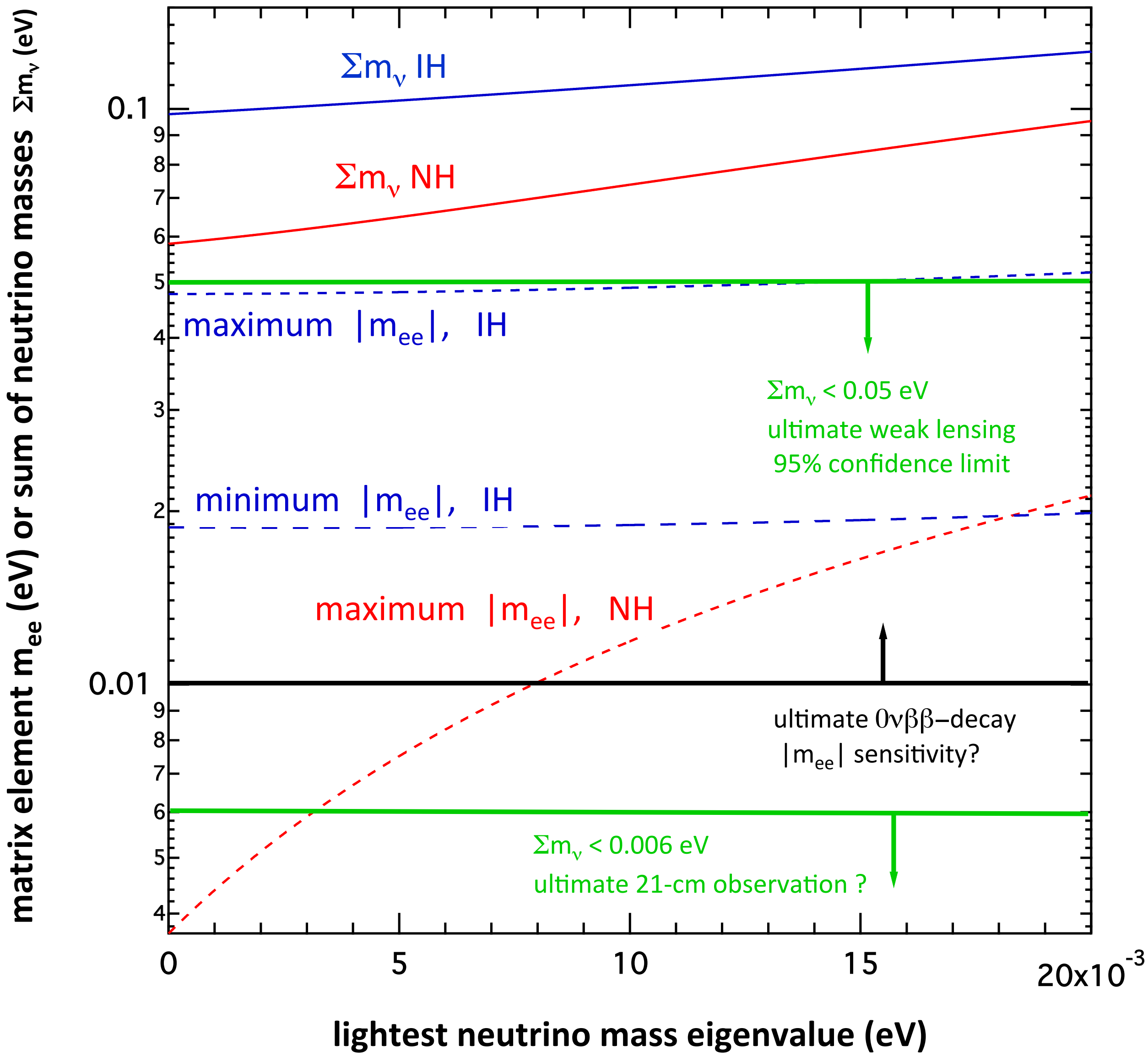
$0\nu\beta\beta$ experiments with sensitivity $|m_{ee}| = 0.01\,\text{eV}$ can probe lightest neutrino mass eigenvalue $m > 0$ in the Inverted Neutrino Mass Hierarchy. In the Normal Mass Hierarchy, they can probe lightest neutrino mass eigenvalue $m > 8\,\text{meV}$.

neutrinoless double beta decay requires Majorana neutrino spin flip



Only way to get at questions of Majorana/Dirac, Majorana mass, and Majorana phases?

$$\left| m_{ee} \right| = \left| m_1 \, c_{12}^2 \, c_{13}^2 \; + \; m_2 \, s_{12}^2 \, c_{13}^2 \, e^{2i\alpha} \; + \; m_3 \, s_{13}^2 \, e^{2i\beta} \right|$$



So what is unique about compact objects (e.g., CC SN, NS-mergers) as a "lab" for studying neutrinos/lepton number violation?

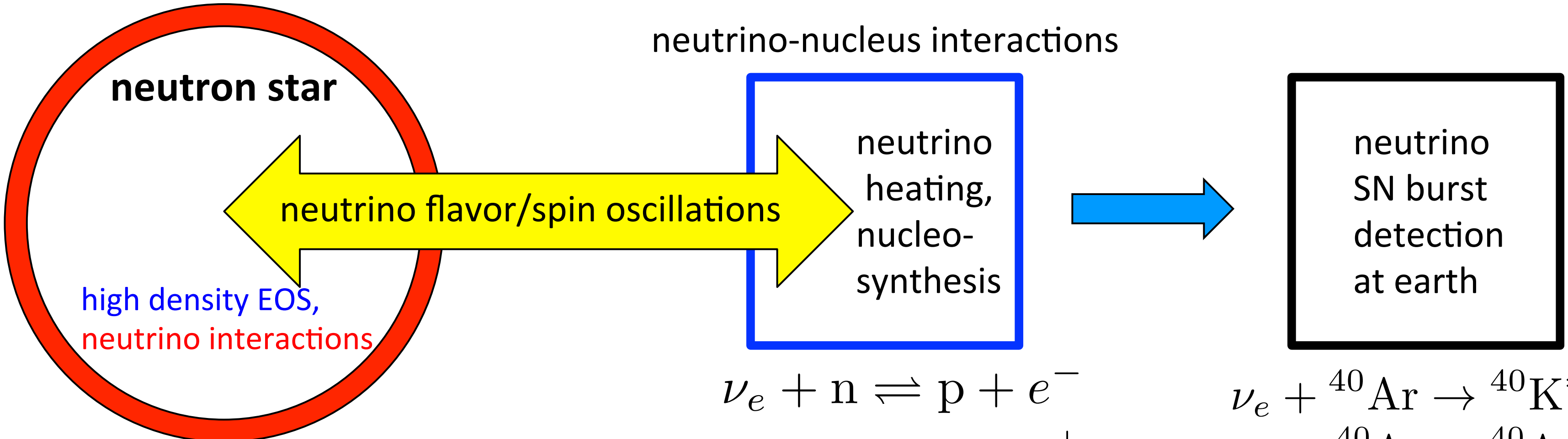
In a nutshell:

Core collapse supernovae and neutron stars are ***cold, highly electron lepton number degenerate systems***.

They are ***exquisitely sensitive*** to lepton number violating processes.

Macroscopic effects in SN physics or signal from:
flavor oscillations: very sensitive to neutrino mass hierarchy;
spin coherence: sensitive to Majorana/Dirac nature of neutrinos
& absolute neutrino masses

Calculating neutrino flavor transformation in the core collapse supernova environment is a vexing problem,
but one whose solution may lie at the heart of many aspects of the physics of stellar collapse, nucleosynthesis, and the ν signal.



$$\nu_e + \mathrm{n} \rightleftharpoons \mathrm{p} + e^-$$
$$\bar{\nu}_e + \mathrm{p} \rightleftharpoons \mathrm{n} + e^+$$

$$\nu_e + {}^{40}\mathrm{Ar} \to {}^{40}\mathrm{K}^* + e^-$$
$$\nu_x + {}^{40}\mathrm{Ar} \to {}^{40}\mathrm{Ar}^* + \nu_x$$

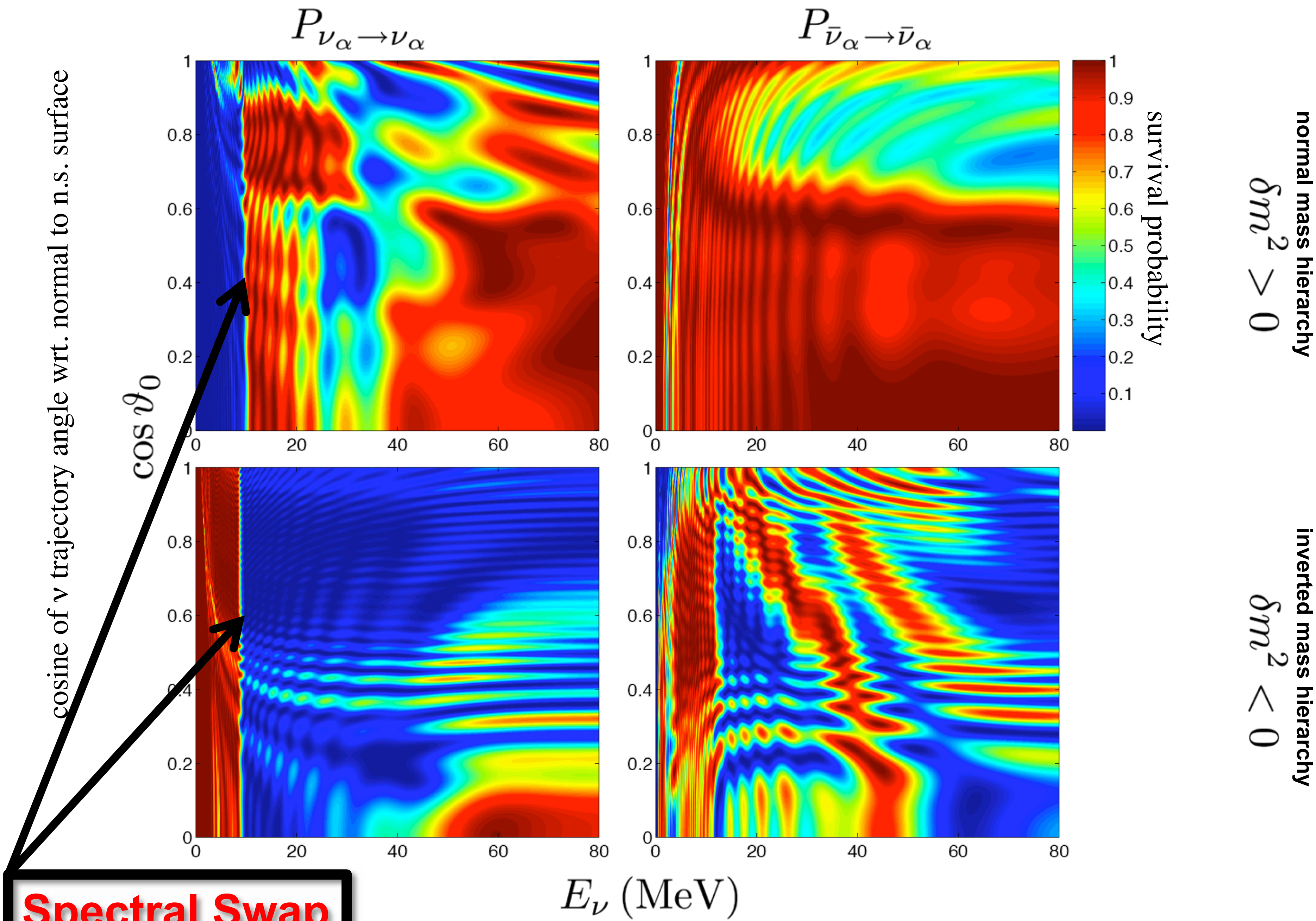
We need the fluxes and energy spectra of each flavor/type of neutrino at all epochs and at all radii.

Unfortunately,
we can’t model the evolution of the neutrino flavor field
in any but idealized geometries and “toy-problem” conditions!

Claims about what happens or doesn’t happen in supernovae should be taken with suspicion!

Coherent Nonlinear Neutrino Flavor Evolution

- *Collective Oscillations*
- *Instability*
- *Matter fluctuations*



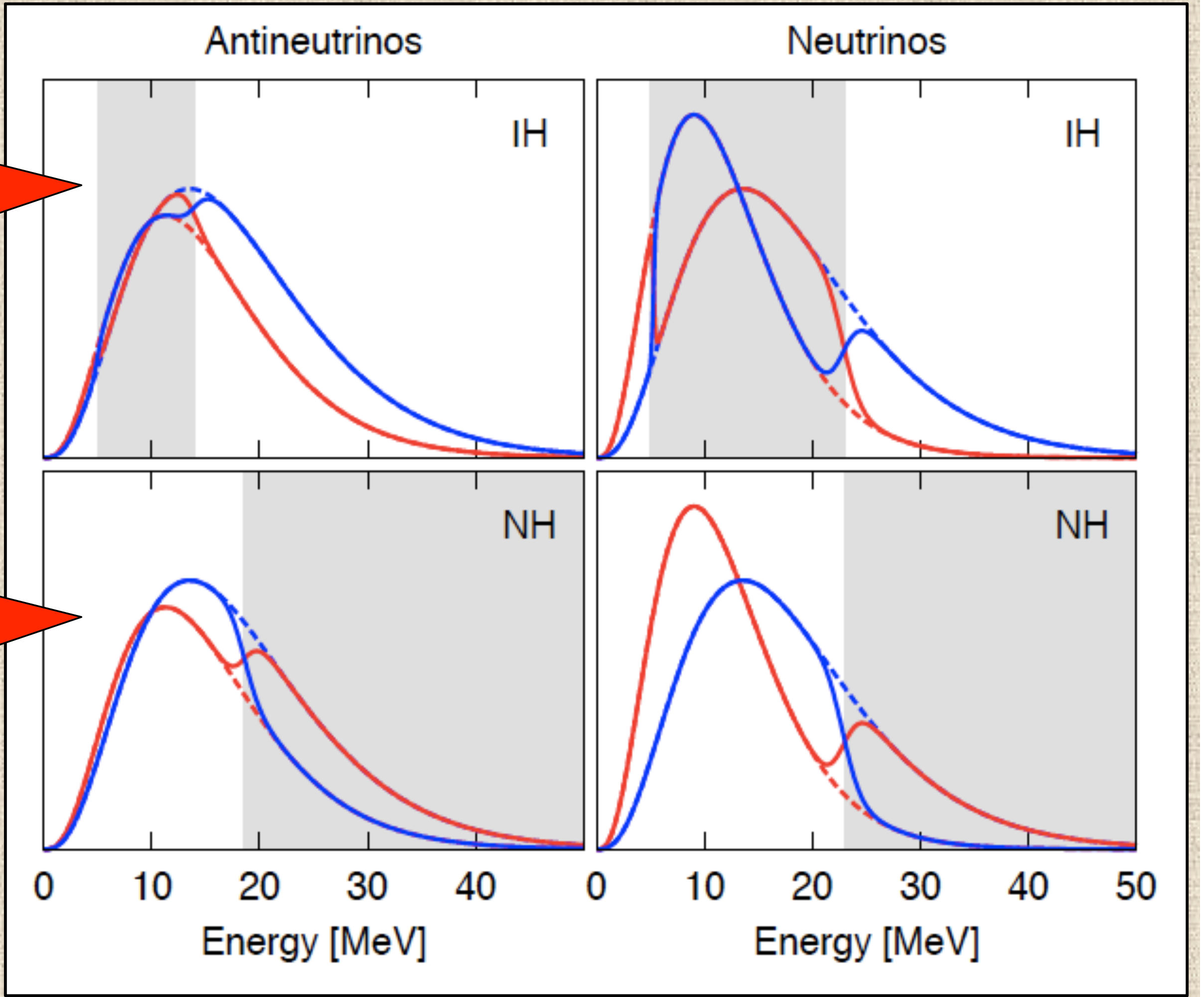
consequences of neutrino mass and quantum coherence in supernovae

H. Duan, G. M. Fuller, J. Carlson, Y.-Z. Qian, Phys. Rev. Lett. **97**, 241101 (2006) **astro-ph/0606616**

Multiple Spectral Splits

Spectral Splits in Inverted Hierarchy

Spectral Splits in Normal Hierarchy



Dasgupta, Dighe, Raffelt and Smirnov, arXiv: 0904.3542 (PRL)

Spatial instability develops in neutrino flavor field above the neutron star

enhanced instability in the neutrino flavor field

– not easily matter-suppressed

nonlinearity:

neutrino flavor field may not retain the symmetry of the neutrino sphere initial conditions

G. Raffelt, S. Sarikas, and D. de Sousa; ArXiv:1308.142

A. Mirrizi; ArXiv:1308.5255

H. Duan and S. Shalgar , *Phys. Lett.,* **B747**, 139 (2015). arXiv:1412.7097

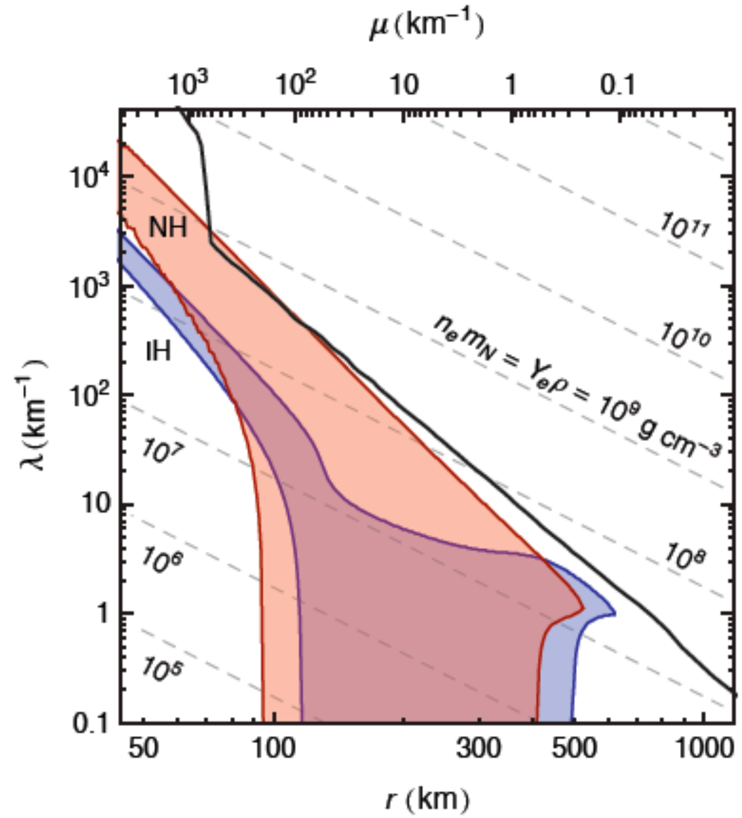


FIG. 2: Region where $\kappa r > 1$ for IH (blue) and NH (red), depending on radius r and multi-angle matter potential λ for our simplified SN model. *Thick black line:* SN density profile. *Thin dashed lines:* Contours of constant electron density, where Y_e is the electron abundance per baryon. (The IH case corresponds to Fig. 4 of Ref. [18], except for the simplified spectrum used here.)

Over the last year or so we have derived, for the first time,
the complete equations that govern the evolution of
neutrino flavor and ***spin*** (left-handed neutrino/right-handed antineutrino)
in the **general** (e.g.**, anisotropic**) **environments** of interest in astrophysics

The ***Neutrino Quantum Kinetic Equations for Flavor & Spin***

Some surprises: *discovery of spin coherence*

Alexey Vlasenko, G. M. Fuller, V. Cirigliano, Phys. Rev. D **89**, 105004 (2014), arXiv:1309.2628

V. Cirigliano, G. M. Fuller, A. Vlasenko Phys. Lett. B, **747**, 27-35 (2015), arXiv:1406.5558

A. Vlasenko, G. M. Fuller, V. Cirigliano, arXiv:1406.6724

C. Volpe, D. Vaanaen, C. Espinoza, , Phys. Rev. D **87**, 113010 (2013) arXiv:1302.2374

J. Serreau & C. Volpe, arXiv:1409.3591

A. Kartavtsev, G. Raffelt, H. Vogel, Phys. Rev D **91**, 125020 (2015) arXiv:1504.03230

A. Vlasenko, V. Cirigliano, G.M.F.

Solving the algebraic constraints and rearranging the remaining integro-differential equations gives the following compact form for the QKEs:

$$D\mathcal{F} = -i\,[\mathcal{H},\mathcal{F}] + \{\mathcal{C}_{\rm GAIN}, 1-\mathcal{F}\} - \{\mathcal{C}_{\rm LOSS},\mathcal{F}\}$$

Vlasov operator (convective derivative plus force terms)

Vacuum + coherent forward scattering Hamiltonian

Flavored Boltzmann terms with gain-loss structure

$$\mathcal{F} = \begin{pmatrix} f & \phi \\ \phi^\dagger & \bar{f}^T \end{pmatrix}$$

6 x 6 matrix containing neutrino and antineutrino densities & spin coherence

$$\mathcal{H} = \begin{pmatrix} H & H_\phi \\ H_\phi^\dagger & -\bar{H}^T \end{pmatrix}$$

Usual Hamiltonian, plus helicity mixing term of O(m/E)

Spin coherence/spin flip (and hence **neutrino-antineutrino)** coherence was a surprise, but perhaps it should not have been!

$$\mathcal{L}_{\rm int} \sim \bar{\nu}_{\rm L}\, m\, \nu_{\rm R} - \bar{\nu}_{\rm L}\, \not{\Sigma}_{\rm R}\, \nu_{\rm L} - \bar{\nu}_{\rm R}\, \not{\Sigma}_{\rm L}\, \nu_{\rm R}$$

$$\Rightarrow \quad \bar{\nu}\, \left(\Sigma^{\mu}_{\rm A}\, \gamma_{\mu}\, \gamma_5\right)\, \nu$$

$$\Big\downarrow$$

$$\vec{\sigma} \cdot \vec{\Sigma}_{\rm A} \qquad \text{Gamow} - \text{Teller}$$

Quantum Kinetic Equations

$$i\,D\hat{f} - \left[\hat{\mathcal{H}}, \hat{f}\right] - \hat{U}\left[\hat{\phi}\right] = \text{collision terms}\left(\hat{f}, \hat{\bar{f}}\right)$$

where $\hat{f}$ and $\hat{\bar{f}}$ are 3×3 Hermitian density operators for neutrinos and antineutrinos, respectively, and $\hat{\phi}$ is a 3×3 complex matrix encoding spin coherence.

and where $\hat{\mathcal{H}}$ & $\hat{U}$ give neutrino interactions with matter and other neutrinos

Schroedinger-like:

$$i\frac{\partial|\psi\rangle}{\partial t} = \hat{H}|\psi\rangle \text{ with } |\psi\rangle = (\psi_e, \psi_\mu, \psi_\tau)$$

$$\hat{H} = \frac{m^2}{2E} + \hat{H}_{e\nu} + \hat{H}_{\nu\nu}$$

@ "low" density where
neutrinos propagate coherently

Boltzmann equation

@ "high" density where
inelastic scattering dominates

A. Vlasenko, G.M.F., V. Cirigliano (2013), arXiv:1309.2628

Toward Quantum Kinetics

G. Sigl & G. Raffelt, Nucl. Phys. B **406**, 423 (1993)
B.H.J. McKellar and M.J. Thomson, Phys. Rev. D **49**, 2710 (1994)
K. Enqvist, K. Kainulainen, J. Maalami, Nucl. Phys. B **349**, 743 (1991)
R. F. Sawyer, Phys. Rev. D**79**,, 105003 (2009).
C. Volpe, D. Vaanaen, C. Espinoza, ArXiv:1302.2374
A. Vlasenko, G. M. Fuller, V. Cirigliano, Phys. Rev. D **89**, 105004 (2014), ArXiv:1309.2628
V. Cirigliano, G. M. Fuller, A. Vlasenko arXiv:1406.5558
J. Serreau & C. Volpe, arXiv:1409.3591

