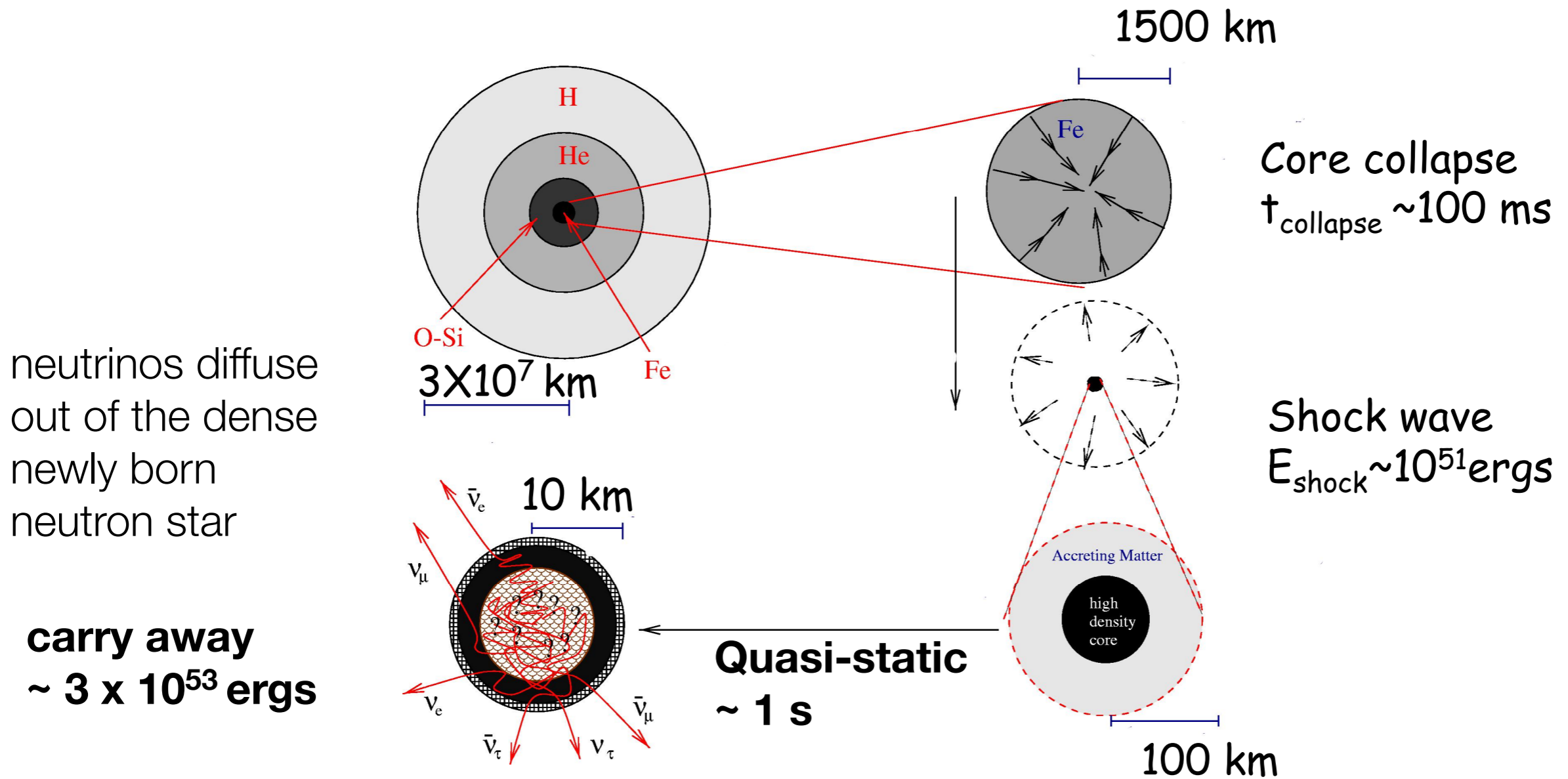


# Supernova Neutrinos: The role of the nuclear equation of state and neutrino-matter interactions in hot and dense matter

- Introduction
- Temporal structure: Neutron star tomography
- Neutrino spectra
- Supernova cooling constraints

Sanjay Reddy  
INT, Univ. of Washington, Seattle

# Supernova Neutrinos




- The time structure of the neutrino signal depends on how heat is transported in the neutron star core ( $10^{13}$ - $10^{15} \text{ g/cm}^3$ ).
- The spectrum is set by scattering in a hot ( $T=5$ - $10 \text{ MeV}$ ) and not so dense ( $10^{11}$ - $10^{13} \text{ g/cm}^3$ ) neutrino-sphere.