(a) Effects of a small amount of direction-changing scattering on neutrino flavor transformation? – The Halo

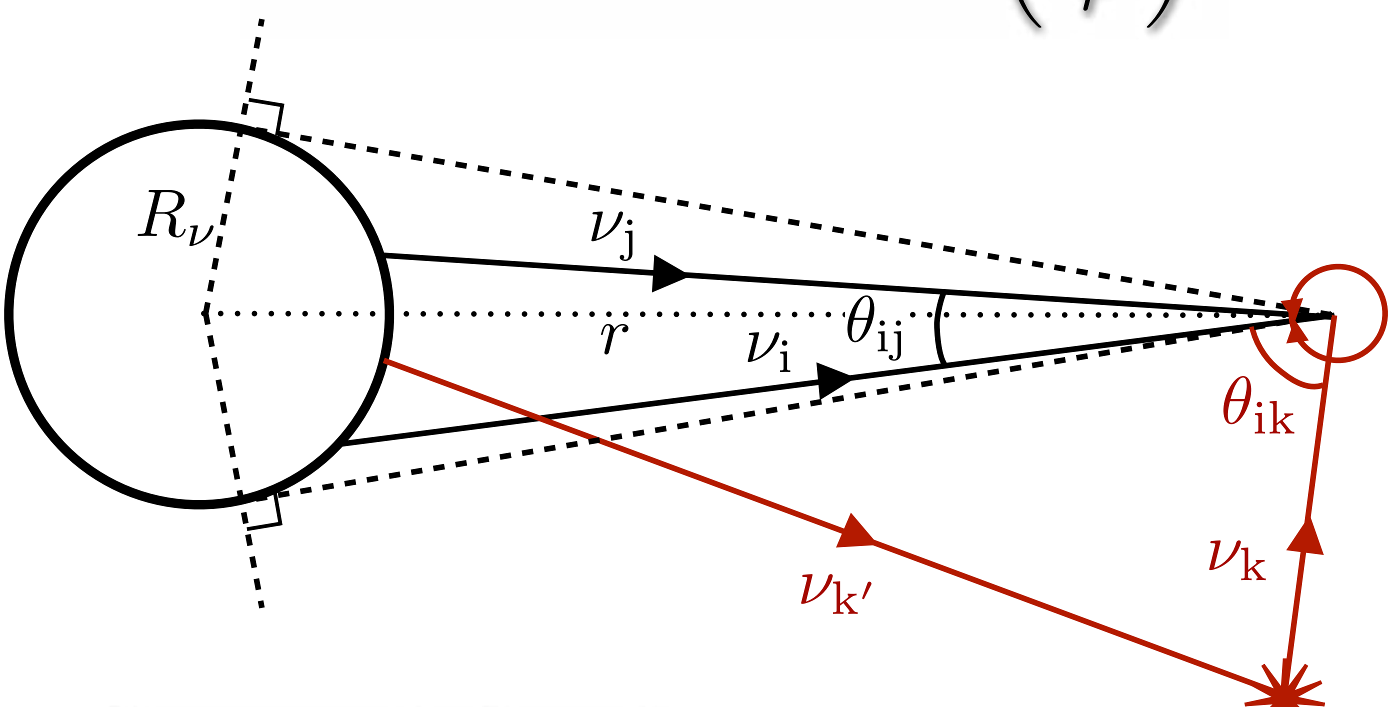
(b) *Spin Coherence*: neutrino-antineutrino inter-conversion

A. Vlasenko, G. M. Fuller, V. Cirigliano, Phys. Rev. D **89**, 105004 (2014), ArXiv:1309.2628

J. Cherry, A. Friedland, G. Fuller, J. Carlson, A. Vlasenko, Phys. Rev. Lett. **108**, 261104 (2012) 1203.1607

The Neutrino Halo

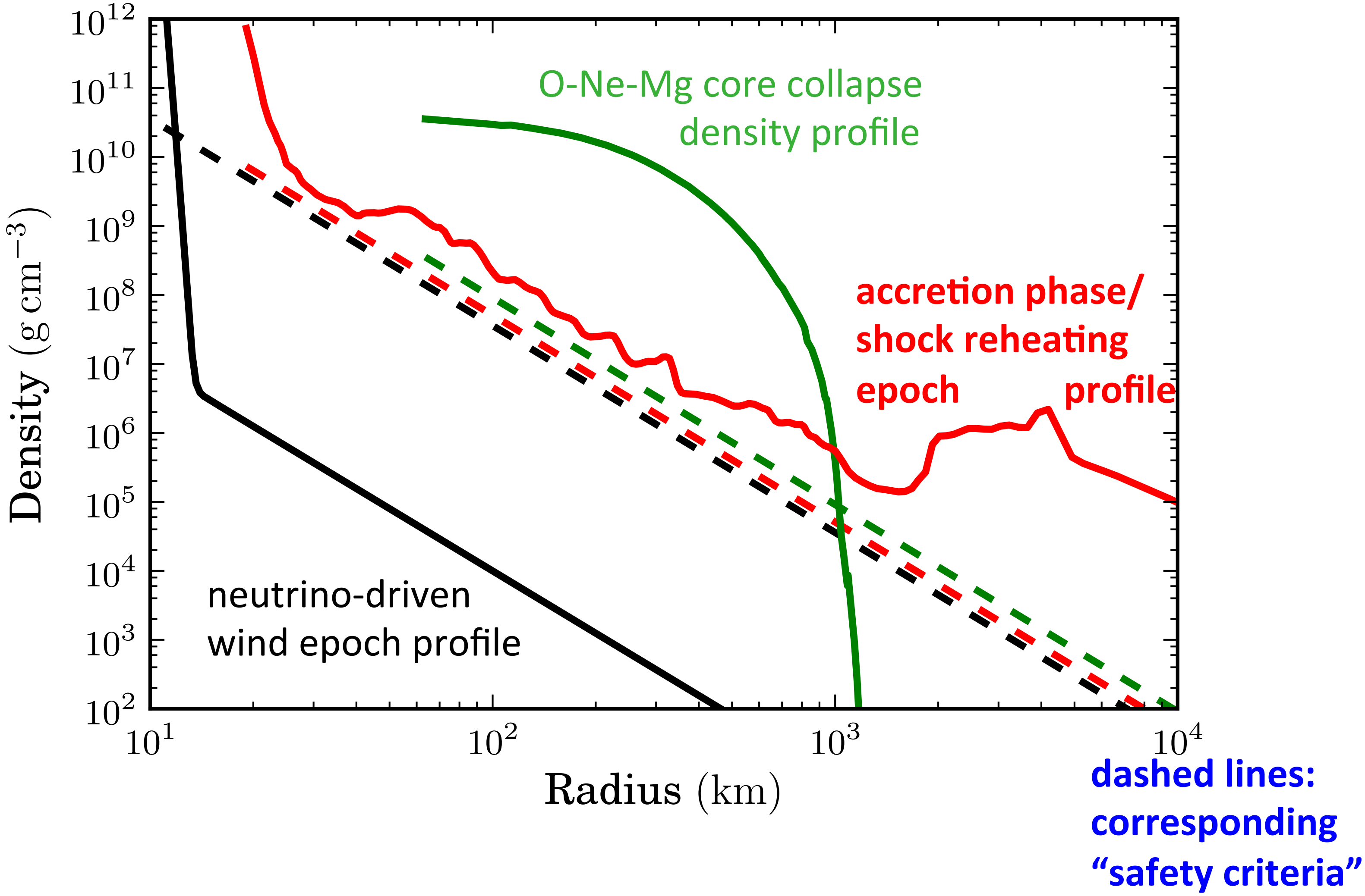
$$r \gg R_\nu \Rightarrow \langle 1 - \cos\theta_{\rm ij} \rangle \propto \left(\frac{R_\nu}{r}\right)^2$$



$$\langle 1 - \cos\theta_{\rm ik} \rangle \approx 1$$

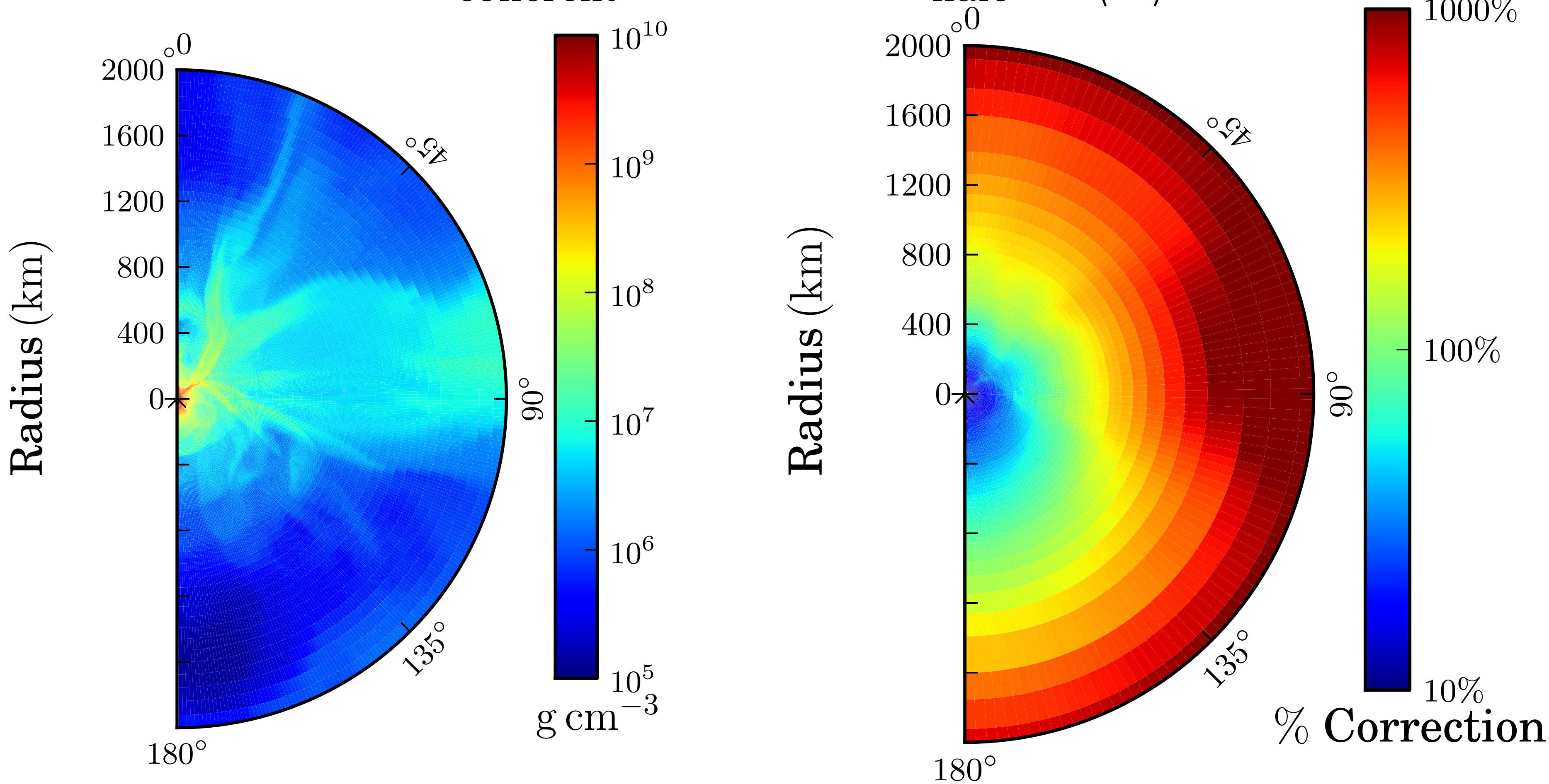
$\sim 10^{-3}$ of all ν's

If the density profile at some epoch falls above the corresponding safety criterion (corresponding color dashed line), the "Halo" effect is important



How large is the Halo effect for free nucleons?

$$\sigma_{\mathrm{coherent}} \propto A^2 \Rightarrow \mathcal{H}_{\mathrm{halo}} \propto \langle A \rangle$$



J. F. Cherry, A. Friedland, G. M. Fuller, J. Carlson, and A. Vlasenko, Phys. Rev. Lett. **108**, 261104 (2012), 1203.1607.

the Halo converts the neutrino flavor evolution problem from an *initial value problem* into a *boundary value problem*

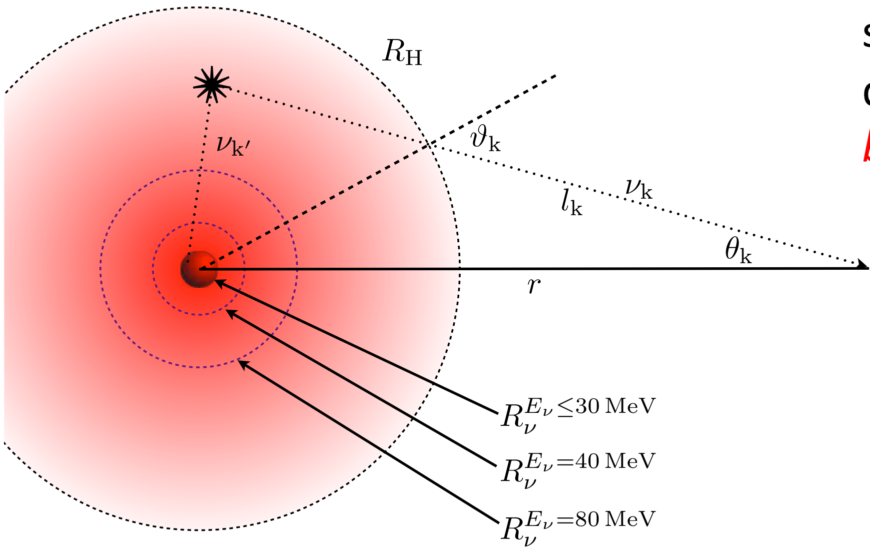
(quantum flavor information *coming down* from outer regions of star)

and moreover couples in nuclear composition in a completely new way

stability analyses suggest little effect from Halo during shock re-heating/accretion phase

(S. Sarikas, I. Tamborra, G. Raffelt, L. Hudepohl, H.T. Janka PRD **85**, 113007 (2012) 1204.0971;

A. Mirizzi & P.D. Serpico, PRD **86**, 085010 (2012) 1208.0157) – But these studies leave out much of the halo and do not capture the composition/inhomogeneous effects

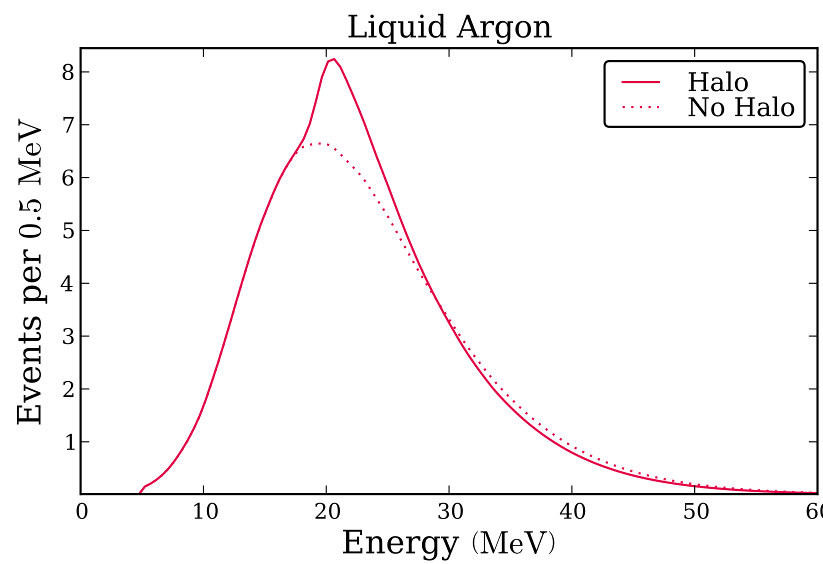


O-Ne-Mg Core Collapse – *very centrally-condensed*, so we *can* model the Halo with our initial value code: quantum mechanical information all coming from *below* region of collective oscillations!

Dispersion/de-coherence in Halo causes neutrino trajectory-dependent swap energy, which could have consequences for a detected neutrino signal

With Halo fewer high energy ν_e's are transformed

$\Rightarrow$ more ν_e-induced events in detector



J. Cherry, J. Carlson, A. Friedland, G.M.F, A. Vlasenko, PRD **87**, 085037 (2013). arXiv:1302.1159

Quantum Kinetic Equations

A. Vlasenko, G.M.F., V. Cirigliano (2013), arXiv:1309.2628
V. Cirigliano, G.M.F, A. Vlasenko (2014), arXiv:1406.5558
A. Vlasenko, G.M.F, V. Cirigliano (2014), arXiv:1406.6724

$$i\,\mathcal{D}\,[\mathcal{F}] - [\mathcal{H}, \mathcal{F}] = i\,\mathcal{C}\,[\mathcal{F}]$$

a 6X6 matrix formulation

$$\mathcal{F} = \begin{bmatrix} f & \phi \\ \phi^\dagger & \bar{f}^{\mathrm{T}} \end{bmatrix}$$

$f(x,p)$ and $\bar{f}(x,p)$ are neutrino/antineutrino density operators, so they are 3×3 matrices $\Rightarrow$ $\begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$

Here ϕ is a new dynamical quantity encoding neutrino spin/helicity

Collision terms $\Rightarrow$ $\mathcal{C} = \begin{bmatrix} C & C_\phi \\ C_\phi^\dagger & \bar{C}^{\mathrm{T}} \end{bmatrix}$

and the Hamiltonian is $\mathcal{H} = \begin{bmatrix} H & H_{\nu\bar{\nu}} \\ H_{\nu\bar{\nu}}^\dagger & -\bar{H}^{\mathrm{T}} \end{bmatrix}$

with $H_{\nu\bar{\nu}} = \frac{-1}{|\vec{p}|}\left(\Sigma^+\, m^* + m^*\, \Sigma^{+\,\mathrm{T}}\right)$

with Σ^+ spacelike potentials orthogonal to neutrino trajectory

$H_{\nu\bar{\nu}} \propto \frac{m}{E}$ (weak magnetic vector potential)

neutrino-antineutrino transformation

$\Rightarrow$ particles must have mass

$\Rightarrow$ need anisotropic matter/neutrino potentials, with components $\perp$ to neutrino momentum

$\Rightarrow$ spin coherence density ϕ must be present

Coherent Limit

low densities $\Rightarrow$ potentials small, $\Sigma \ll m$, collision term smaller still, so drop Σ^2 and maybe $m\Sigma$ terms.

Then ϕ decouples and we have

$$i\,\frac{p^\mu}{E}\partial_\mu f - [H, f] = 0 \quad \text{with} \quad H = \Sigma^\kappa - \frac{m^\dagger\, m}{2E}$$

$$i\,\frac{p^\mu}{E}\partial_\mu \bar{f} - [\bar{H}, \bar{f}] = 0 \quad \text{with} \quad \bar{H} = \Sigma^\kappa - \frac{m^\dagger\, m}{2E}$$

This is the Schrödinger Equation for the wave functions $|\psi\rangle$ given earlier.

Boltzmann limit $\Rightarrow$

$$\begin{bmatrix} f_{11} & 0 & 0 \\ 0 & f_{22} & 0 \\ 0 & 0 & f_{33} \end{bmatrix}$$

$\Rightarrow$ collision terms mix different energy & flavor states, averaging out the off-diagonal terms, so f diagonal

$\Rightarrow$ no coherence $[f, H] = 0$

$\Rightarrow$ mass terms small enough to discard, but must be careful because of nonlinearity

$$\frac{p^\mu}{E}\partial_\mu f_\alpha = \Pi^+_\alpha\,(1 - f_\alpha) - \Pi^-_\alpha\, f_\alpha \quad \text{with} \quad \alpha = e, \mu, \tau$$

Π functions $\Rightarrow$ usual Boltzmann gain-loss terms (depend on matter and ν densities)

Coherent *neutrino-antineutrino* transformation?

as neutrinos propagate through an anisotropic medium
their energy eigenstates (propagation states)
are coherent superpositions of right- and left-handed states
(i.e., superpositions of neutrino and antineutrino states)

consider an example: *propagation of a single neutrino through a supernova envelope*

Take our single neutrino to have electron flavor.
We then have a 2×2 problem, mixing ν_e and $\bar{\nu}_e$.
In the ν_e-$\bar{\nu}_e$ basis the "spin-coherence" Hamiltonian is:

$$\mathcal{H}_{\text{single neutrino}} \equiv \begin{bmatrix} H_{\nu_e} & H_{\nu_e\bar{\nu}_e} \\ H^{\dagger}_{\nu_e\bar{\nu}_e} & -\bar{H}^{\mathrm{T}}_{\bar{\nu}_e} \end{bmatrix}$$

write this 2X2 Hamiltonian as $\mathcal{H} \equiv \begin{bmatrix} H & H_{\nu\bar{\nu}} \\ H^{\dagger}_{\nu\bar{\nu}} & -\bar{H}^{\mathrm{T}}_{\bar{\nu}} \end{bmatrix} \Rightarrow \begin{bmatrix} X & C \\ C^* & -X \end{bmatrix}$

Where the diagonal neutrino potentials are: $X = A + B$

"matter" potential: $A = \frac{3\sqrt{2}}{2} G_{\mathrm{F}}\, n_{\mathrm{b}} \left[Y_e - \frac{1}{3} \right]$

"neutrino" potential: $B = \sqrt{2} G_{\mathrm{F}} \left[2\left(\text{“}n_{\nu_e}\text{”} - \text{“}n_{\bar{\nu}_e}\text{”}\right) + \left(\text{“}n_{\nu_\mu}\text{”} - \text{“}n_{\bar{\nu}_\mu}\text{”}\right) + \left(\text{“}n_{\nu_\tau}\text{”} - \text{“}n_{\bar{\nu}_\tau}\text{”}\right) \right]$

and the off-diagonal "spin coherence" potential is $C = -\frac{m_\nu}{E_\nu} \cdot \Sigma_\perp$ e.g., a matter current gives $\Sigma_\perp \sim G_{\mathrm{F}}\, n_{\mathrm{b}} \cdot \frac{v}{c}$

so at time t a neutrino in state: $|\Psi(t)\rangle = |\nu_e\rangle\langle\nu_e|\Psi(t)\rangle + |\bar{\nu}_e\rangle\langle\bar{\nu}_e|\Psi(t)\rangle$

evolves in time according to: $i\frac{\partial}{\partial t}\begin{bmatrix} \langle\nu_e|\Psi\rangle \\ \langle\bar{\nu}_e|\Psi\rangle \end{bmatrix} = \begin{bmatrix} X & C \\ C^* & -X \end{bmatrix}\begin{bmatrix} \langle\nu_e|\Psi\rangle \\ \langle\bar{\nu}_e|\Psi\rangle \end{bmatrix}$

A. Vlasenko, GMF, V. Cirigliano arXiv:1406.6724

to solve this equation,
project into instantaneous energy states

$$|\nu_e\rangle = \cos\theta_s(t)\;|E_+(t)\rangle + \sin\theta_s(t)\;|E_-(t)\rangle$$

$$|\bar{\nu}_e\rangle = -\sin\theta_s(t)\;|E_+(t)\rangle + \cos\theta_s(t)\;|E_-(t)\rangle$$

$$\begin{bmatrix} \langle\nu_e|\Psi(t)\rangle \\ \langle\bar{\nu}_e|\Psi(t)\rangle \end{bmatrix} = U_s(t) \begin{bmatrix} \langle E_+(t)|\Psi(t)\rangle \\ \langle E_-(t)|\Psi(t)\rangle \end{bmatrix}$$

Unitary transformation between these bases is $U_s(t) = \begin{pmatrix} \cos\theta_s(t) & \sin\theta_s(t) \\ -\sin\theta_s(t) & \cos\theta_s(t) \end{pmatrix}$

$$i\frac{\partial}{\partial t}\begin{bmatrix} \langle\nu_e|\Psi\rangle \\ \langle\bar{\nu}_e|\Psi\rangle \end{bmatrix} = \begin{bmatrix} X & C \\ C^* & -X \end{bmatrix}\begin{bmatrix} \langle\nu_e|\Psi\rangle \\ \langle\bar{\nu}_e|\Psi\rangle \end{bmatrix}$$

Transform this equation into the instantaneous energy basis

$$i\frac{\partial}{\partial t}\begin{bmatrix} \langle E_+(t)|\Psi\rangle \\ \langle E_-(t)|\Psi\rangle \end{bmatrix} = \left\{ \Delta\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} 0 & -i\dot{\theta}_s \\ +i\dot{\theta}_s & 0 \end{bmatrix} \right\} \begin{bmatrix} \langle E_+(t)|\Psi\rangle \\ \langle E_-(t)|\Psi\rangle \end{bmatrix}$$

where we define

$$\Delta \equiv \sqrt{X^2 + |C|^2}$$

$$\cos 2\theta_s = \frac{X}{\Delta} = \frac{X}{\sqrt{X^2+|C|^2}}$$

$$\sin 2\theta_s = \frac{-C}{\Delta} = \frac{-C}{\sqrt{X^2+|C|^2}}$$

adiabatic when $\dot{\theta}_s \rightarrow 0$, *i.e.*, $\gamma \gg 1$

with $\gamma \equiv \left.\frac{\Delta}{|\dot{\theta}_s|}\right|_{\text{resonance}} = \left.\left(\frac{2|C|^2}{\dot{X}}\right)\right|_{X=0}$

"resonance" at $X = 0 \quad \Rightarrow \theta_s = \frac{\pi}{4}$

X large, **positive** (*i.e.*, $Y_e > \frac{1}{3}$) $\Rightarrow \theta_s \rightarrow 0$, $|E_+\rangle \rightarrow |\nu_e\rangle$, $|E_-\rangle \rightarrow |\bar{\nu}_e\rangle$

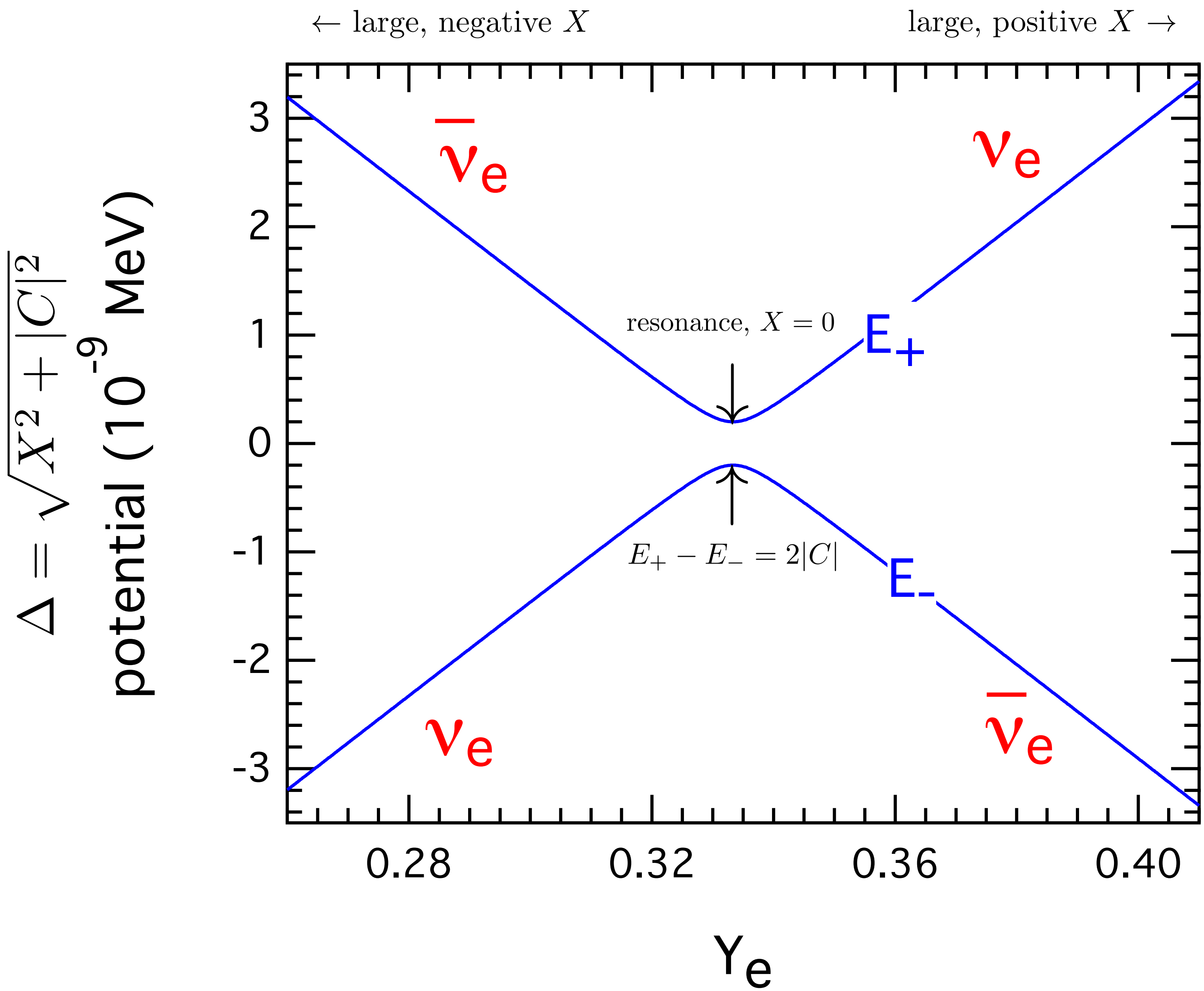
X large, **negative** (*i.e.*, $Y_e < \frac{1}{3}$) $\Rightarrow \theta_s \rightarrow \frac{\pi}{2}$, $|E_+\rangle \rightarrow -|\bar{\nu}_e\rangle$, $|E_-\rangle \rightarrow |\nu_e\rangle$

in adiabatic limit,
instantaneous energy eigenvalues are

$$E_+ = +\Delta = +\sqrt{X^2+|C|^2}$$

$$E_- = -\Delta = -\sqrt{X^2+|C|^2}$$

A. Vlasenko, GMF, V. Cirigliano arXiv:1406.6724



at relatively high matter (baryon) density

Spin Coherence –feedback crucial

neutrino-antineutrino inter-conversion?

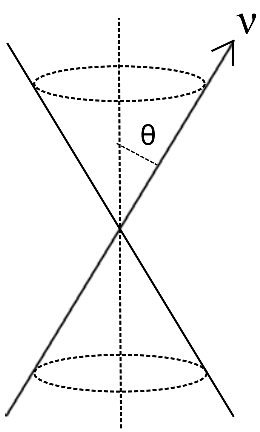
Not adiabatic in linear approximation,
but nonlinear feedback could be important

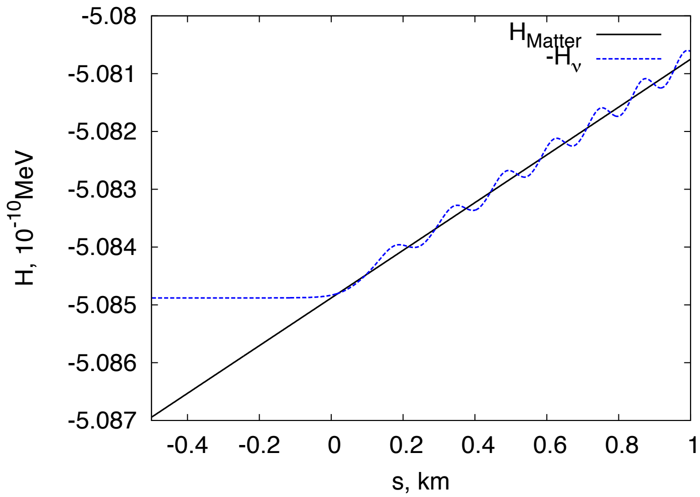
A. Vlasenko, G. M. Fuller, V. Cirigliano ArXiv:1406.6724

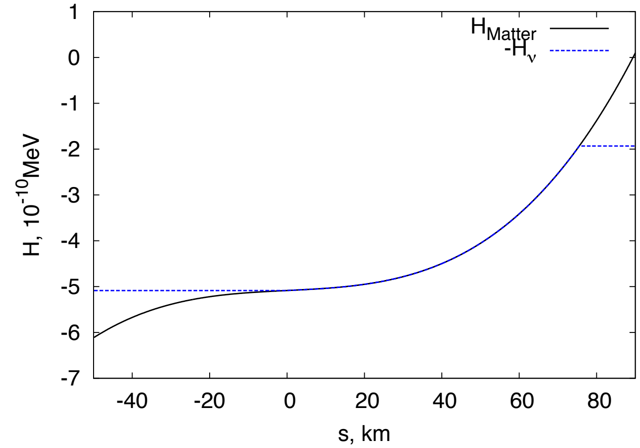
Need:
Neutrino mass; Majorana neutrinos; anisotopy in environment; Majorana phases

nonlinear feedback can *increase* adiabaticity

A. Vlasenko, G. M. Fuller, V. Cirigliano ArXiv:1406.6724





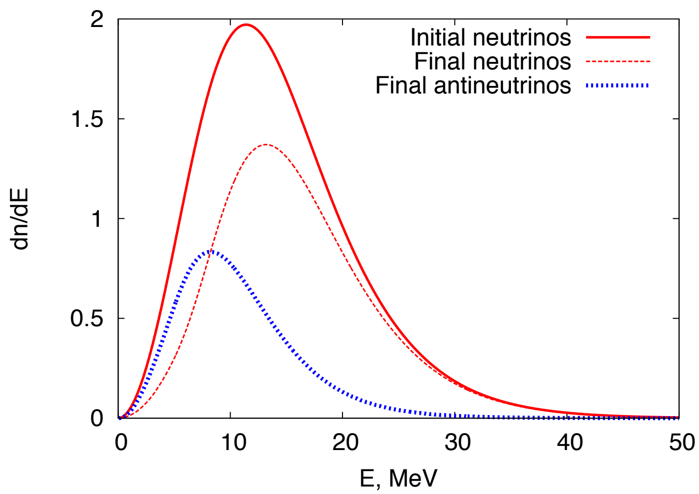


resonance where
$H = H_{\mathrm{matter}} + H_\nu = 0$

onset and **tracking** (and cancelation) of matter potential by the neutrino potential over the course of coherent neutrino-antineutrino transformation

Result for m_ν = 1 eV
Scale height 10 X 10^3 km

= coverts initial neutrino-only distribution
to neutrinos and antineutrinos



Now do 2X2

Structure of the complete Hamiltonian and density matrix in a 2-flavor bulb model:

$$\mathcal{F} = \begin{pmatrix} f & \phi \\ \phi^\dagger & \bar{f}^t \end{pmatrix} = \begin{pmatrix} \rho_{ee} & \rho_{ex} & \rho_{e\bar{e}} & \rho_{e\bar{x}} \\ \rho^\star_{ex} & \rho_{xx} & \rho_{x\bar{e}} & \rho_{x\bar{x}} \\ \rho^\star_{e\bar{e}} & \rho^\star_{x\bar{e}} & \rho_{\bar{e}\bar{e}} & \rho^\star_{\bar{e}\bar{x}} \\ \rho^\star_{e\bar{x}} & \rho^\star_{x\bar{x}} & \rho_{\bar{e}\bar{x}} & \rho_{\bar{x}\bar{x}} \end{pmatrix}$$

$$\mathcal{H} = \begin{pmatrix} H_{\rm VAC} + H_{\rm MAT} + H_\nu & H_{\rm spin-flip} \\ H^\dagger_{\rm spin-flip} & H_{\rm VAC} - H_{\rm MAT} - H^T_\nu \end{pmatrix}$$

$$H_{\rm VAC} = \frac{\Delta m^2}{2E} \begin{pmatrix} -c_{2\theta} & s_{2\theta} \\ s_{2\theta} & c_{2\theta} \end{pmatrix}$$

$$H_{\rm MAT} = \frac{G_F n_B}{\sqrt{2}} \left(1 - u V_{\rm OUT}\right) \begin{pmatrix} 3Y_e - 1 & 0 \\ 0 & Y_e - 1 \end{pmatrix} \qquad u = \hat{k} \cdot \hat{r}$$

Using the compact notation $\partial^\kappa \equiv n(p) \cdot \partial$, $\partial^i \equiv x^i(p) \cdot \partial$, the generalized Vlasov operators are

$$D_{\vec{p},x} F(\vec{p}, x) = \partial^\kappa F + \frac{1}{2|\vec{p}|} \left\{\Sigma^i, \partial^i F\right\} - \frac{1}{2} \left\{ \frac{\partial \Sigma^\kappa}{\partial \vec{x}}, \frac{\partial F}{\partial \vec{p}} \right\} \tag{1}$$

$$\bar{D}_{\vec{p},x} \bar{F}(\vec{p}, x) = \partial^\kappa \bar{F} - \frac{1}{2|\vec{p}|} \left\{\Sigma^i, \partial^i \bar{F}\right\} + \frac{1}{2} \left\{ \frac{\partial \Sigma^\kappa}{\partial \vec{x}}, \frac{\partial \bar{F}}{\partial \vec{p}} \right\}. \tag{2}$$

The physical meaning of D and $\bar{D}$ becomes more transparent by noting that they can be re-written as

$$\partial_t + 1/2\{\partial_{\vec{p}}\omega_\pm, \partial_{\vec{x}} \;\} - 1/2\{\partial_{\vec{x}}\omega_\pm, \partial_{\vec{p}} \;\}, \tag{3}$$

with $\omega_+ = |\vec{p}| + \Sigma^\kappa$ in D and $\omega_- = |\vec{p}| - \Sigma^\kappa$ in $\bar{D}$. Recalling that $\omega_\pm(\vec{p}) = |\vec{p}| \pm \Sigma^\kappa$ are the $O(\epsilon)$ neuutrino (+) and anti-neutrinos (−) hamiltonian operators, one sees that D and $\bar{D}$ generalize the total time-derivative operator $d_t = \partial_t + \dot{\vec{x}}\, \partial_{\vec{x}} + \dot{\vec{p}}\, \partial_{\vec{p}}$, with $\dot{\vec{p}} = -\partial_{\vec{x}}\, \omega$ and $\dot{\vec{x}} = \partial_{\vec{p}}\, \omega$, thus encoding drift and force term.

Use the following notation for the 2×2 blocks of the Majorana density matrix:

$$f = \begin{pmatrix} \rho_{ee} & \rho_{ex} \\ \rho_{ex}^{*} & \rho_{xx} \end{pmatrix} \qquad \phi = \begin{pmatrix} \rho_{e\bar{e}} & \rho_{e\bar{x}} \\ \rho_{x\bar{e}} & \rho_{x\bar{x}} \end{pmatrix} \tag{1}$$

with this the flavor $\times$ spin density matrix for two flavors $(\nu_e,\, \nu_x)$ is given by

$$\mathcal{F} = \begin{pmatrix} f & \phi \\ \phi^{\dagger} & \bar{f}^{T} \end{pmatrix} = \begin{pmatrix} \rho_{ee} & \rho_{ex} & \rho_{e\bar{e}} & \rho_{e\bar{x}} \\ \rho_{ex}^{\star} & \rho_{xx} & \rho_{x\bar{e}} & \rho_{x\bar{x}} \\ \rho_{e\bar{e}}^{\star} & \rho_{x\bar{e}}^{\star} & \rho_{\bar{e}\bar{e}} & \rho_{\bar{e}\bar{x}}^{\star} \\ \rho_{e\bar{x}}^{\star} & \rho_{x\bar{x}}^{\star} & \rho_{\bar{e}\bar{x}} & \rho_{\bar{x}\bar{x}} \end{pmatrix} . \tag{2}$$

Neglecting small corrections to the derivative (redshift terms, trajectory bending) and corrections to the hamiltonian of order G_F^2, the coherent QKEs for the density matrix are

$$D_{\vec{p},x}\, \mathcal{F}(\vec{p}, x) = -i\, [\mathcal{H}, \mathcal{F}] \tag{3}$$

The Hamiltonian contains vacuum, matter and neutrino contributions. In the presence of spacelike currents, the matter and neutrino contributions give a spin flip term. We break up the terms in the Hamiltonian as follows:

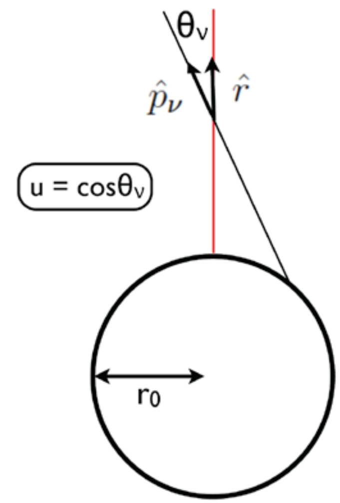
$$\mathcal{H} = \mathcal{H}_{\text{vac}} + \mathcal{H}_{\text{matter}} + \mathcal{H}_{\nu} + \mathcal{H}_{\text{sf}} \tag{4}$$

The traceless part of the vacuum Hamiltonian is

$$\mathcal{H}_{\text{vac}} = \frac{\delta m^2}{2E} \begin{pmatrix} -c_{2\theta} & s_{2\theta} & 0 & 0 \\ s_{2\theta} & c_{2\theta} & 0 & 0 \\ 0 & 0 & -c_{2\theta} & s_{2\theta} \\ 0 & 0 & s_{2\theta} & c_{2\theta} \end{pmatrix} \tag{1}$$

Assuming spherical symmetry and the absence of muons, the matter Hamiltonian is

$$\mathcal{H}_{\text{matter}} = \frac{G_F n_B}{\sqrt{2}} (1 - uV_{\text{out}}) \begin{pmatrix} 3Y_e - 1 & 0 & 0 & 0 \\ 0 & Y_e - 1 & 0 & 0 \\ 0 & 0 & -(3Y_e - 1) & 0 \\ 0 & 0 & 0 & -(Y_e - 1) \end{pmatrix} \tag{2}$$



Here, n_B is the baryon number density, $Y_e = (n_e - n_{\bar{e}})/n_B$ is the electron lepton number fraction, $u = \cos\theta_\nu$ where θ_ν is the angle between the neutrino trajectory and the outward radial direction and V_{out} is the (radial) matter outflow speed as a fraction of the speed of light.

The neutrino Hamiltonian is

$$
\begin{aligned}
\mathcal{H}_\nu &= 2\sqrt{2}G_F \begin{pmatrix} 2J^0_{ee}+J^0_{xx} & J^0_{ex} & 0 & 0 \\ J^{0\star}_{ex} & 2J^0_{xx}+J^0_{ee} & 0 & 0 \\ 0 & 0 & -2J^0_{ee}-J^0_{xx} & -J^{0\star}_{ex} \\ 0 & 0 & -J^0_{ex} & -2J^0_{xx}-J^0_{ee} \end{pmatrix} \\
&- 2\sqrt{2}G_F u \begin{pmatrix} 2J^r_{ee}+J^r_{xx} & J^r_{ex} & 0 & 0 \\ J^{r\star}_{ex} & 2J^r_{xx}+J^r_{ee} & 0 & 0 \\ 0 & 0 & -2J^r_{ee}-J^r_{xx} & -J^{r\star}_{ex} \\ 0 & 0 & -J^r_{ex} & -2J^r_{xx}-J^r_{ee} \end{pmatrix} \qquad (1)
\end{aligned}
$$

The time-like and radial components of the neutrino current J^0_{IJ} and J^r_{IJ} are:

$$
J^0_{IJ} = \int \frac{E'^2 dE'}{2\left(2\pi\right)^2} \int_{u_{\min}}^{1} du' \left[\rho_{IJ}\left(E',u'\right) - \rho_{\bar{I}\bar{J}}\left(E',u'\right)\right] \qquad (2)
$$

$$
J^r_{IJ} = \int \frac{E'^2 dE'}{2\left(2\pi\right)^2} \int_{u_{\min}}^{1} du' \; u' \left[\rho_{IJ}\left(E',u'\right) - \rho_{\bar{I}\bar{J}}\left(E',u'\right)\right] \; + \; \delta J^r_{IJ} \qquad (3)
$$

with

$$
\delta J^r = -\int \frac{E' dE'}{4\left(2\pi\right)^2} \int_{u_{\min}}^{1} du' \sqrt{1-u'^2} \left[m^\dagger \left(\phi^\dagger + \phi^\star\right) + \left(\phi + \phi^T\right) m\right] \; . \qquad (4)
$$

The last term is proportional to m/E times the spin coherence.

2×2 blocks in the 4×4 operator $\mathcal{H}_{\rm sf}$ that determines the evolution of neutrino flavor and spin depend on matter and neutrino potentials and on the neutrino mass matrix

$$\mathcal{H}_{\rm sf} = \begin{pmatrix} 0 & \left(H_{\rm sf}^{\rm matter} + H_{\rm sf}^{\nu}\right) \frac{m^\star}{E} + \frac{m^\star}{E} \left(H_{\rm sf}^{\rm matter} + H_{\rm sf}^{\nu}\right)^T \\ \left(\left(H_{\rm sf}^{\rm matter} + H_{\rm sf}^{\nu}\right) \frac{m^\star}{E} + \frac{m^\star}{E} \left(H_{\rm sf}^{\rm matter} + H_{\rm sf}^{\nu}\right)^T\right) & 0 \end{pmatrix}$$

where the 2×2 **Majorana mass matrix** satisfies $m = m^T$ and the 2×2 matrices $H_{\rm s.f.}^{\rm matter}$ and $H_{\rm s.f.}^{\nu}$ are

$$H_{\rm sf}^{\rm matter} = -\frac{G_F n_B}{2\sqrt{2}} V_{\rm out} \sqrt{1-u^2} \begin{pmatrix} 3Y_e - 1 & 0 \\ 0 & Y_e - 1 \end{pmatrix} \quad (1)$$

$$H_{\rm sf}^{\nu} = -\sqrt{2} G_F \sqrt{1-u^2} \begin{pmatrix} 2J_{ee}^r + J_{xx}^r & J_{ex}^r \\ J_{ex}^{r\star} & 2J_{xx}^r + J_{ee}^r \end{pmatrix} . \quad (2)$$

Neutrino Mass Matrix $m = U^* \, m_d \, U^\dagger$

where $m_d = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \qquad U = \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix} \times \begin{pmatrix} 1 & 0 \\ 0 & e^{i\alpha/2} \end{pmatrix}$

In terms of the observable parameters $\delta m^2 = m_2^2 - m_1^2 > 0$
and $m_0 \equiv (1/2)(m_1 + m_2)$ (so that $m_{1,2} = m_0 \mp \delta m^2/(4m_0)$) we find:

$$m = m_0 \begin{pmatrix} c_\theta^2 + e^{-i\alpha} s_\theta^2 & (e^{-i\alpha} - 1) s_\theta c_\theta \\ (e^{-i\alpha} - 1) s_\theta c_\theta & s_\theta^2 + e^{-i\alpha} c_\theta^2 \end{pmatrix} + \frac{\delta m^2}{4m_0} \begin{pmatrix} -(c_\theta^2 - e^{-i\alpha} s_\theta^2) & (e^{-i\alpha} + 1) s_\theta c_\theta \\ (e^{-i\alpha} + 1) s_\theta c_\theta & c_\theta^2 e^{-i\alpha} - s_\theta^2 \end{pmatrix}$$

This shows the dependence of the mass matrix on the absolute neutrino mass scale, m_0, the effective vacuum mixing angle θ, and the Majorana phase α. Clearly the overall size of $\mathcal{H}_{\rm sf}$ grows with m_0. Moreover, in the degenerate limit $m_0 \gg \sqrt{\delta m^2}$, in which the first term in this equation dominates, a non-zero Majorana phase α can induce a significant new source of flavor (and spin) mixing.