# A broad brush description of SN neutrino emission


Neutrino luminosity:

$$L_\nu(t) \approx 4\pi\sigma_\nu R_\nu^2(t) T_\nu^4(t)$$

Radius of the neutrino  
decoupling surface.



Temperature of the neutrino  
decoupling surface.



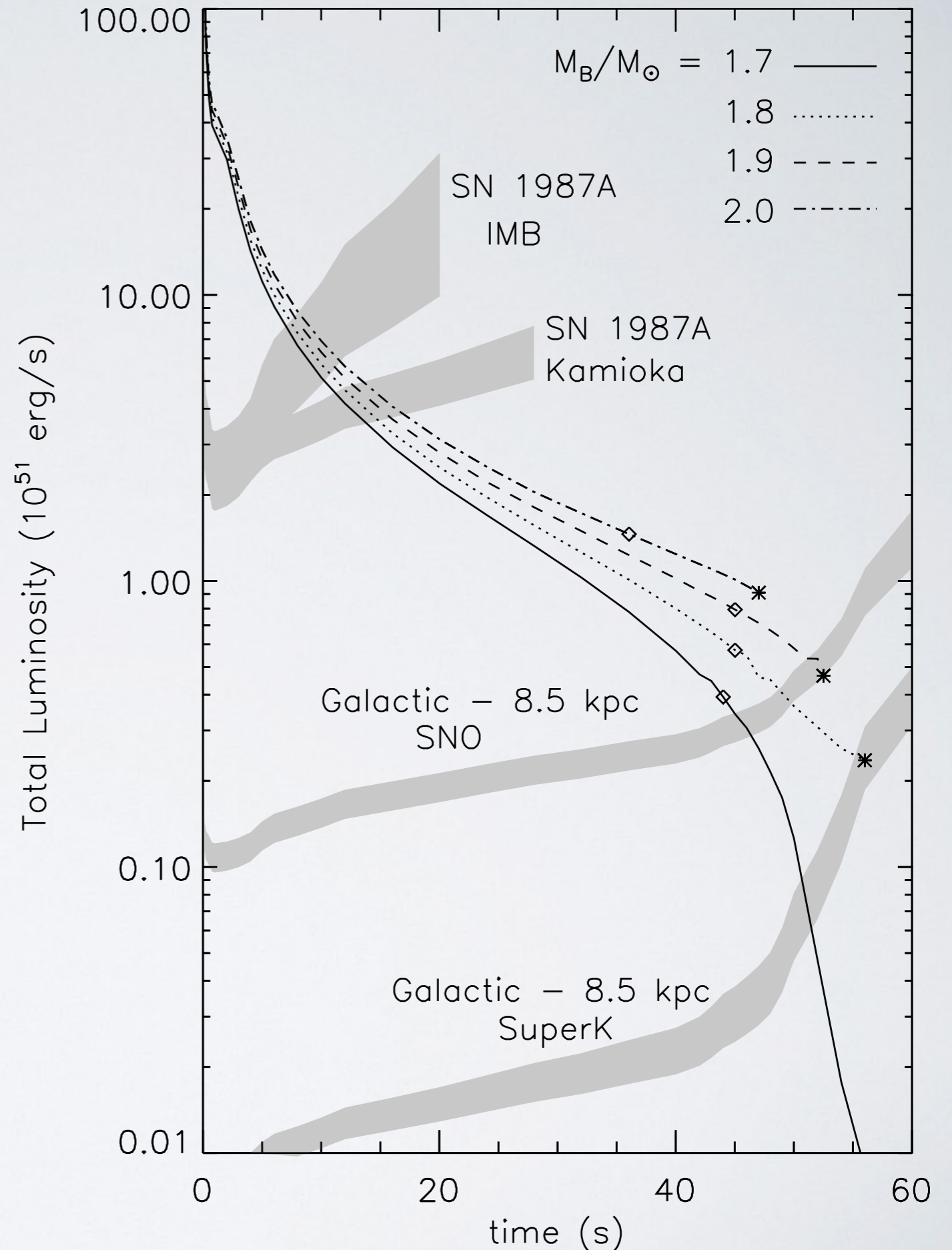
For given neutron star mass:

- The equation of state determines R.
- Neutrino interactions near the PNS surface determines T.
- Time evolution is determined by neutrino opacities in the core, equation of state of dense matter and the stability of the final neutron star.