The neutrino mass eigenvalues m_1 and m_2 come into the amplitude for neutrino spin flip ($\nu \rightleftharpoons \bar{\nu}$) in the 2×2 coherent spin transformation case.

The single majorana phase (α) in the 2×2 case also figures in this amplitude.

Neutrino Mass Matrix $m = U^* \, m_d \, U^\dagger$

where $m_d = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \qquad U = \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix} \times \begin{pmatrix} 1 & 0 \\ 0 & e^{i\alpha/2} \end{pmatrix}$

In terms of the observable parameters $\delta m^2 = m_2^2 - m_1^2 > 0$
and $m_0 \equiv (1/2)(m_1 + m_2)$ (so that $m_{1,2} = m_0 \mp \delta m^2/(4m_0)$) we find:

$$m = m_0 \begin{pmatrix} c_\theta^2 + e^{-i\alpha} s_\theta^2 & (e^{-i\alpha} - 1) s_\theta c_\theta \\ (e^{-i\alpha} - 1) s_\theta c_\theta & s_\theta^2 + e^{-i\alpha} c_\theta^2 \end{pmatrix} + \frac{\delta m^2}{4m_0} \begin{pmatrix} -(c_\theta^2 - e^{-i\alpha} s_\theta^2) & (e^{-i\alpha} + 1) s_\theta c_\theta \\ (e^{-i\alpha} + 1) s_\theta c_\theta & c_\theta^2 e^{-i\alpha} - s_\theta^2 \end{pmatrix}$$

This shows the dependence of the mass matrix on the absolute neutrino mass scale, m_0, the effective vacuum mixing angle θ, and the Majorana phase α.

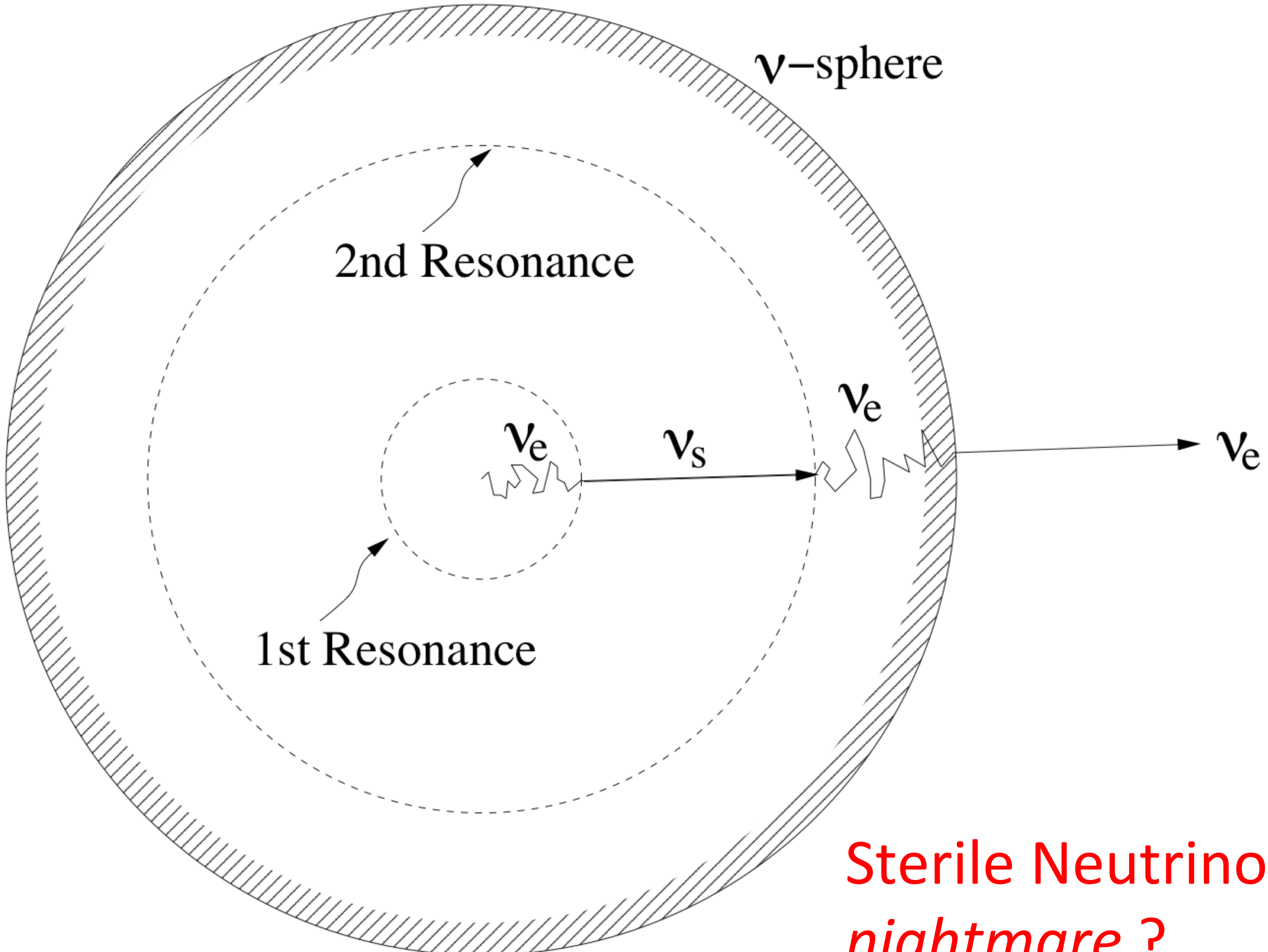
Note the dependence on absolute neutrino rest mass and Majorana phases

Could there be a ν-$\bar{\nu}$ transformation signature in a detected supernova neutrino burst signal?

Could tell if neutrinos are Majorana or Dirac

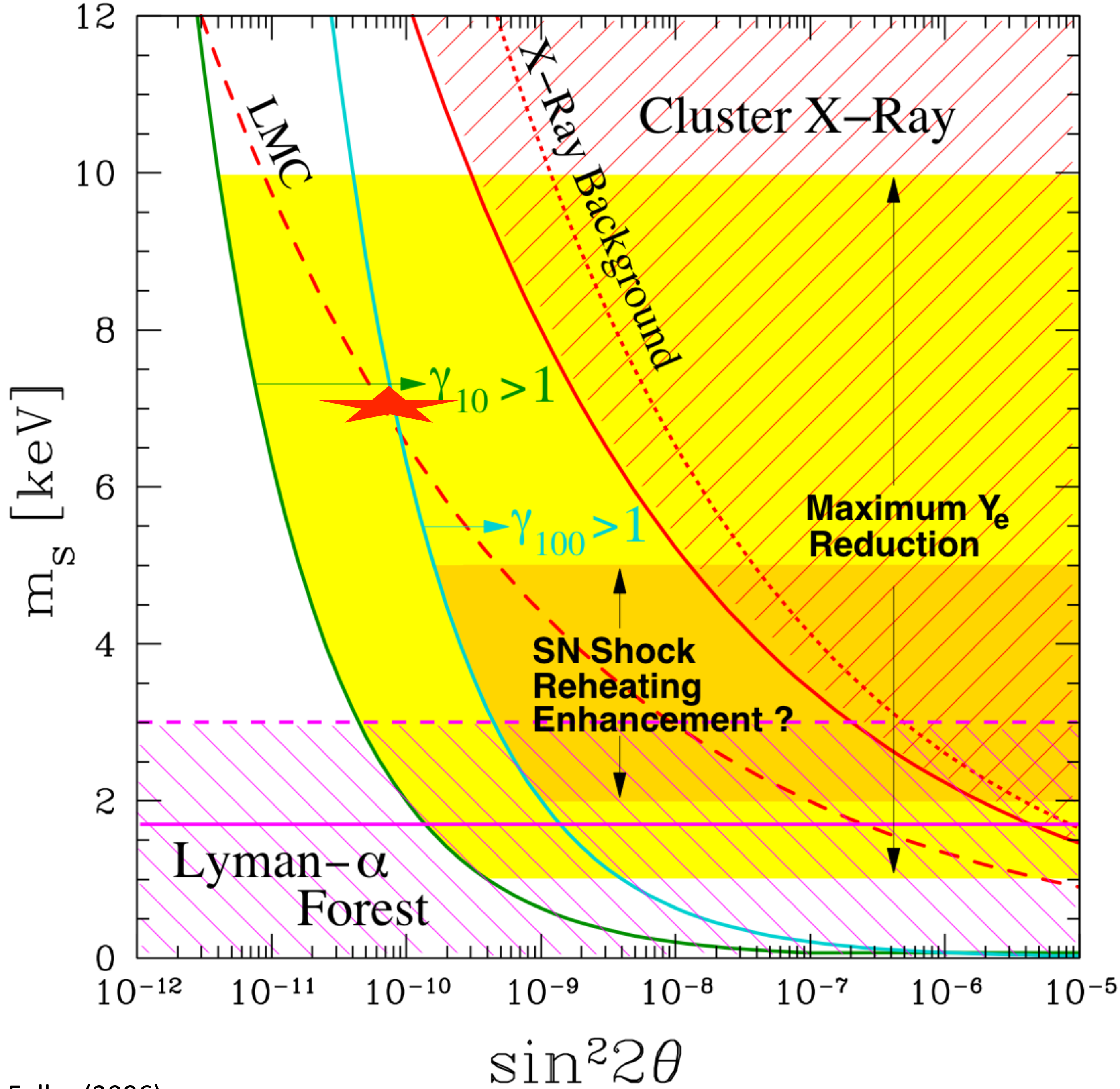
Probe neutrino absolute masses and Majorana Phases

(both complementary to $0\nu2\beta$-decay experiments and cosmological observations)



Sterile Neutrino
nightmare ?

J. Hidaka & G. M. Fuller (2006)



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see review by Alex Kusenko: *Physics Reports* **481**, 1 (2009)

active-active neutrino scattering-induced decoherence

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→ Largely eliminated by the X-ray observations

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M. Shaposhnikov, Nucl. Phys. B **763**, 49 (2007)

~~**(1) Quantum Mechanical Limit:** Dodelson & Widrow 1994~~

~~active neutrino scattering-induced de-coherence produces a relic density of sterile neutrinos -- *picks out keV scale rest masses, small vacuum mixing angles*~~

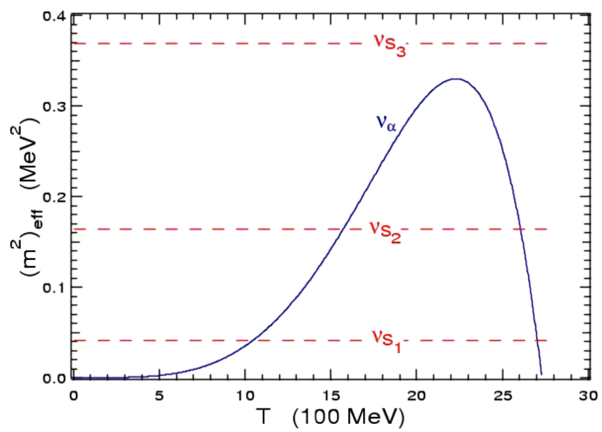
(2) Lepton number-driven resonant production: Shi & Fuller 1998; Abazajian, Fuller, Patel 2001; Abazajian '14

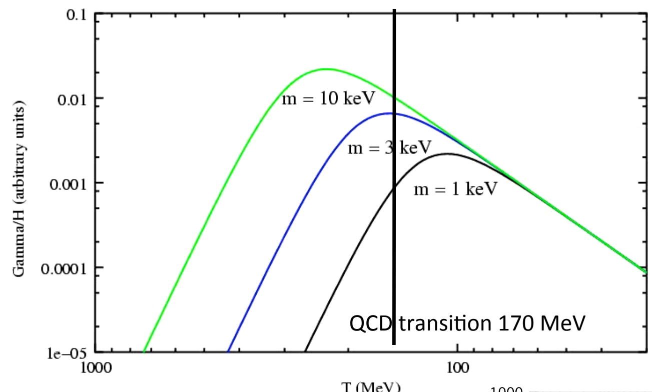
Like MSW, initial lepton number partially converted to a relic sterile neutrino population

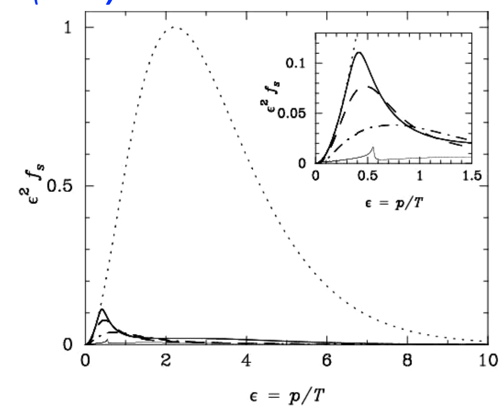
-- *can work for smaller mixing angles, colder sterile neutrino relic energy spectrum*

-- *sterile neutrinos may allow you to make the lepton number*

*e.g., Asaka & Shaposhnikov , "The nuMSM, dark matter, and baryon asymmetry", PLB **620**, 17 (2005)*







(3) Higgs decay; Dilution:

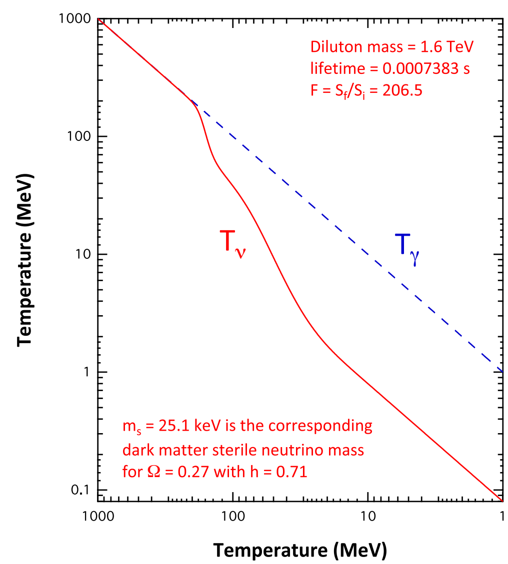
e.g., Asaka, Shaposhnikov, Kusenko (2006);

F. Bezrukov, D. Gorbunov arXiv:1403.4638

Fuller, Kishimoto, Kusenko, Patwardhan (2014)

thermalize or partially thermalize steriles very early, then dilute them down to a DM relic density

-- *can produce relic sterile neutrino populations which are CDM for rest masses ~ 1 keV to ~ 10 MeV, with extremely small vacuum mixing angles*

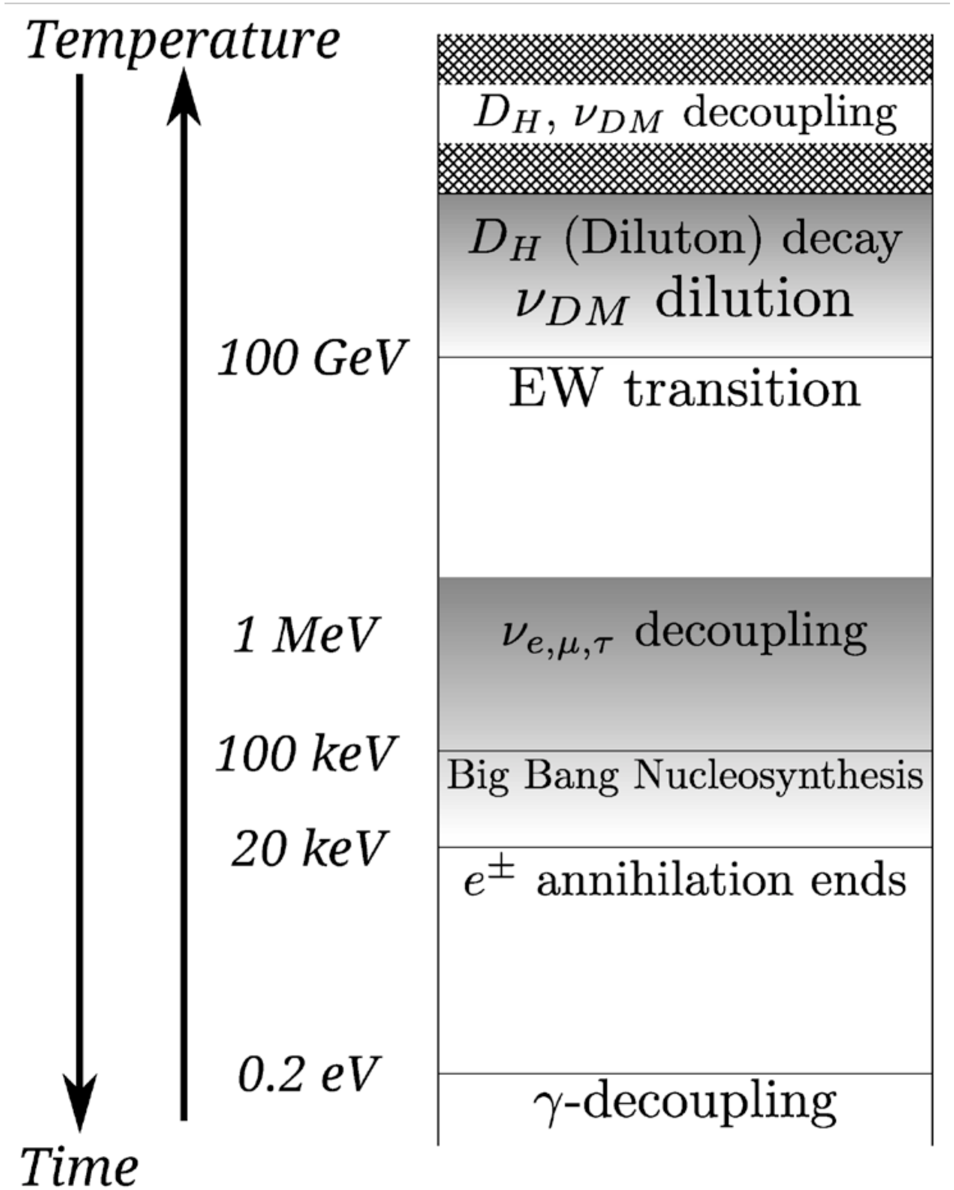


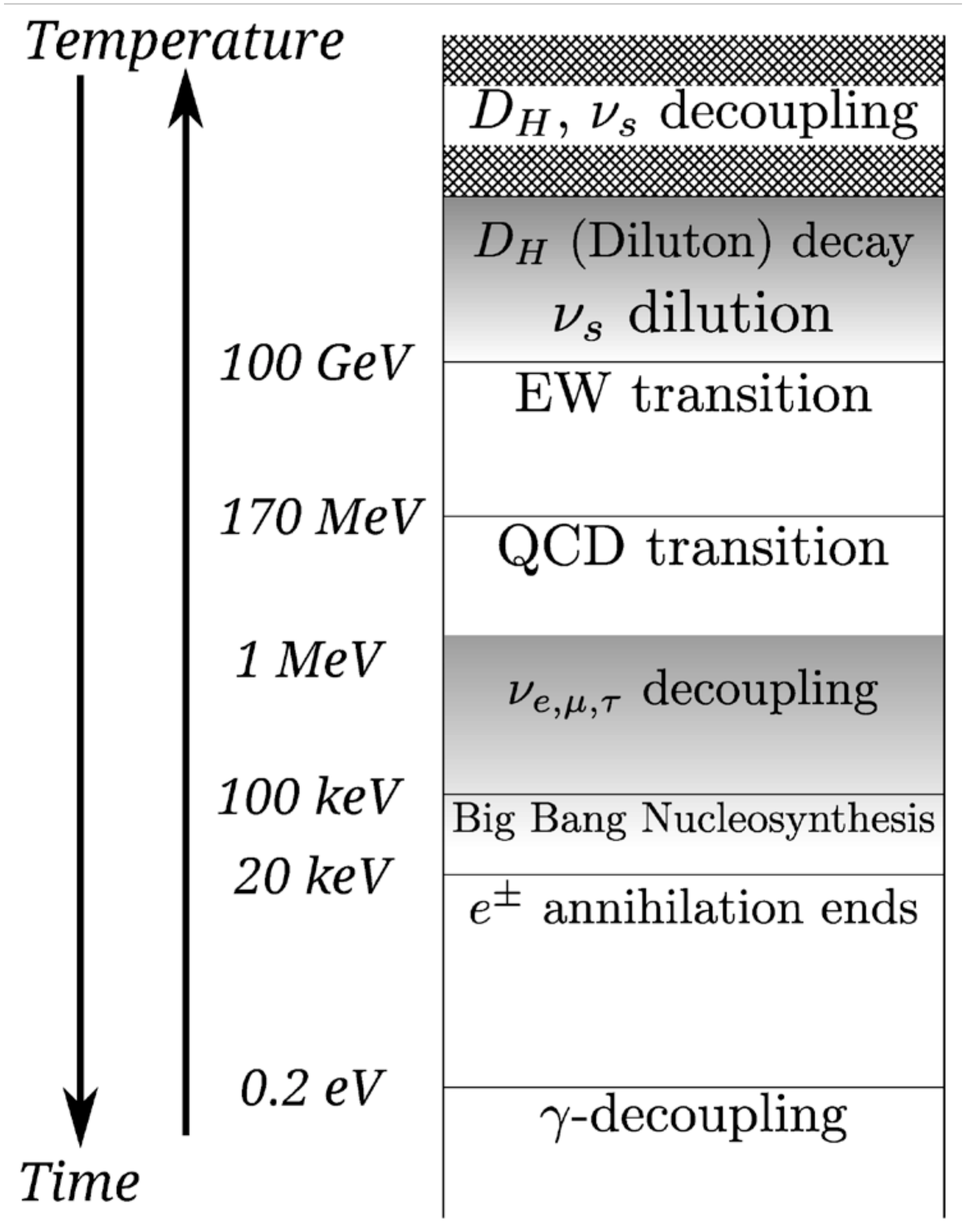
Dark Matter Sterile Neutrino Production Mechanisms

Production mechanism	Dodelson Widrow	Resonant (medium enhanced) Shi & Fuller	ν MSM (involves resonant porduction)	CDM sterile mass: $\sim 1\,\mathrm{keV}$ to $\sim 1\,\mathrm{MeV}$ Dilution GMF, Patwardhan Kusenko 2014	Gauge symmetry breaking (involves dilution)	Scalar decay
DM character	Warm	Cool	Cool	Cold	Cold	Cold
Tooth fairies	None	Lepton number $L_{\nu_e,\nu_\mu,\nu_\tau} \sim 10^{-4}$?	Heavy steriles N_2, N_3, nearly degenerate	heavy particle Additional sterile ? (diluton)	Heavy steriles N_2, N_3, additional gauge symmetry	SU(2) scalar
Warts	Cannot constitute all of DM	Large early Lepton # affecting baryogenesis?	Collider considerations?	1. Requires thermalization, 2. Issues with bayrogenesis	What happens to N_3?	Collider considerations?
Predictions/ handles	Constrained by X-ray + Ly-α data	Lepton # constraints from BBN, CMB ?	1. N_2, N_3 detection, 2. Lepton# constraints from BBN, CMB	??? PeV scale Physics ?	???	SU(2) scalar detection

Table 1: Different models for sterile neutrino DM production in the early universe

A. Patwardhan, GMF, C. T. Kishimoto, A. Kusenko (2015)


Temperature
Time
100 GeV
1 MeV
100 keV
20 keV
0.2 eV
D_H, ν_{DM} decoupling
D_H (Diluton) decay
ν_{DM} dilution
EW transition
$\nu_{e,\mu,\tau}$ decoupling
Big Bang Nucleosynthesis
$e^{\pm}$ annihilation ends
γ-decoupling

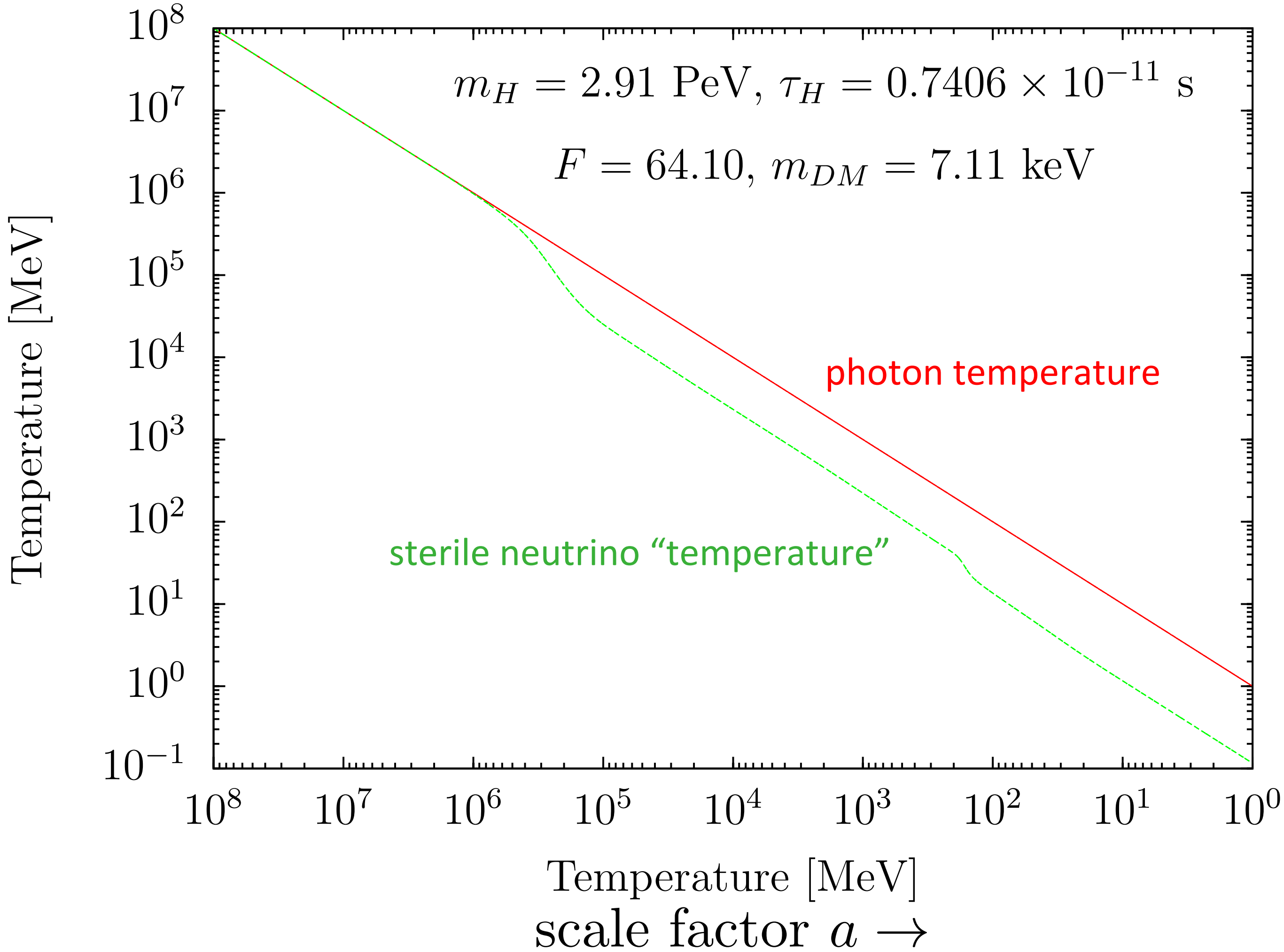


Two particles:

(1) A heavy "*diluton*" D_H that decays out of equilibrium and generates entropy, diluting the number density to entropy ratio of any decoupled particles

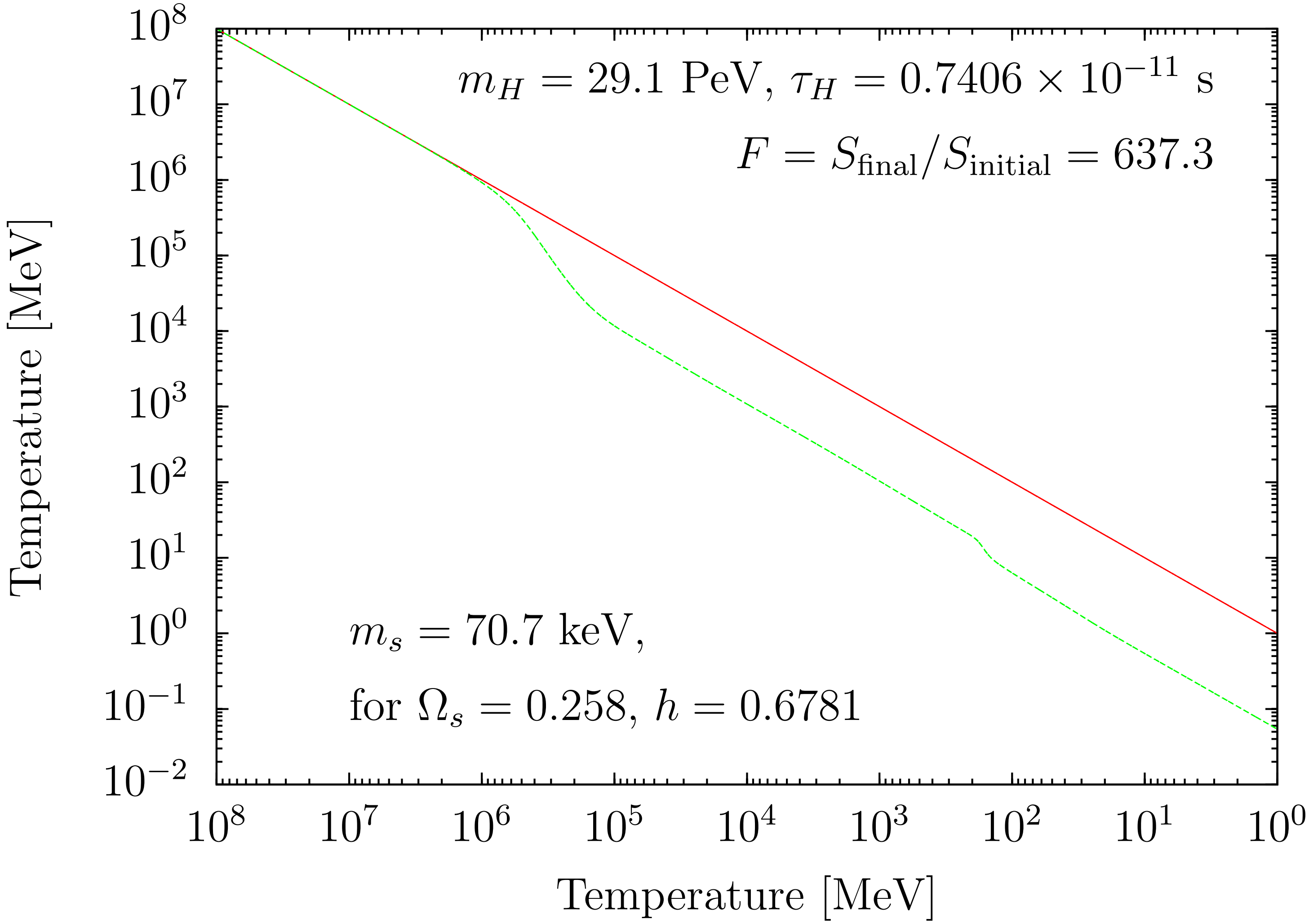
(2) A decoupled sterile neutrino ν_s which will become the dark matter parrticle

Dilution by decay of long-lived 2.91 PeV rest mass particle (supersymmetric?)
- *already-decoupled CDM sterile neutrino has mass 7.11 keV (mixing angle arbitrary)*



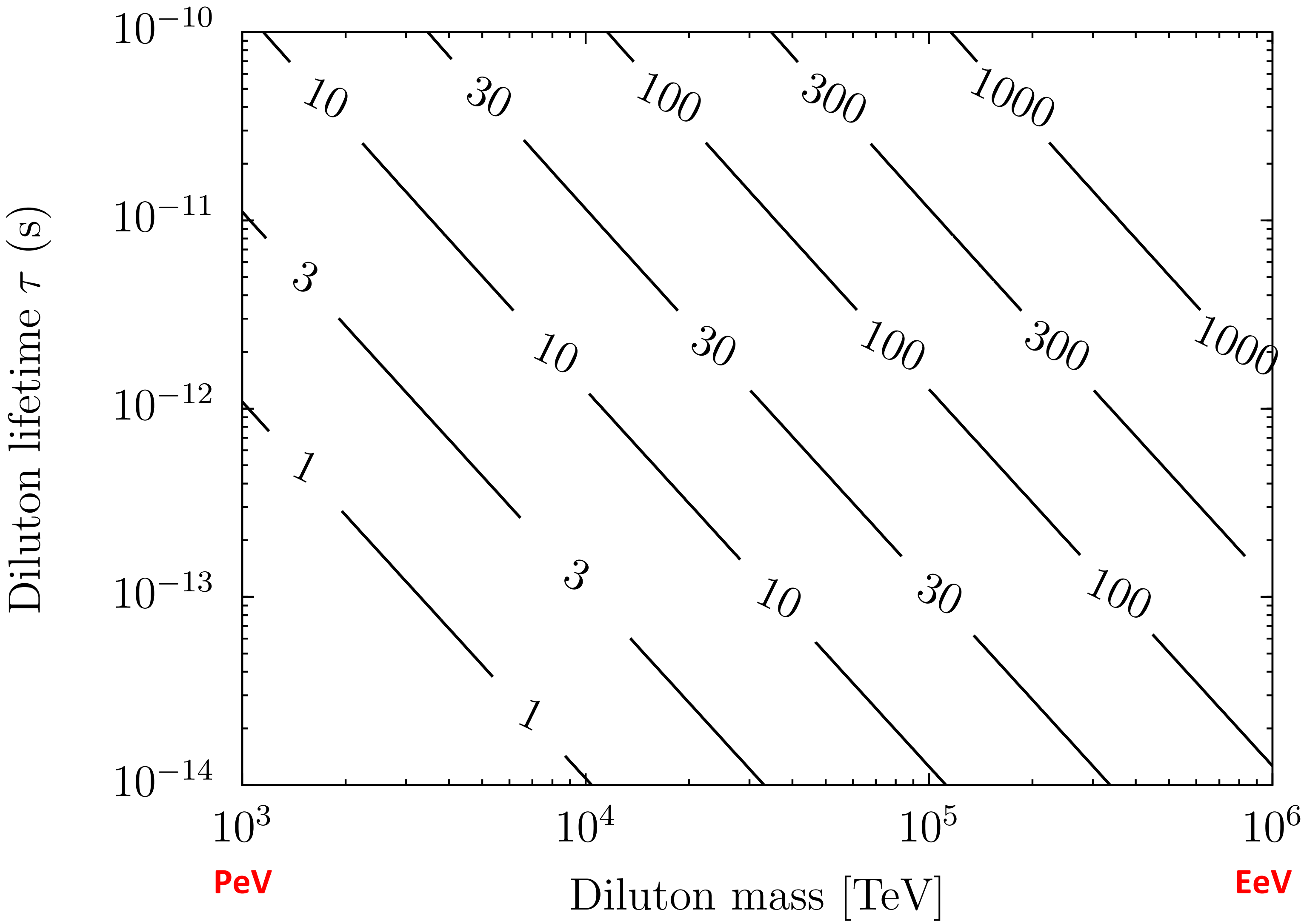
A. Patwardhan, GMF, C. Kishimoto, A. Kusenko (2015)

. . . can make higher rest mass sterile neutrino dark matter



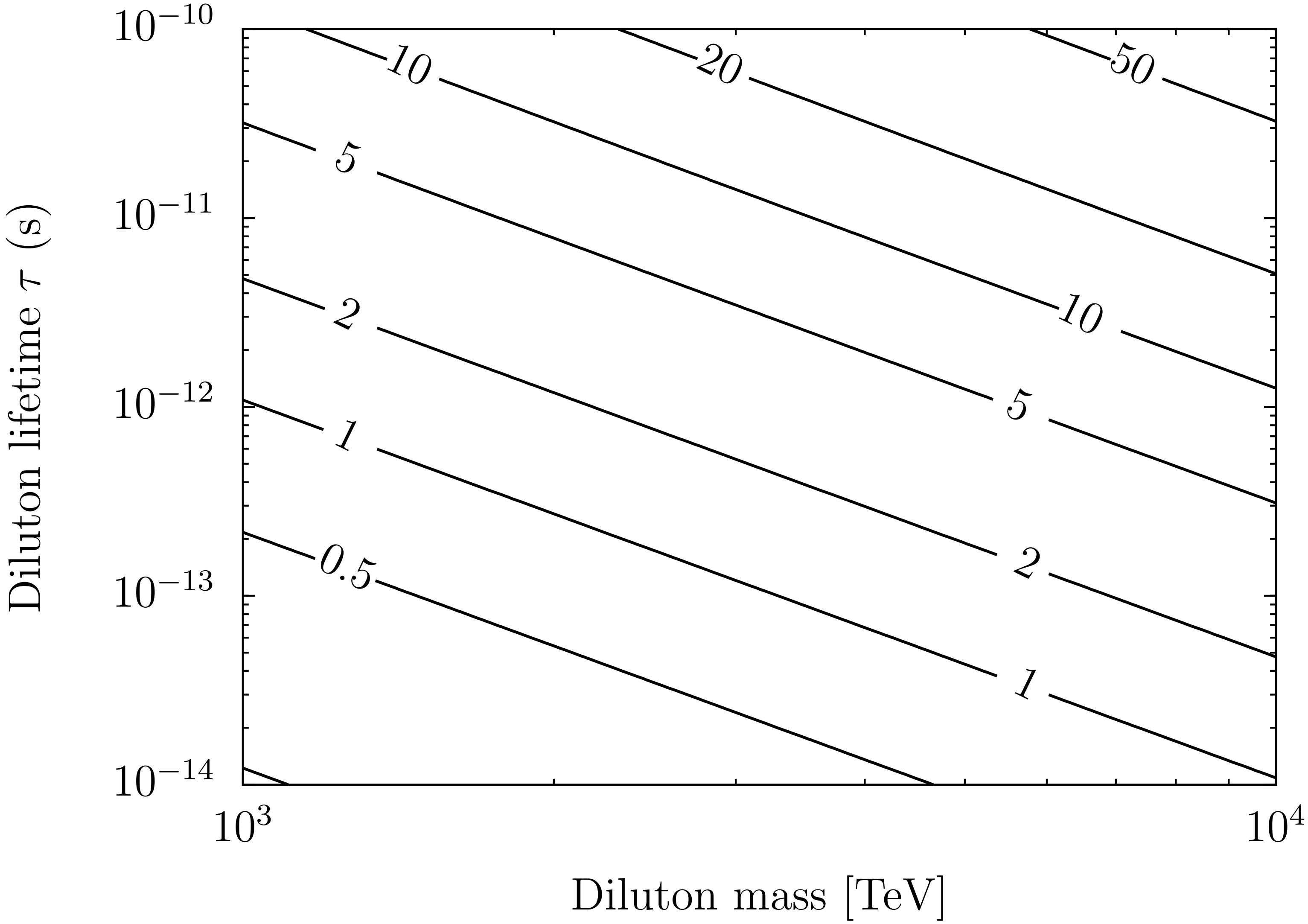
A. Patwardhan, GMF, C. Kishimoto, A. Kusenko (2015)

Contours of sterile neutrino rest mass, m_s , in keV, in the Diluton mass—lifetime plane, when the sterile neutrinos comprise *all* of the dark matter

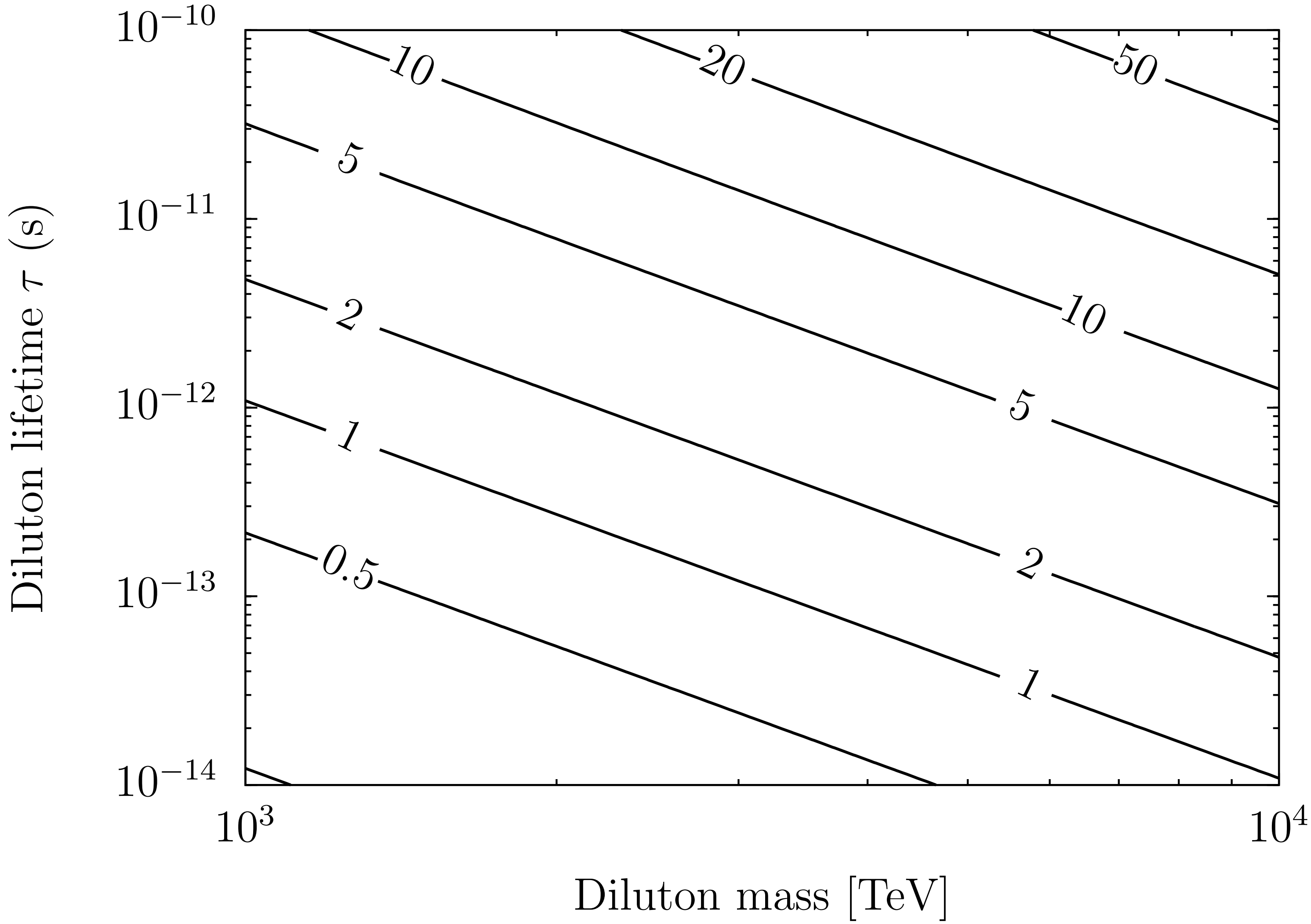


PeV **EeV**

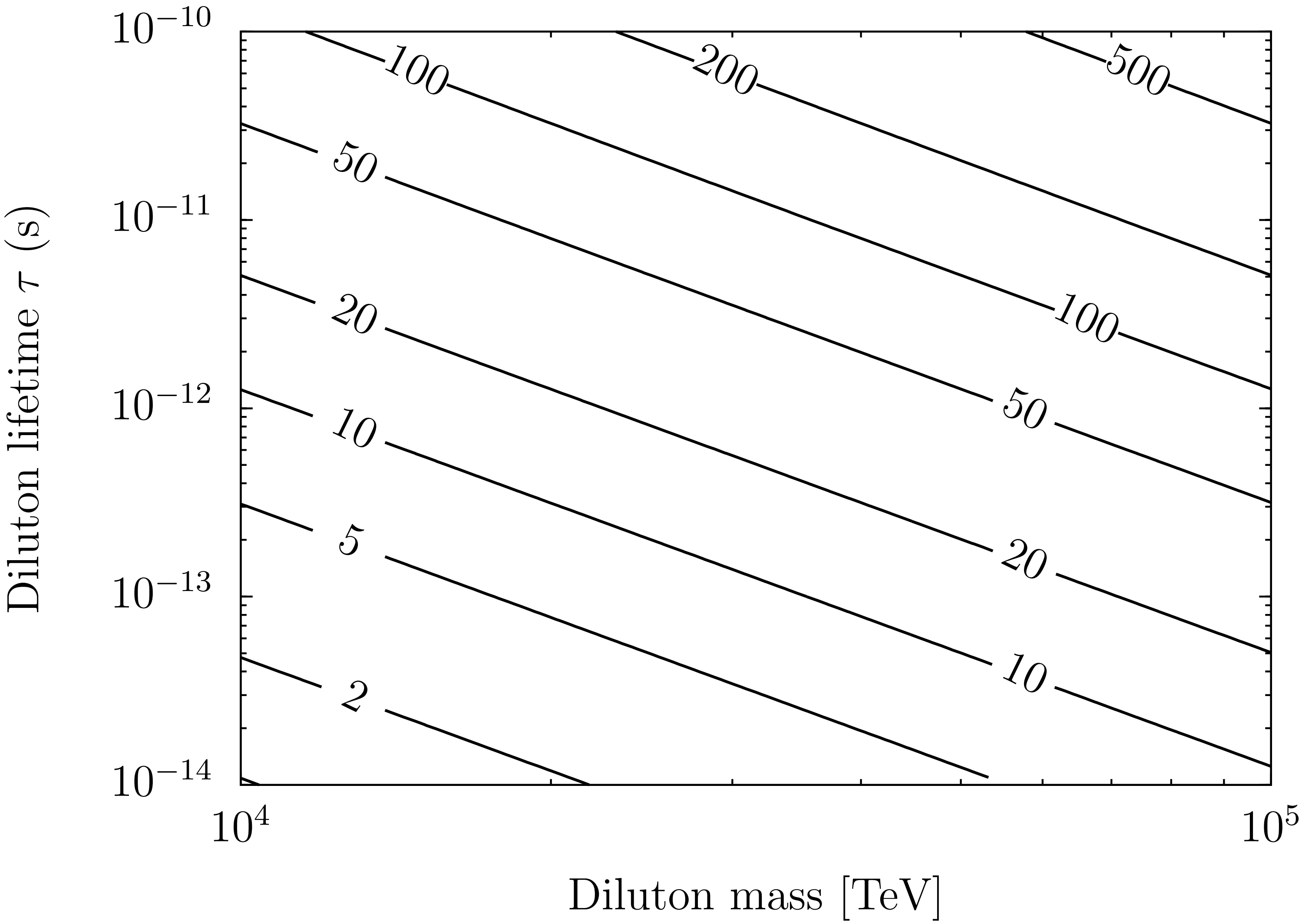
A. Patwardhan, GMF, C. T. Kishimoto, A. Kusenko (2015)