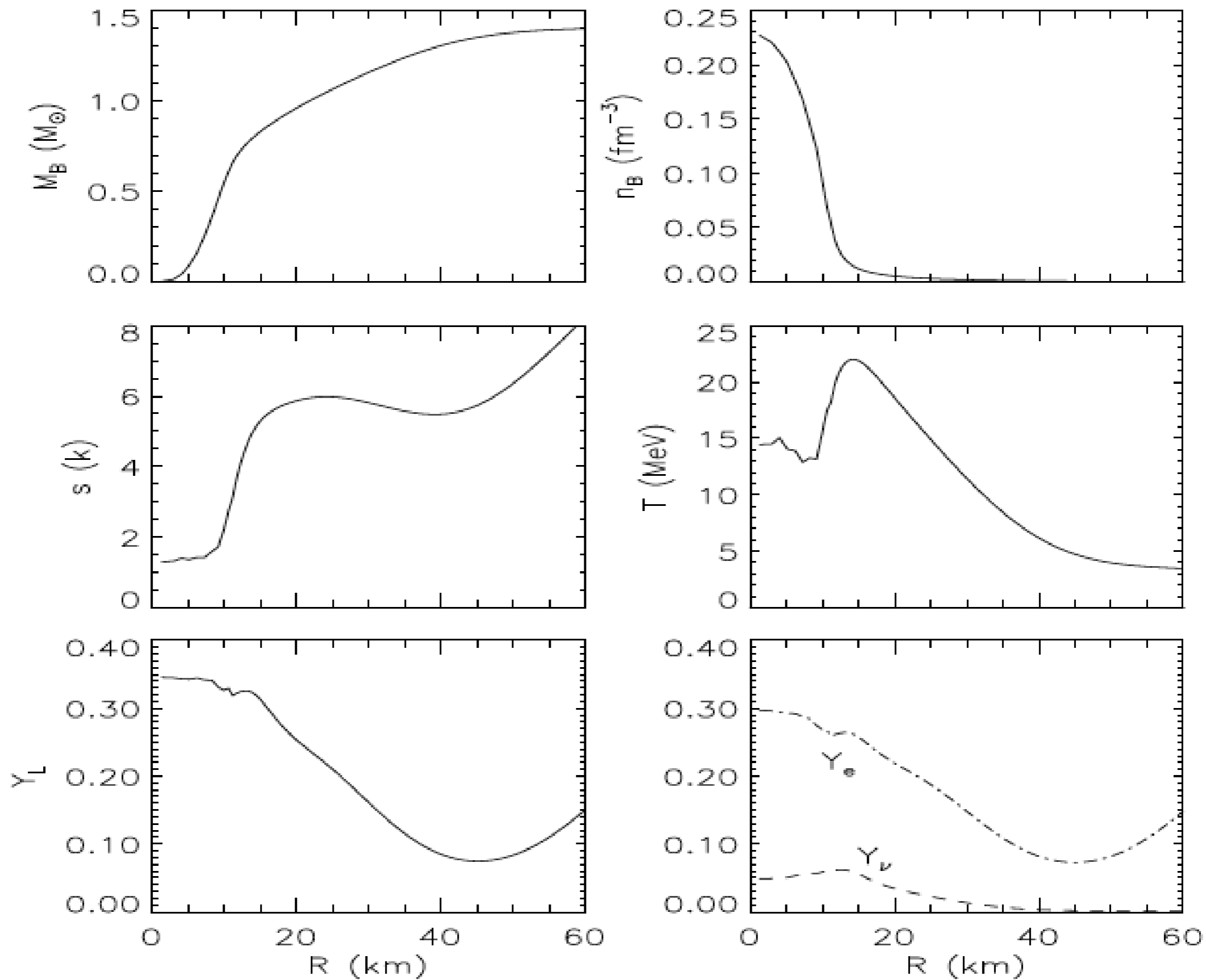
# Neutrino Emission: Baseline Model

Generic expectations:

- For a galactic SN we should see neutrinos for about 40 s
- Luminosity increases with NS mass.
- Heavier masses can collapse to a black-hole if the high density equation of state is soft.