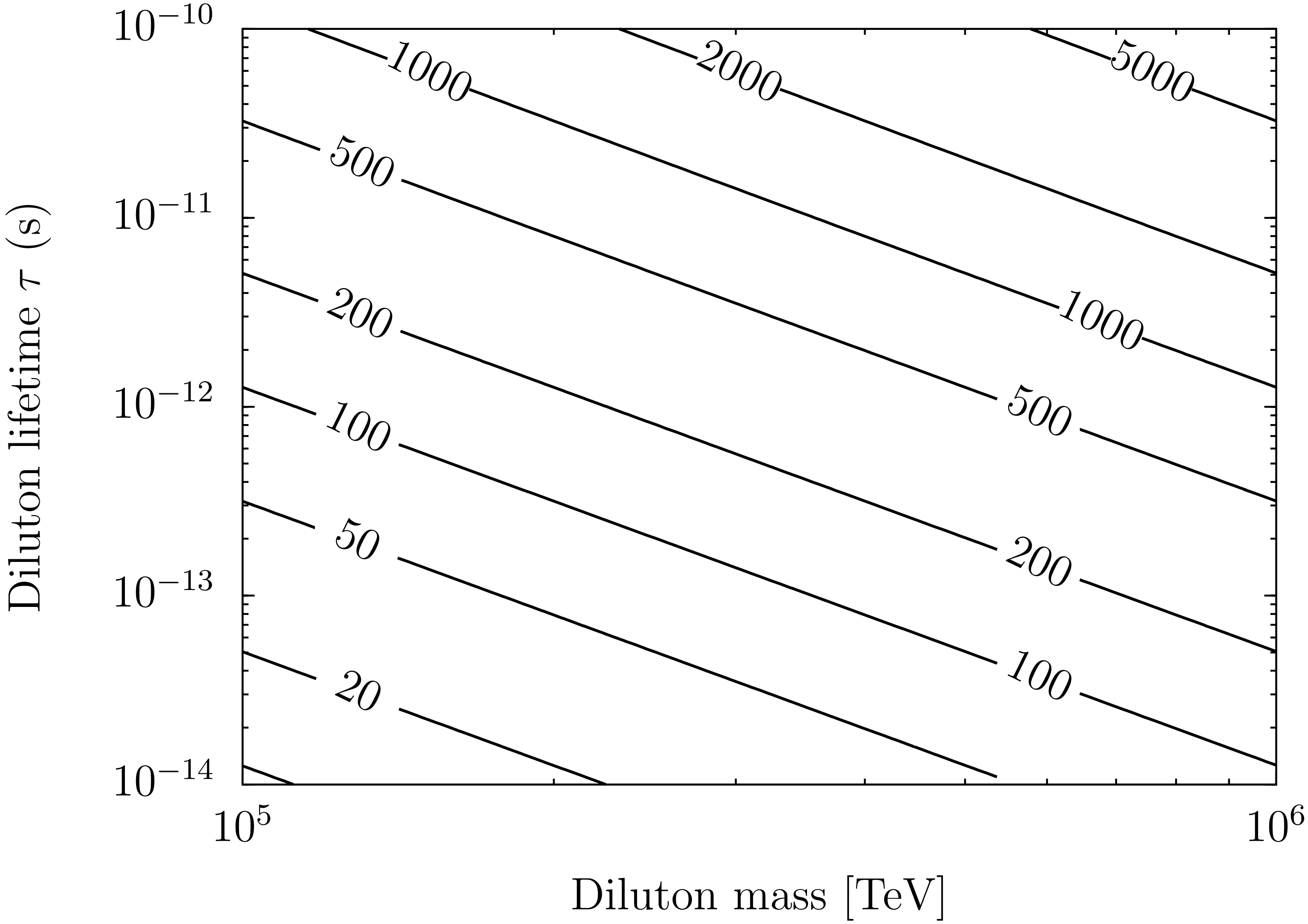
Diluton lifetime τ (s)
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
10^{3}
10^{4}
Diluton mass [TeV]
10
20
50
5
10
2
5
1
2
0.5
1



10
20
50
5
10
2
5
1
2
0.5
1
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
10^{3}
10^{4}
Diluton lifetime τ (s)
Diluton mass [TeV]

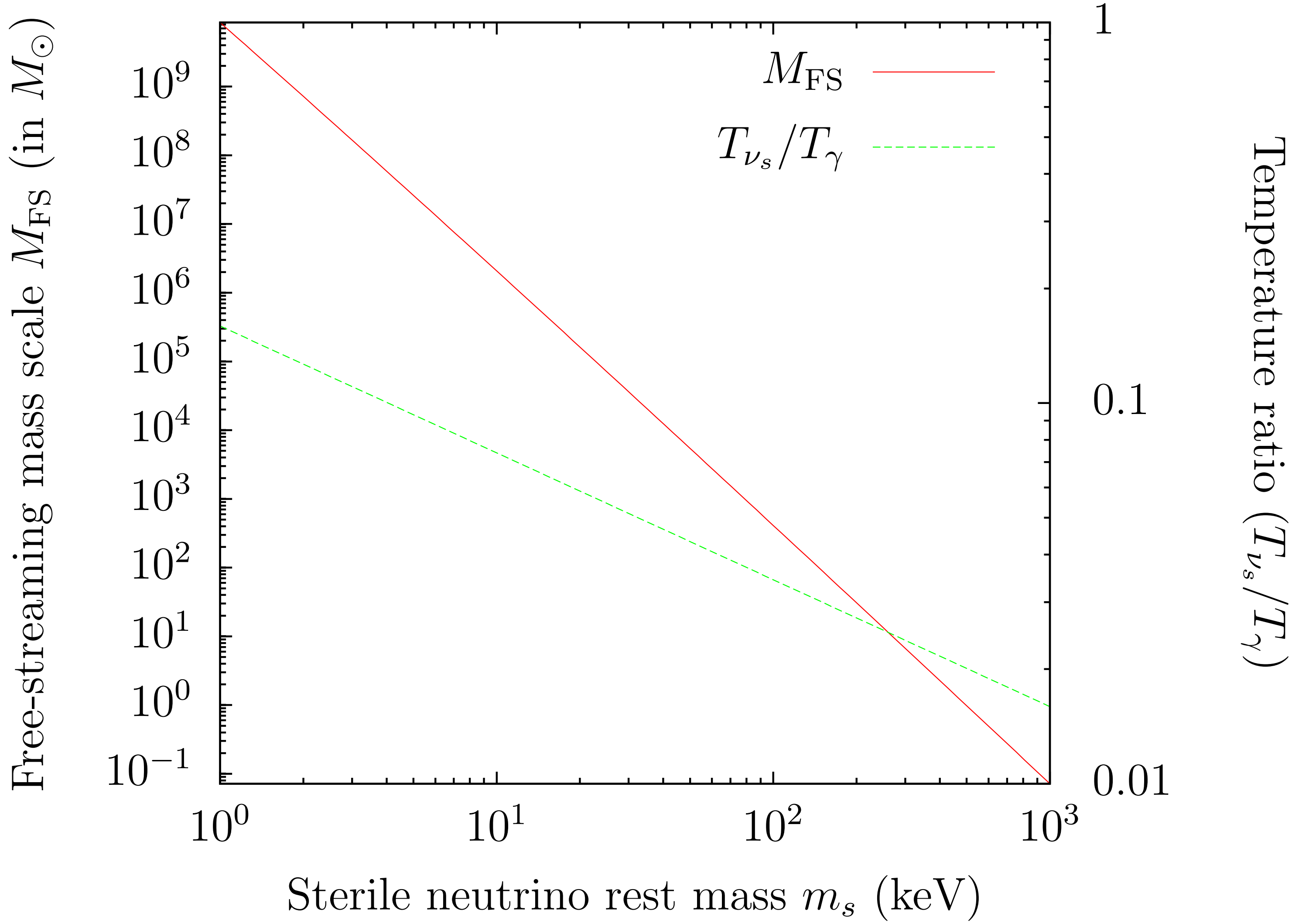


Diluton lifetime τ (s)
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
10^{4}
10^{5}
Diluton mass [TeV]
100
200
500
50
100
20
50
10
20
5
10
2

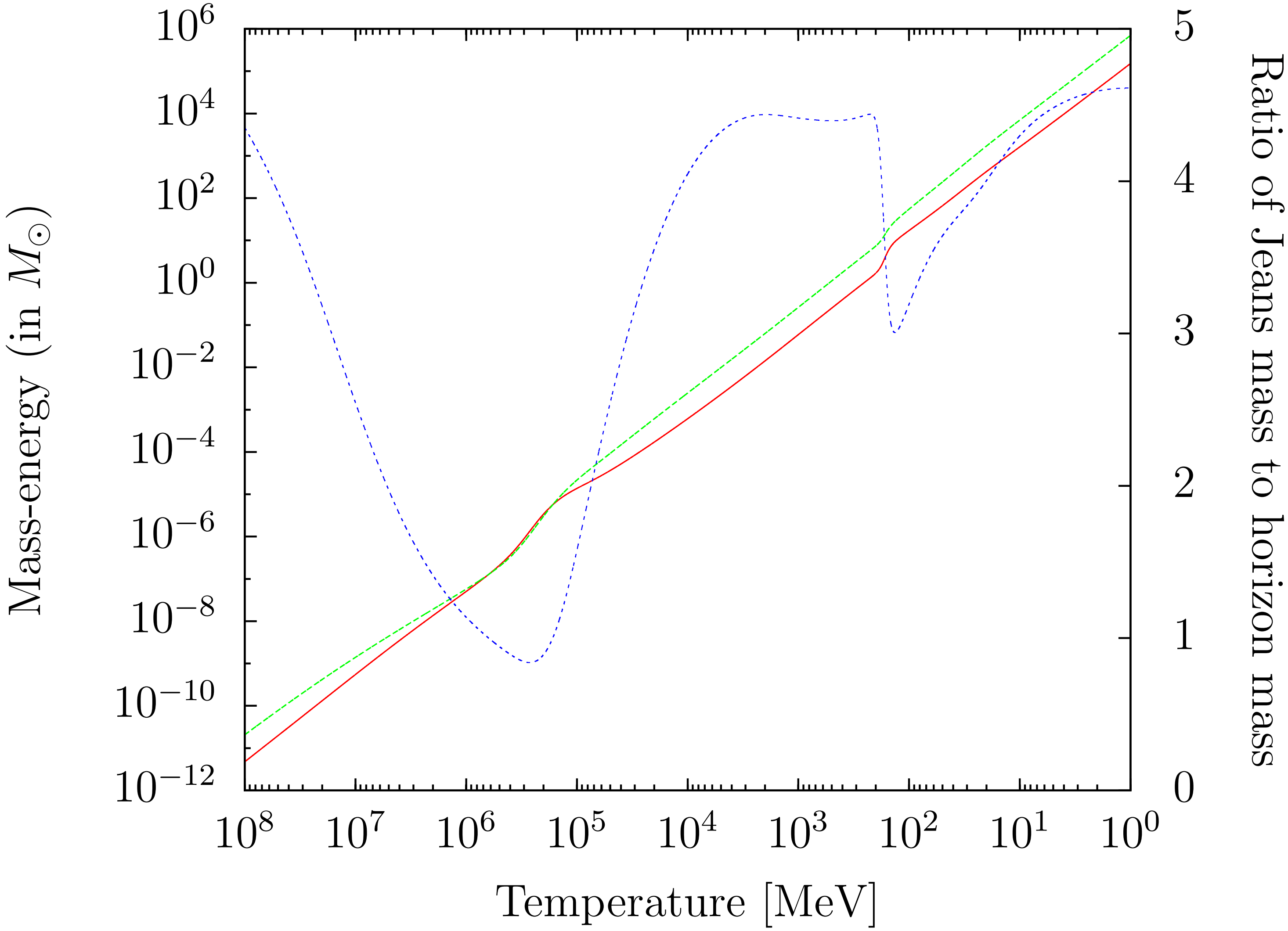


10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
10^{5}
10^{6}
Diluton lifetime τ (s)
Diluton mass [TeV]
1000
2000
5000
500
1000
200
500
100
200
50
100
20

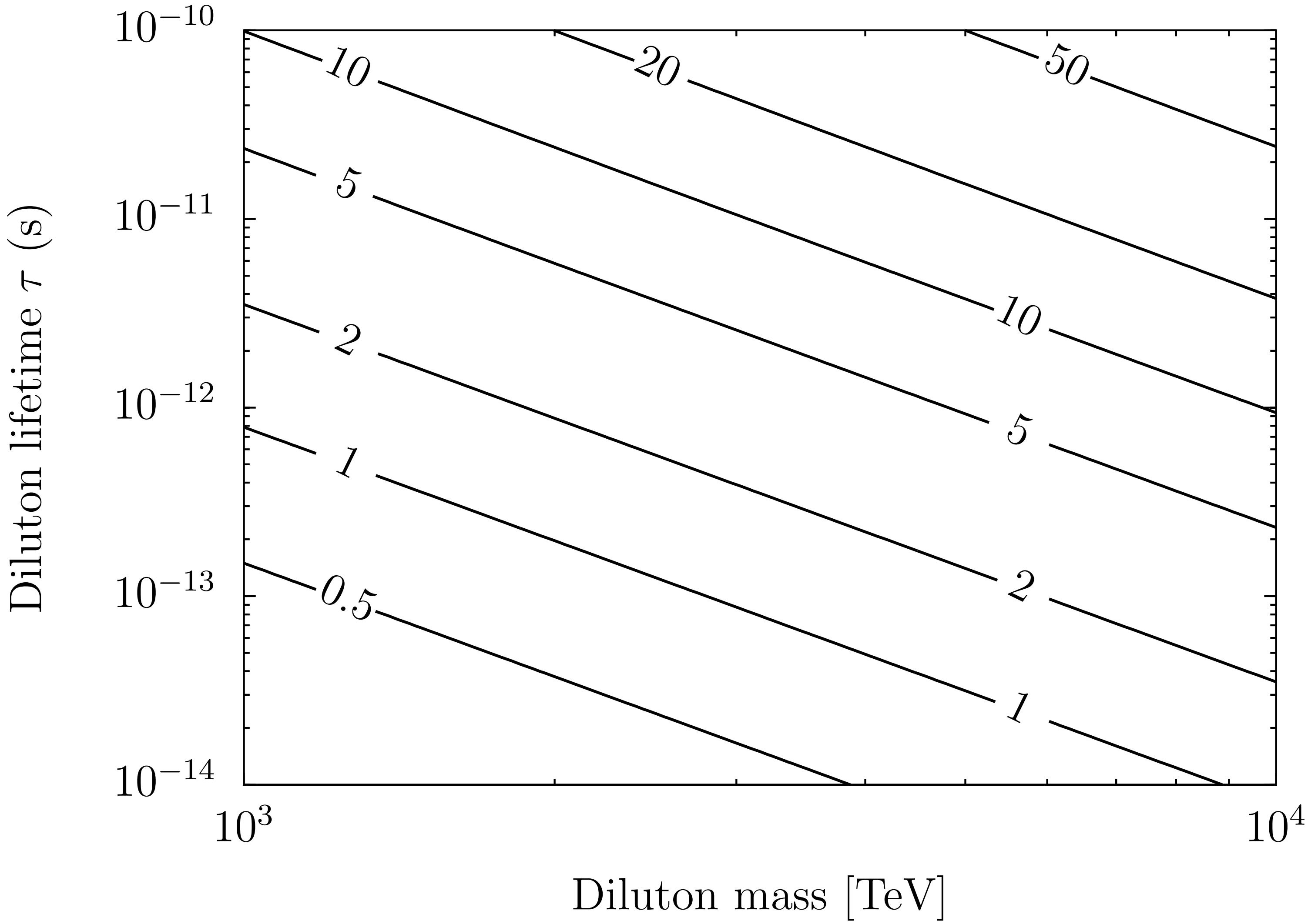
Although these sterile neutrinos have FD-Black-body *shaped* relic energy spectra, they are inherently very ***COLD*** as a consequence of dilution



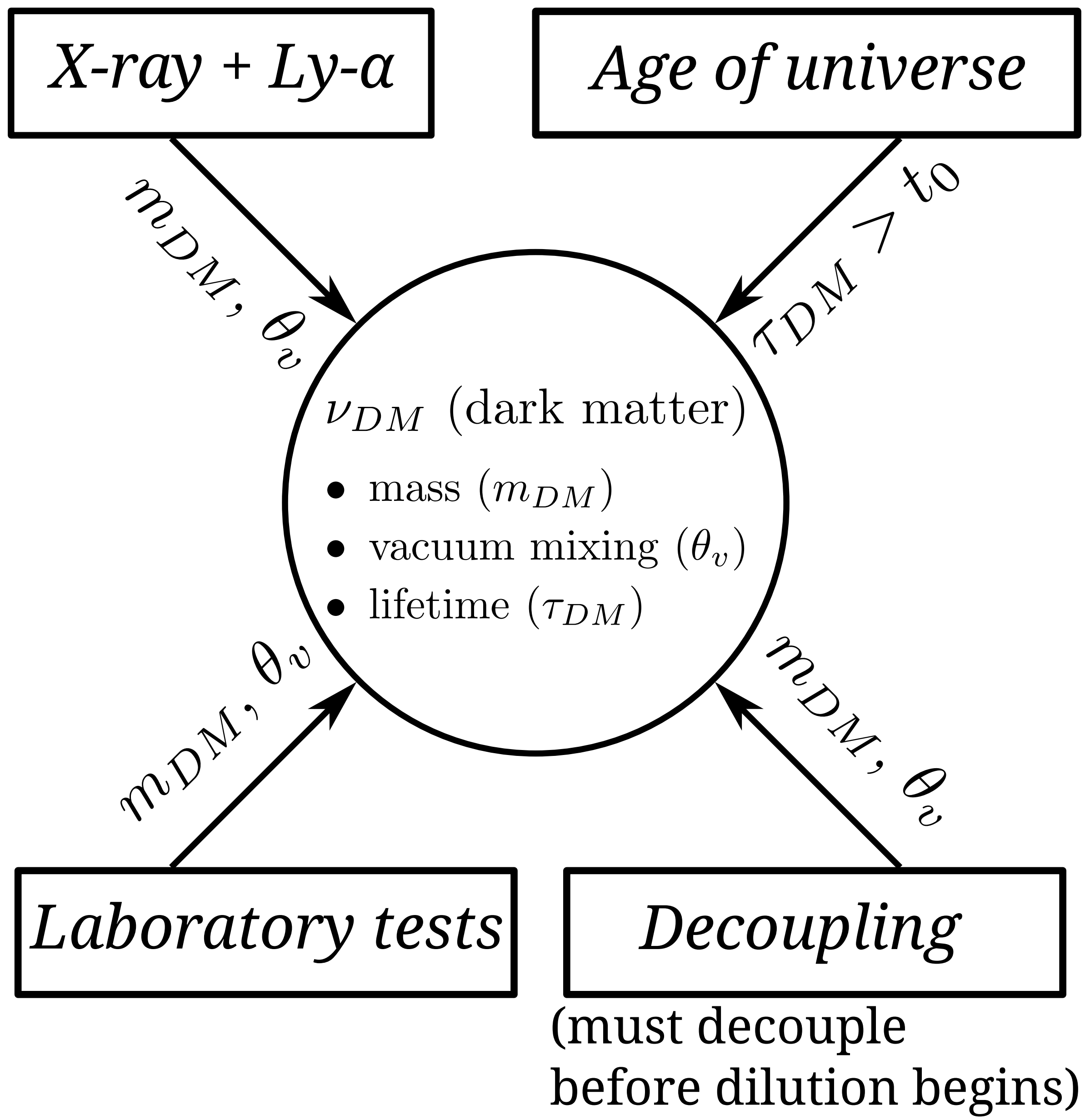
A. Patwardhan, GMF, C. T. Kishimoto, A. Kusenko (2015)