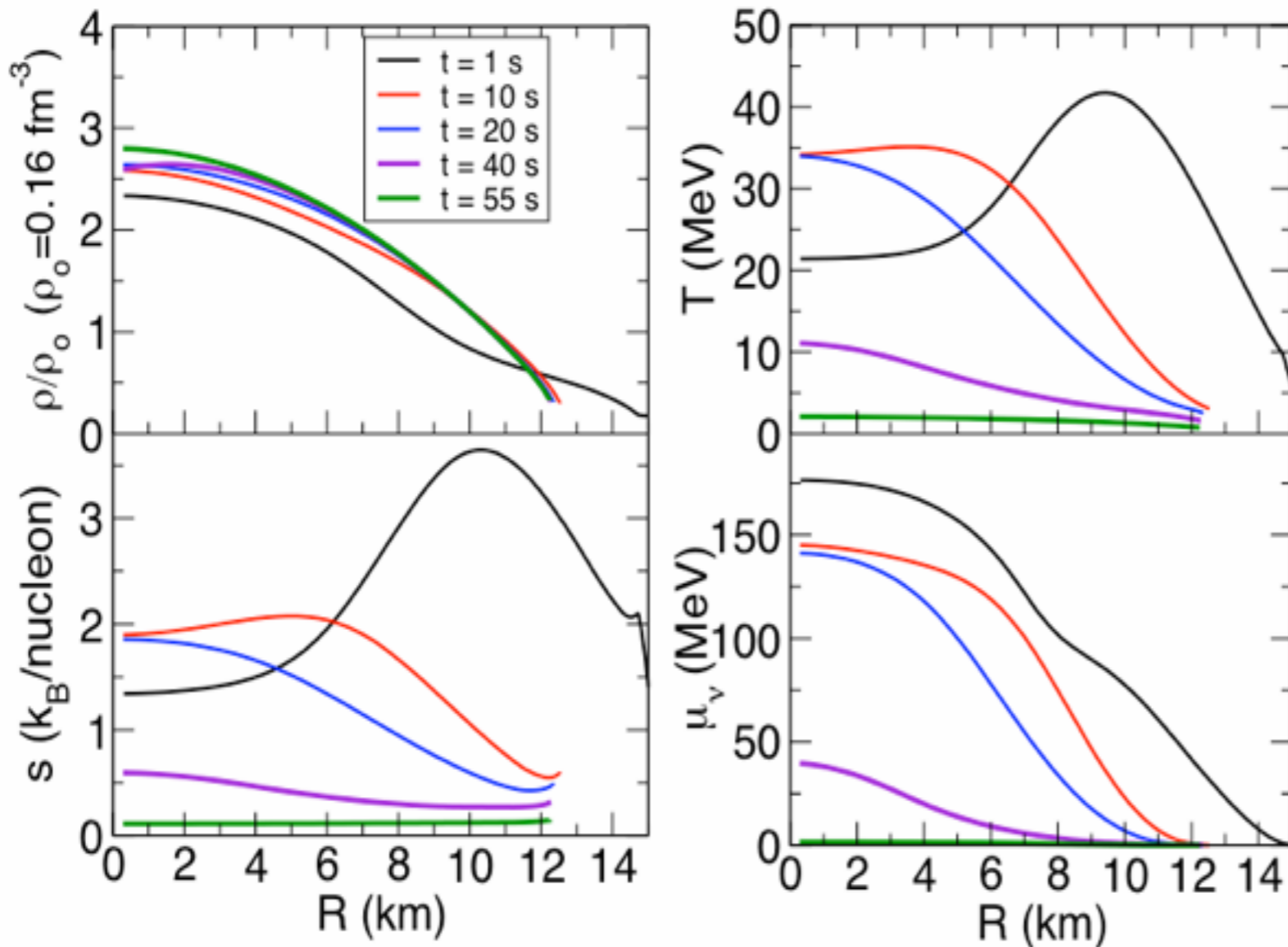
# Protoneutron Star Evolution



Initial state of the PNS.

# Neutrino diffusion cools and deleptonizes the PNS.

Time scales are set by properties of high density matter



Typical time-scales:

$$T(t) \approx T(t=0) \left(1 - \frac{t}{\tau_C}\right)$$

$$\tau_C \approx C_V \frac{R^2}{c \langle \lambda_\nu \rangle}$$

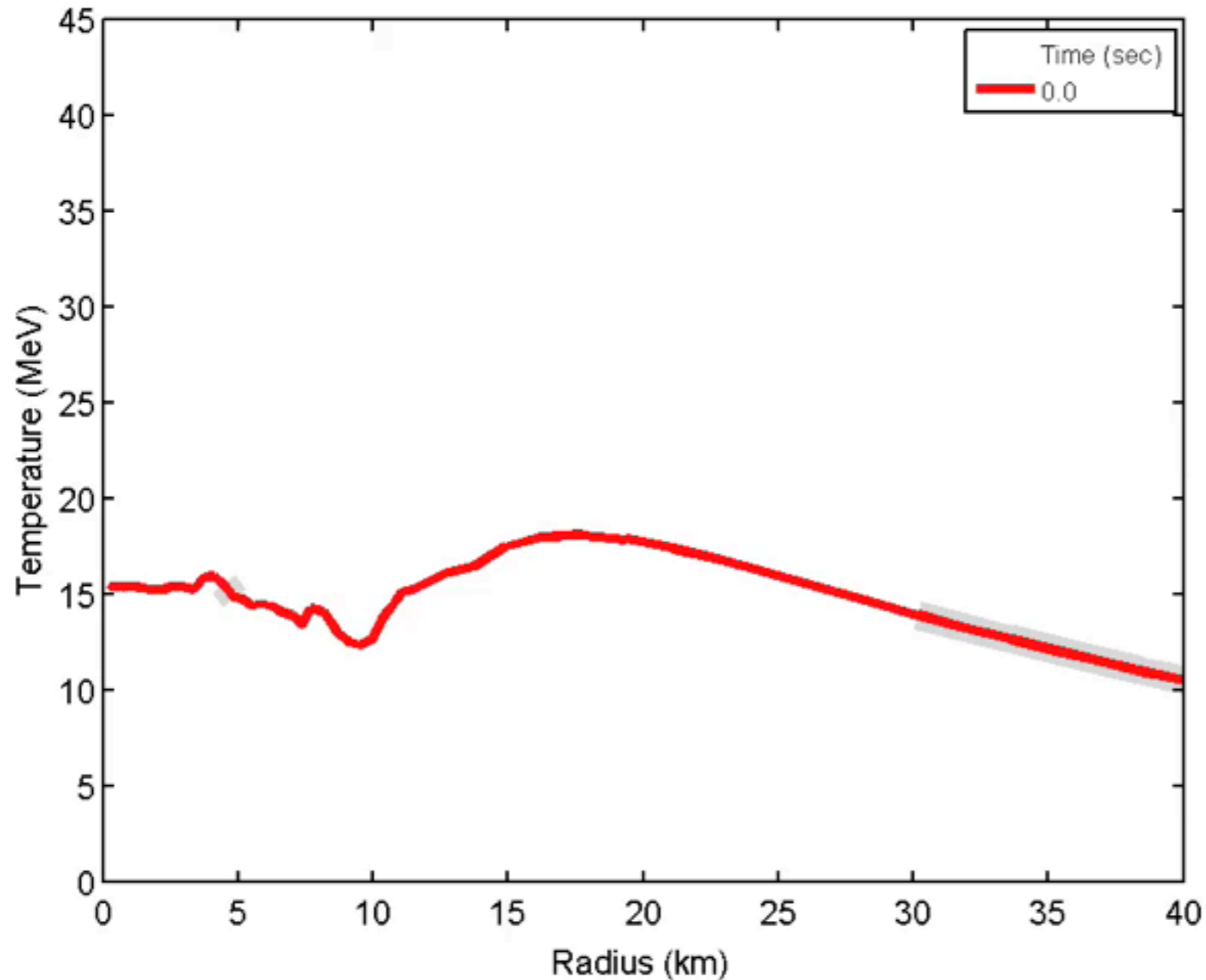
$$Y_\nu(t) \approx \frac{\mu_\nu^3}{6\pi^2}$$

$$\approx Y(t=0) \exp\left(-\frac{t}{\tau_D}\right)$$

$$\tau_D = \frac{3}{\pi^2} \frac{\partial Y_L}{\partial Y_\nu} \frac{R^2}{c \langle \lambda_{\nu e} \rangle}$$