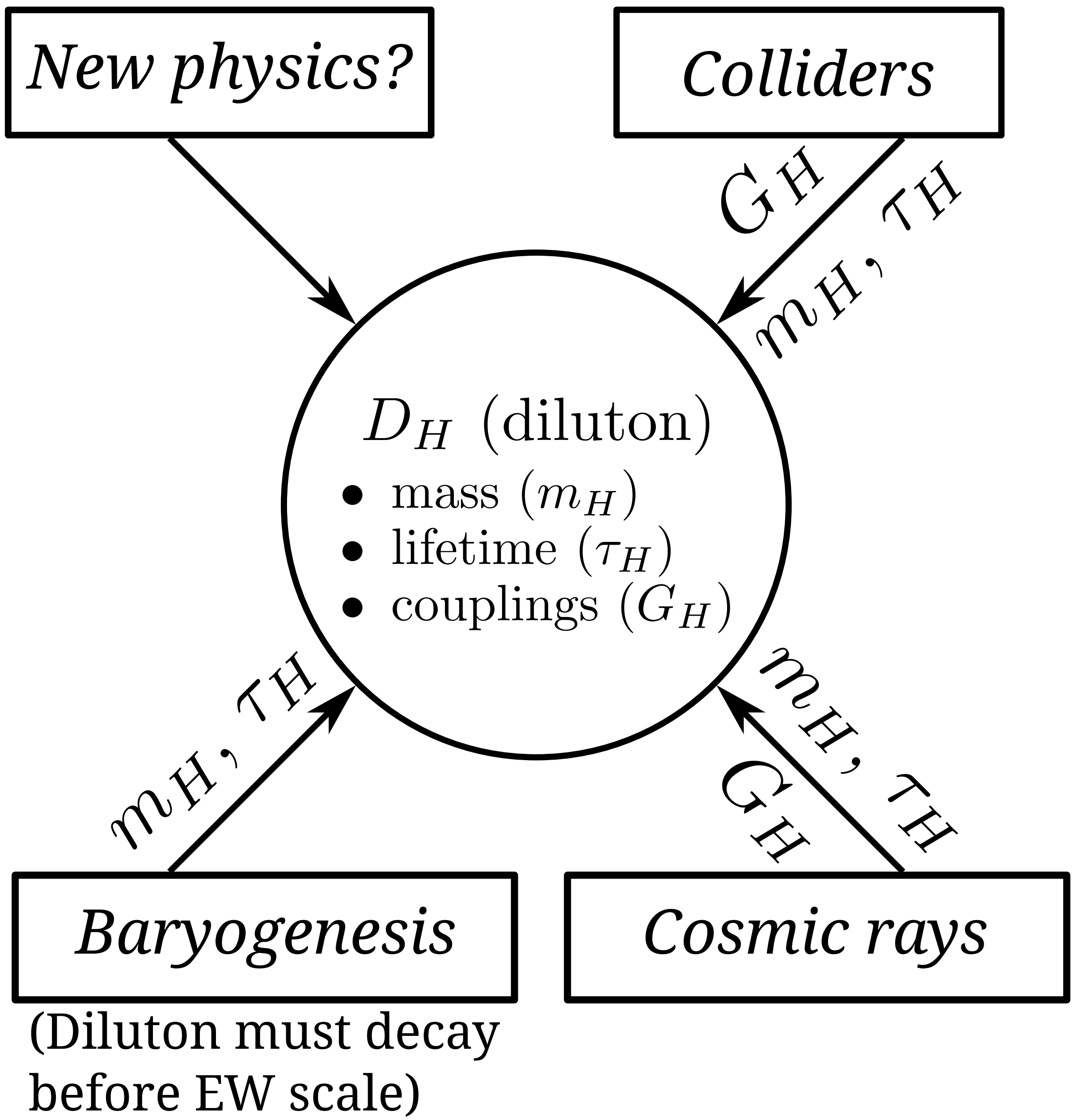


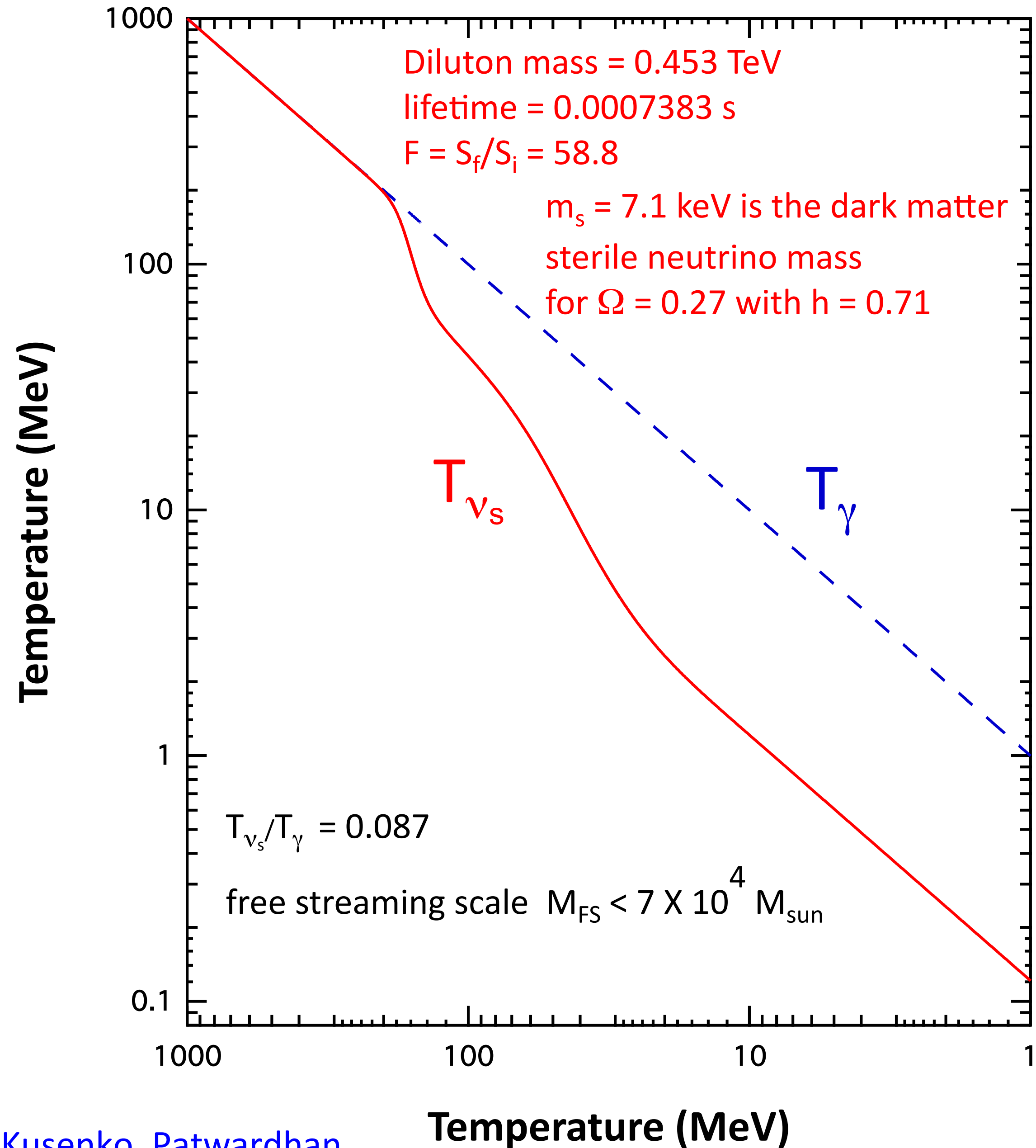
Mass-energy (in $M_\odot$)
10^6
10^4
10^2
10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^8
10^7
10^6
10^5
10^4
10^3
10^2
10^1
10^0
Temperature [MeV]
5
4
3
2
1
0
Ratio of Jeans mass to horizon mass



10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
10^{3}
10^{4}
Diluton lifetime τ (s)
Diluton mass [TeV]
10
20
50
5
10
2
5
1
2
0.5
1

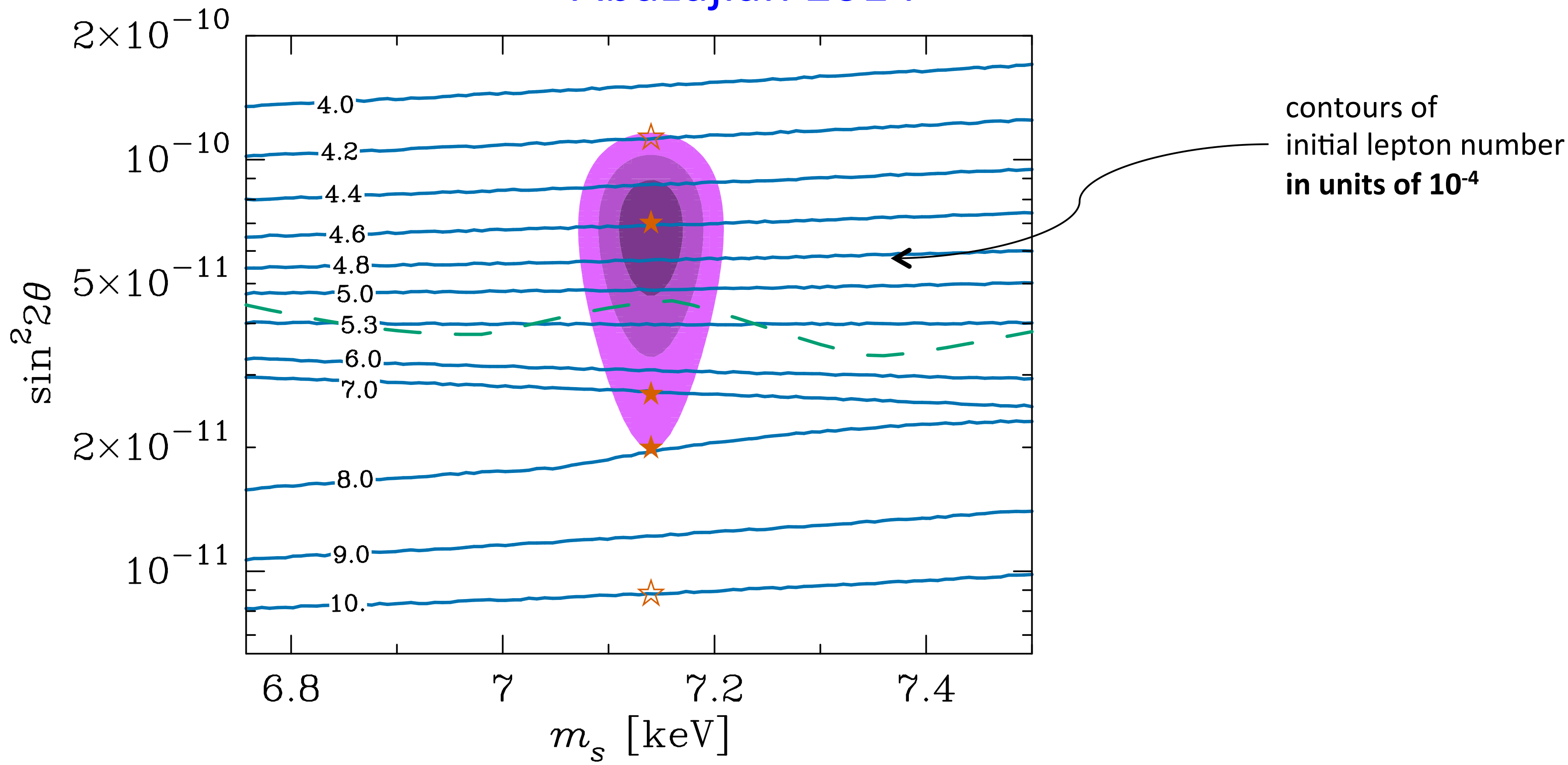

X-ray + Ly-α
Age of universe
m_{DM}, θ_v
$\tau_{DM} > t_0$
ν_{DM} (dark matter)
• mass (m_{DM})
• vacuum mixing (θ_v)
• lifetime (τ_{DM})
m_{DM}, θ_v
m_{DM}, θ_v
Laboratory tests
Decoupling
(must decouple before dilution begins)


New physics?
Colliders
G_H
m_H, τ_H
D_H (diluton)
• mass (m_H)
• lifetime (τ_H)
• couplings (G_H)
m_H, τ_H
m_H, τ_H
G_H
Baryogenesis
Cosmic rays
(Diluton must decay before EW scale)



Fuller, Kishimoto, Kusenko, Patwardhan (2014)

Abazajian 2014

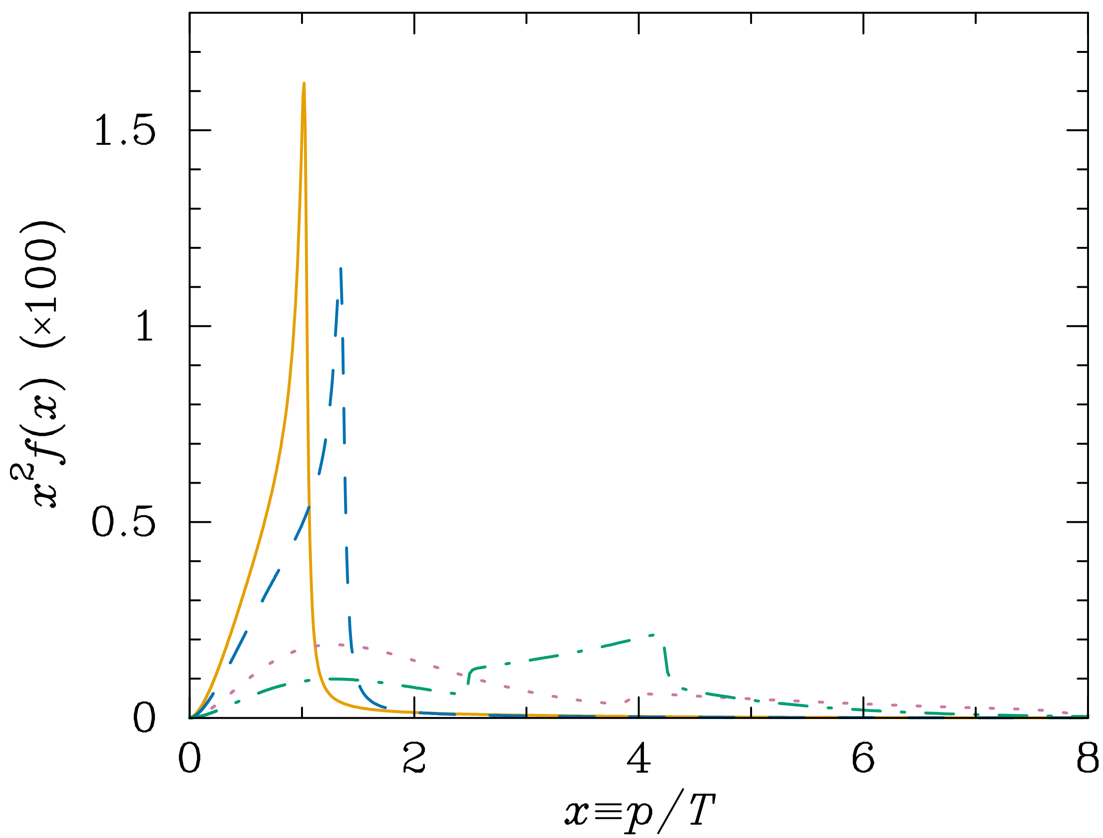


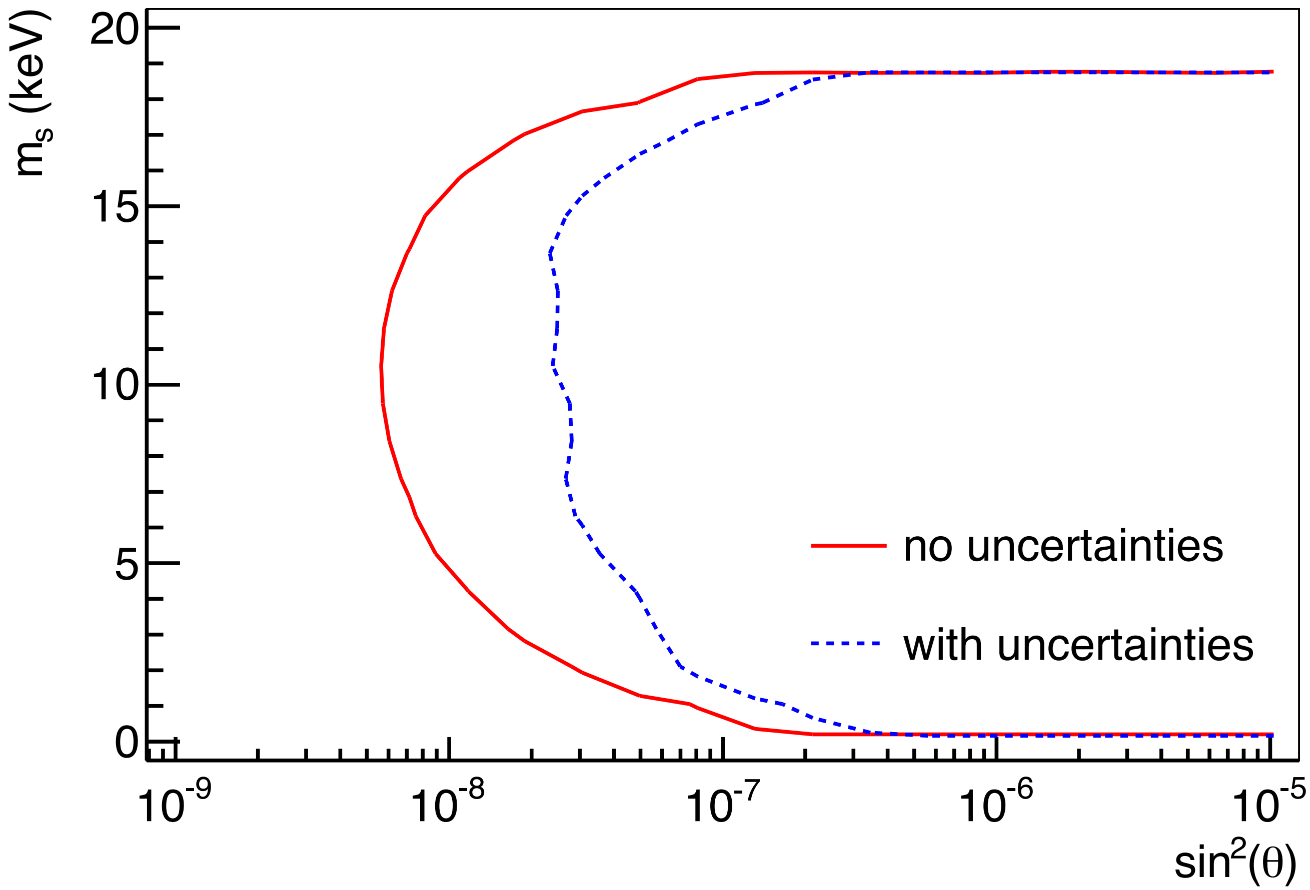
Lepton number is
$\mathcal{L} = 2\,L_{\nu_\alpha} + \sum_{\beta\neq\alpha} L_{\nu_\beta}$, where $\alpha, \beta = \mathrm{e}, \mu, \tau$

where individual lepton numbers are, *e.g.*,
$L_{\nu_\alpha} = (n_{\nu_\alpha} - n_{\bar{\nu}_\alpha})/n_\gamma$

and the baryon number is
$\eta = (n_{\mathrm{baryon}} - n_{\mathrm{anti-baryon}})/n_\gamma = 6.11 \times 10^{-10}$

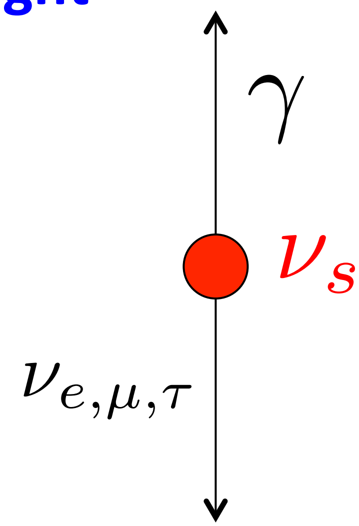
Abazajian 2014 – use $m_s = 7\,\mathrm{keV}$, $\sin^2\theta \sim 5\times10^{-11}$, lepton number 5×10^{-4}



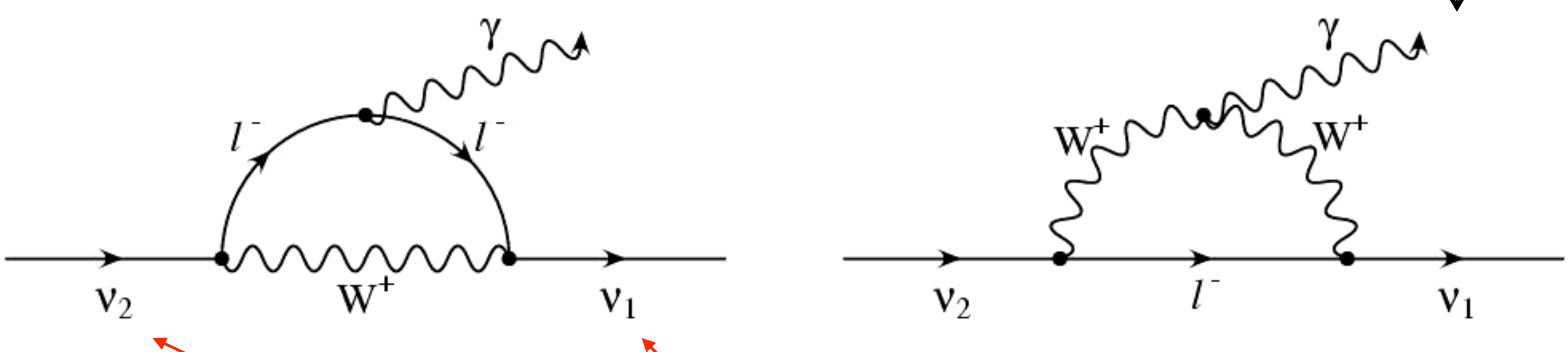


KATRIN – theoretical limits (S. Mertens *et al.,* arXiv:1409.0920)

A heavy "sterile" neutrino can decay into a light "active" neutrino and a photon.



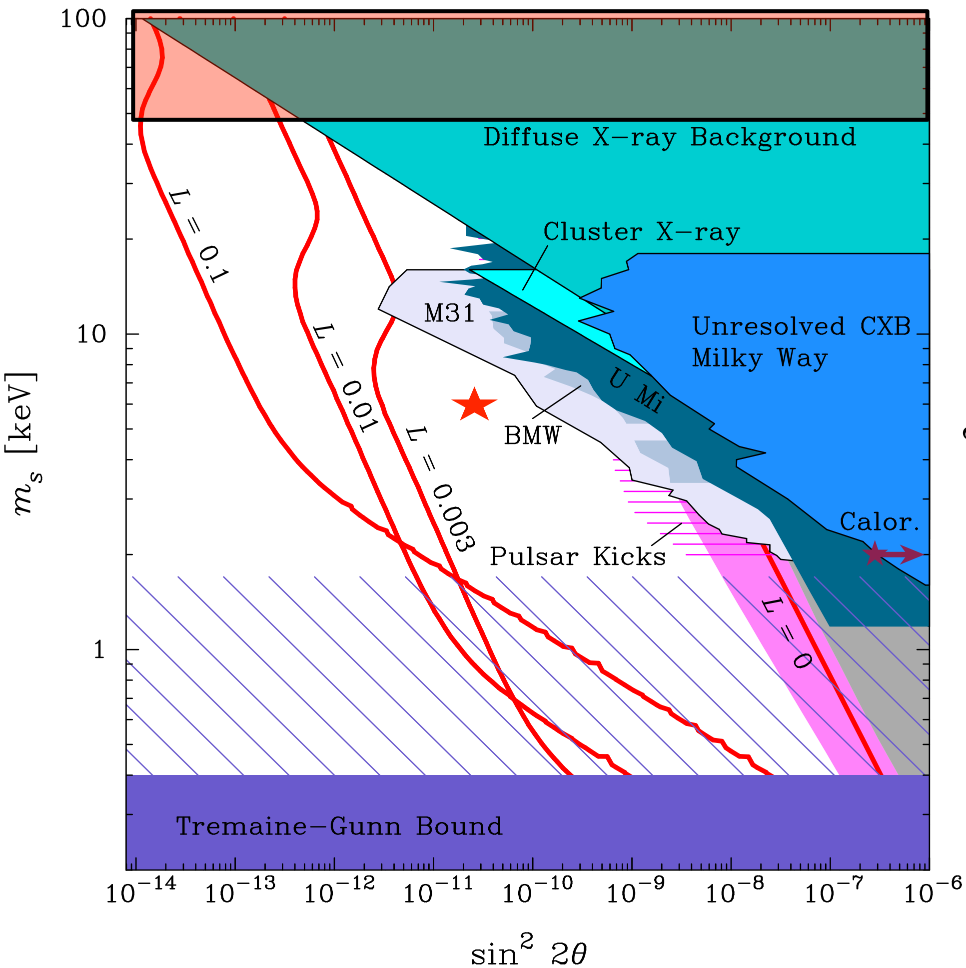
The final state light neutrino and photon *equally share* the rest mass energy of the initial heavy neutrino.

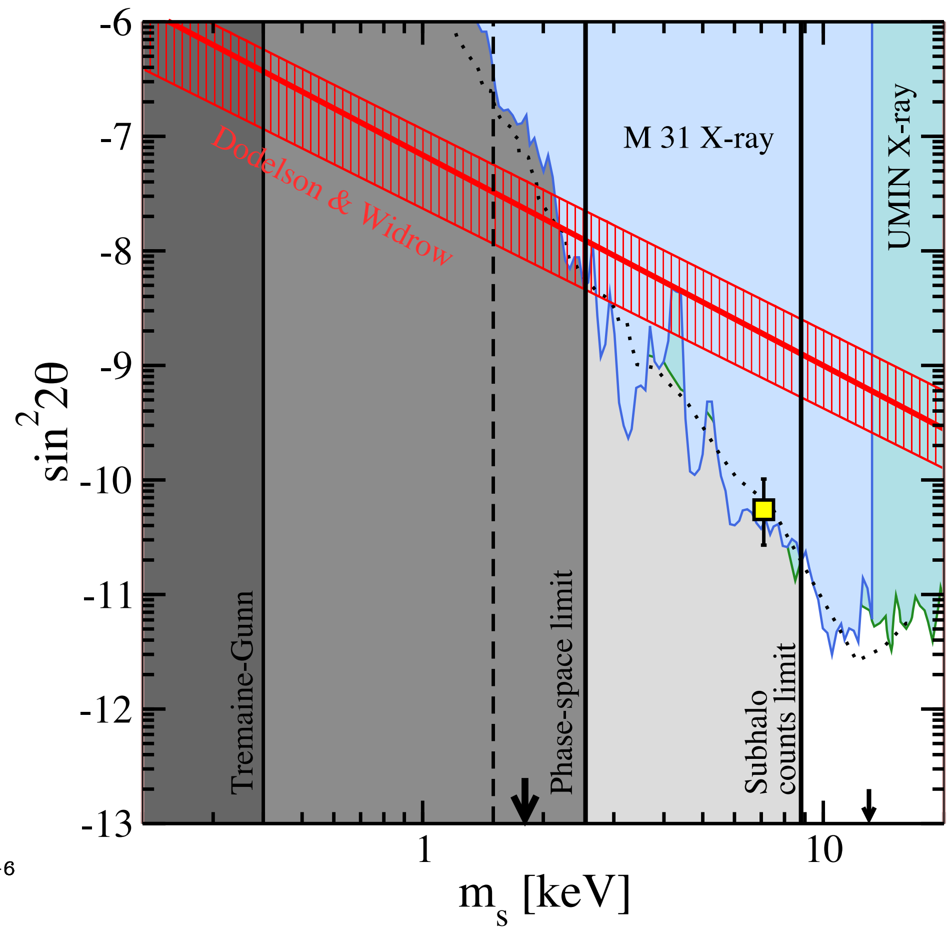


$$\nu_s \longrightarrow \nu_{e,\mu,\tau} + \gamma$$

photon line $E_\gamma = m_s/2$

Sterile Nu decay line

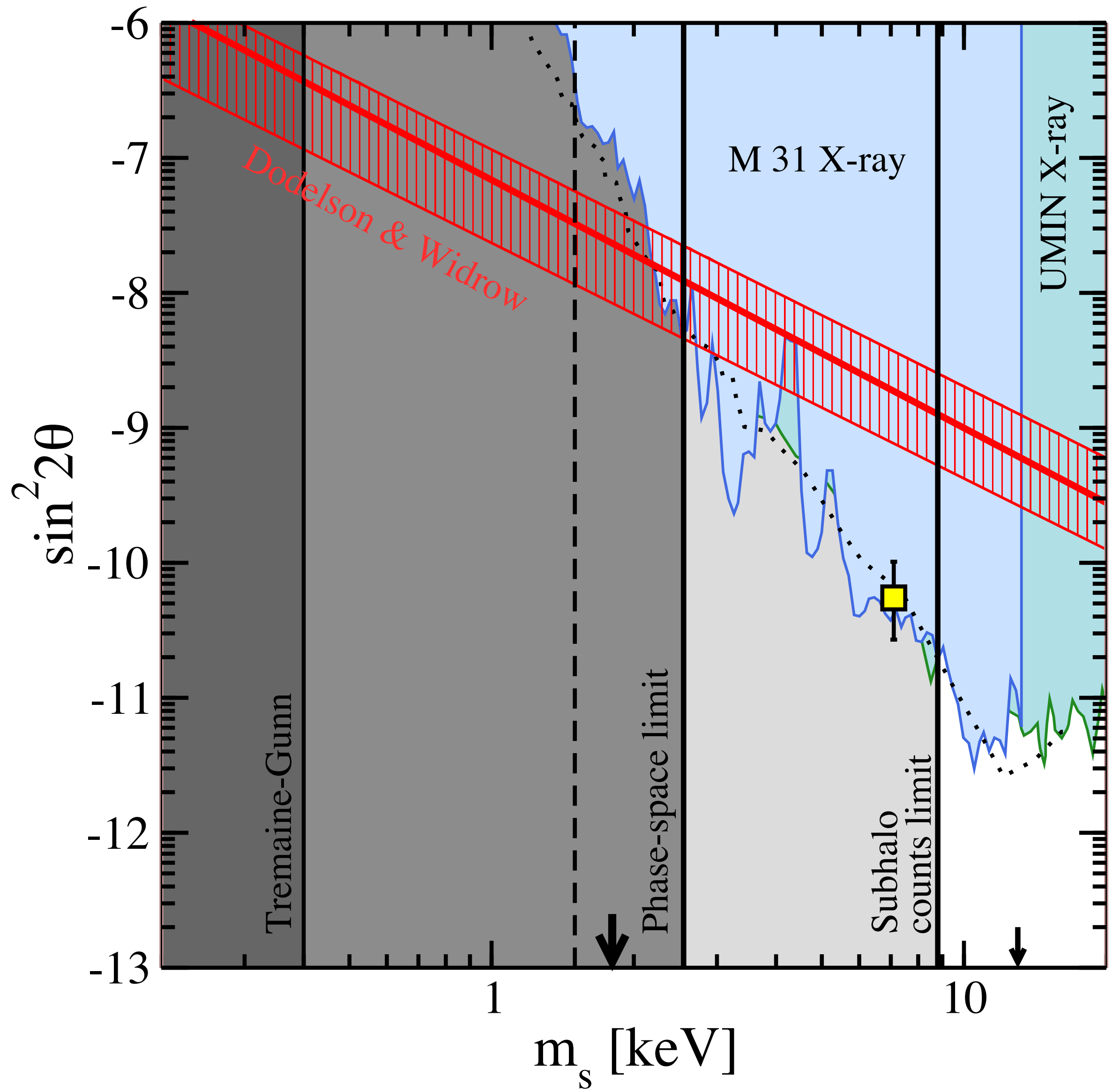




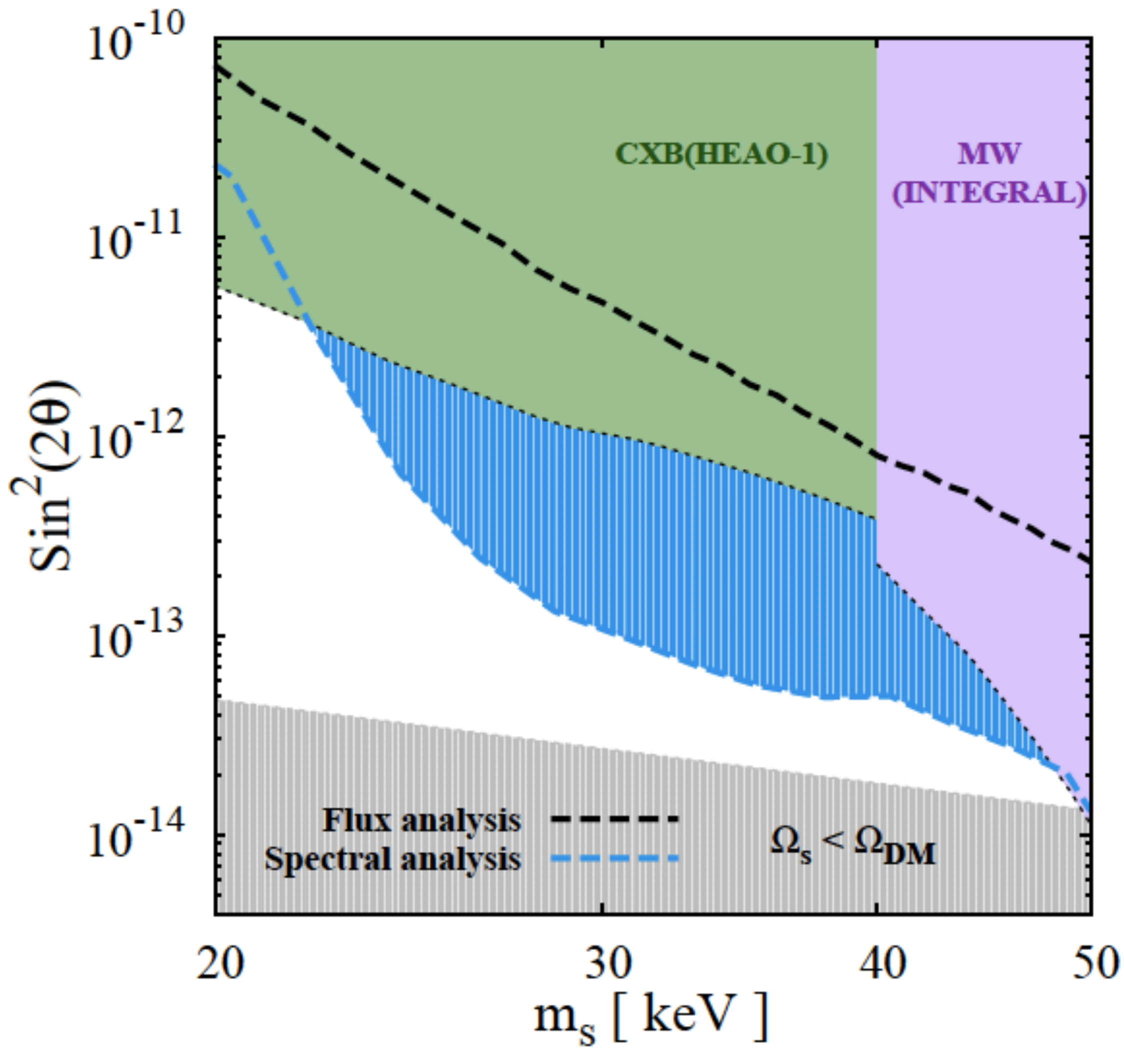
Abazajian, K. (2012)
Yüskel, H., Beacom, J. F., Watson, C.

Horiuchi, Abazajian, K., S., Kaplighat, M. (2014)

Horiuchi, Abazajian, K., S., Kaplighat, M. (2014



new *Fermi* constraints



K. Ng, S. Horiuchi, J. M. Gaskins, M. Smith, R. Preece arXiv:1504.04027

Possible Detections ??

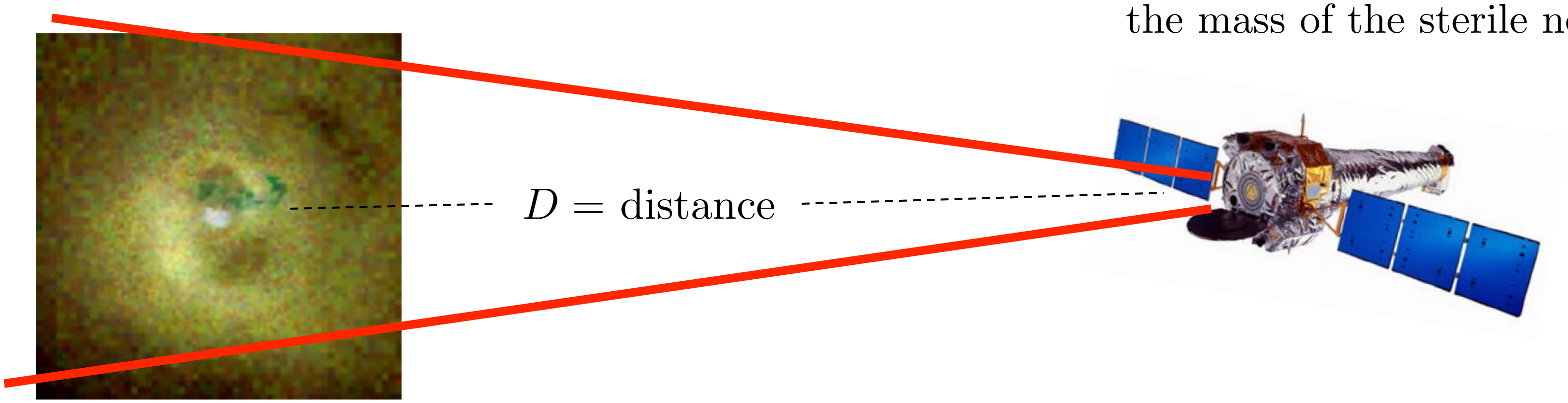
two different X-ray astronomy groups see a **3.5 keV** line in **clusters of galaxies** and in M31,
and this line is ***consistent with a dark matter decay origin***,
corresponding to a **7 keV rest mass sterile neutrino**
with vacuum mixing with active neutrinos $\sin^2 2\theta = (2 - 20) \times 10^{-11}$

E. Bulbul, M. Markevitch, A. Foster, R. Smith, M. Lowenstein, S. Randall
"*Detection of an unidentified emission line in the stacked X-ray spectrum of Galaxy Clusters*" arXiv:1402.2301

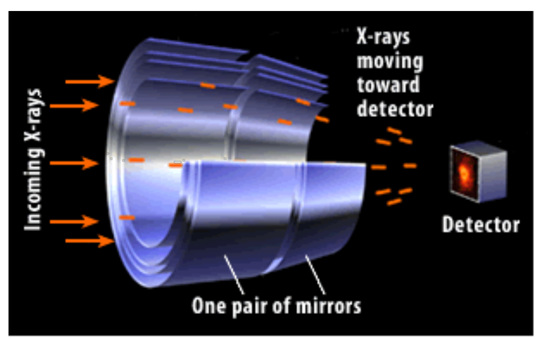
A. Boyarsky, O. Ruchayskiy, D. Iakubovskyi, J. Franse
"An unidentified line in the X-ray spectrum of the Andromeda galaxy and Perseus galaxy cluster" arXiv:1402.4119

M = mass of dark matter
in field of view of X-ray telescope

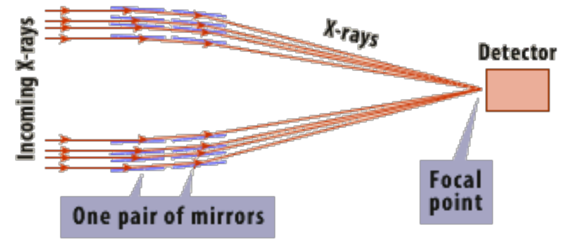
energy of X-ray line:
$E_\gamma = \frac{1}{2}\, m_\mathrm{s}$,
measuring this gives
the mass of the sterile neutrino m_s



measured X-ray flux in line
$\propto M \cdot D^{-2} \cdot m_\mathrm{s}^5 \cdot \sin^2\theta$



Courtesy Chandra mission website:
http://chandra.harvard.edu



Chandra's mirrors are positioned so they're almost parallel to the entering X-rays. The mirrors look like open cylinders, or barrels. The X-rays skip across the mirrors much like stones skip across the surface of a pond.

. . . ah, but there is a debate about the nature of the photon line . . .

Is the 3.5 keV line really from sterile neutrino decay?

Or is it an atomic line – perhaps 17 times ionized potassium?

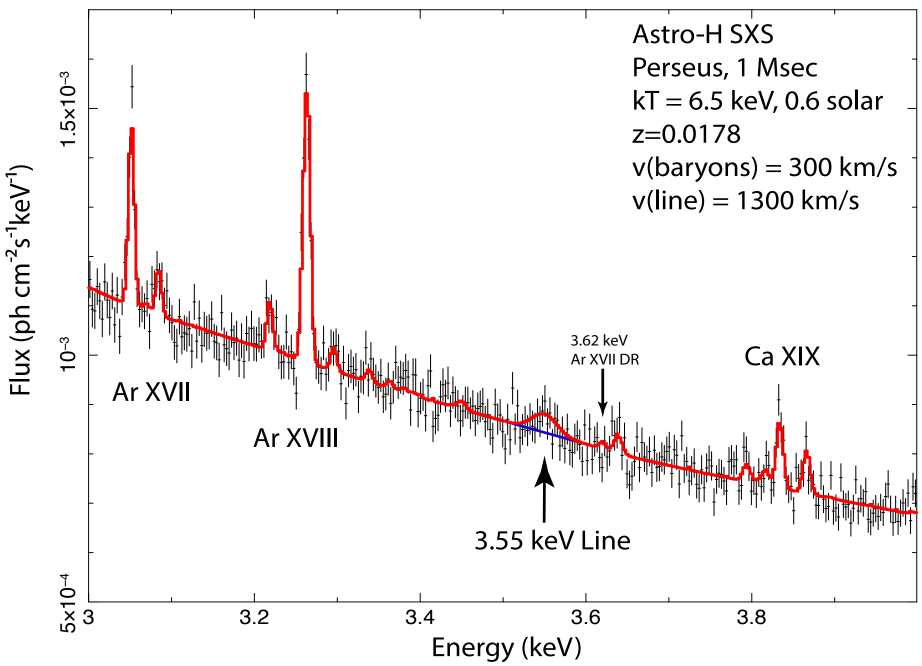
T. E. Jeltma & *S. Profumo* ArXiv:1408.1699

see the rebuttal by Boyarsky *et al.* arXiv:1408:4388

Can also look for X-rays from the dark matter halos of field galaxies:
Do not see the line – do we know how much dark matter is in the field of view?

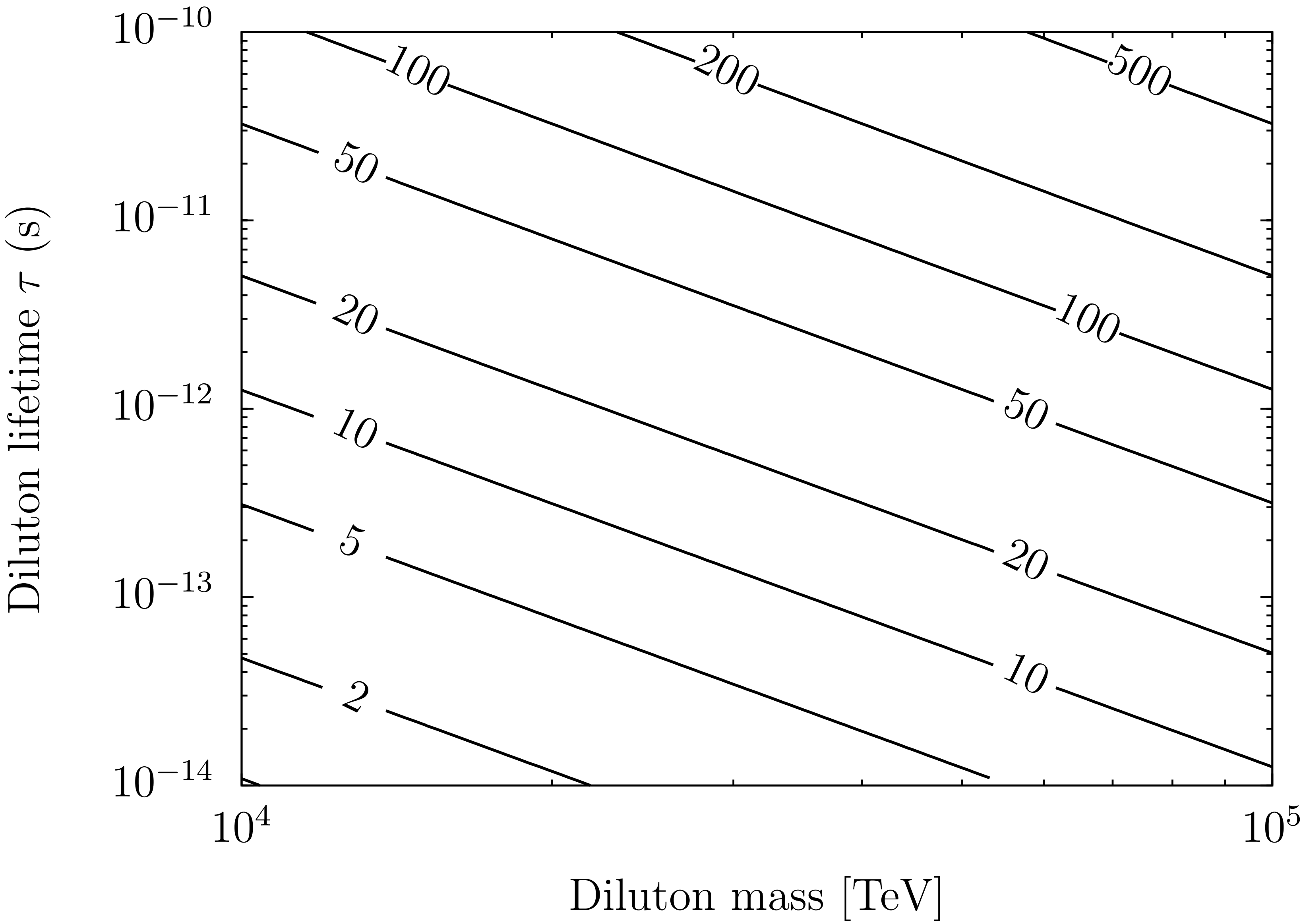
Future smoking gun? -- **Astro-H will have ~ few eV energy resolution**

Bulbul et al.

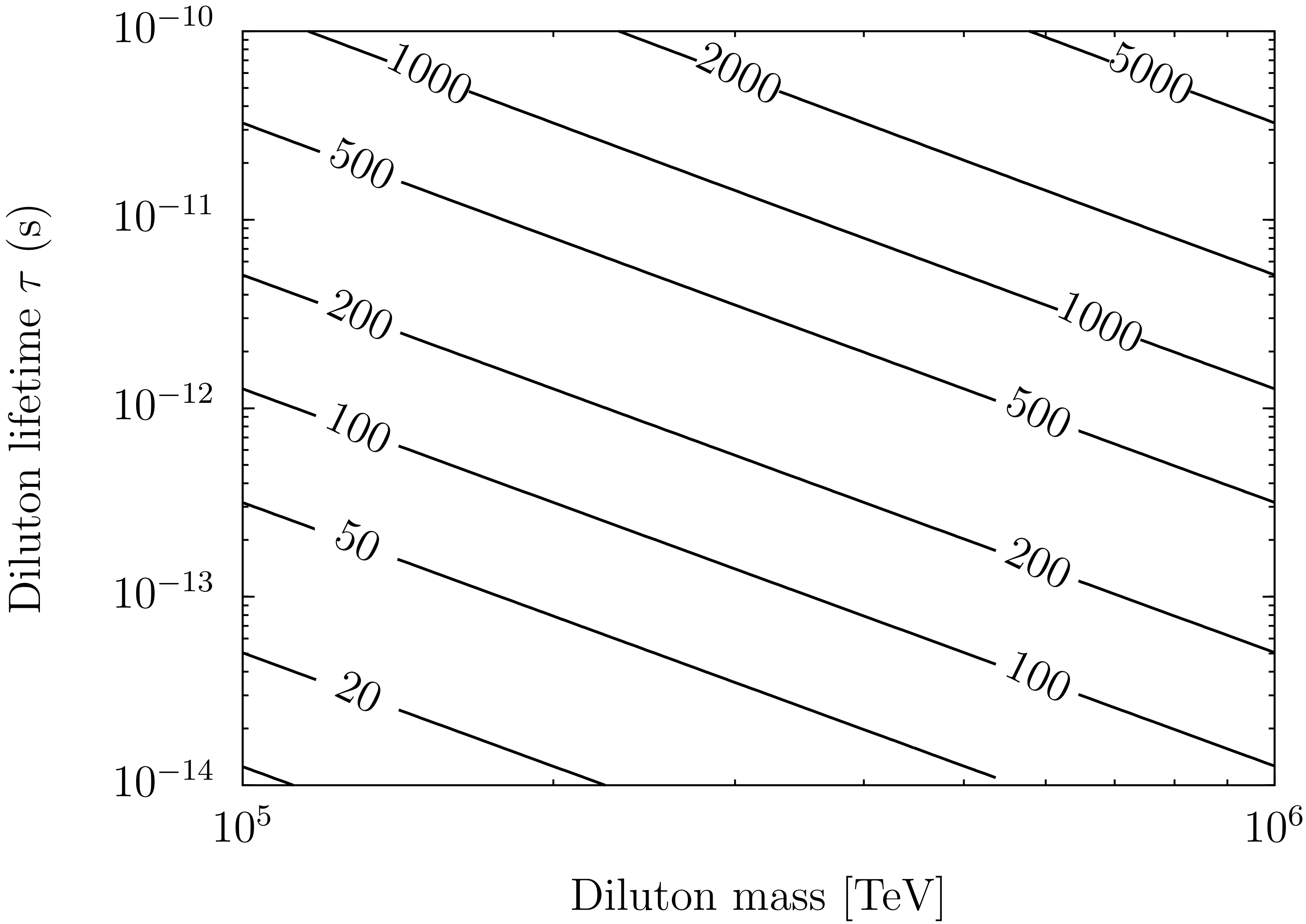


resolve the Virial width?

see Lowenstein & Kusenko



10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
Diluton lifetime τ (s)
10^4
10^5
Diluton mass [TeV]
100
200
500
50
20
10
5
2
100
50
20
10



10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
Diluton lifetime τ (s)
10^5
10^6
Diluton mass [TeV]
1000
2000
5000
500
1000
200
500
100
200
50
100
20