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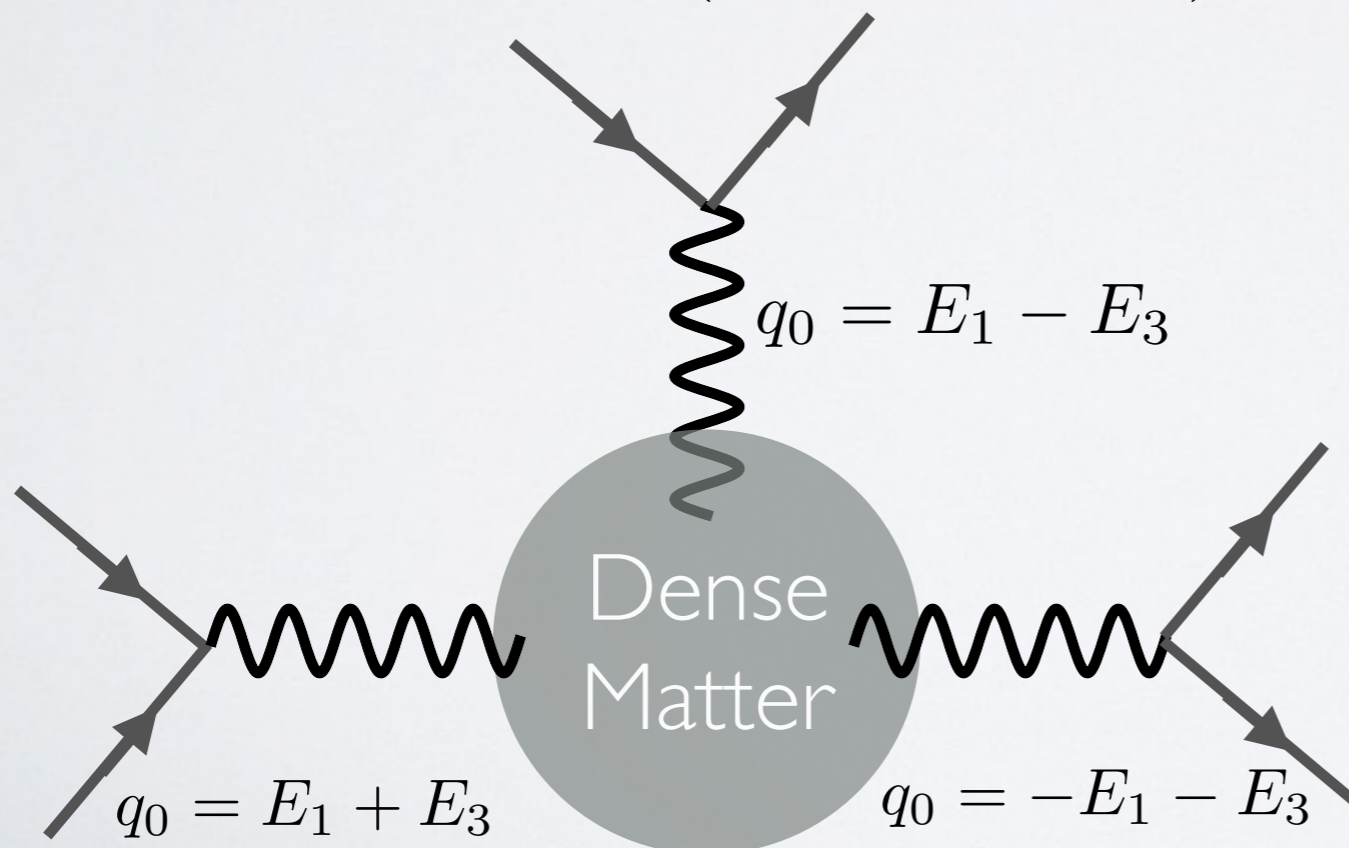
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# Neutrino Transport

- RHS of the Boltzmann Equation.

$$\begin{aligned} \frac{\partial f(E_1)}{\partial t} = & \int \frac{d^3 k_3}{(2\pi)^3} R(E_1, E_3, \cos \theta) f_3 (1 - f_1) \\ & - R(E_3, E_1, \cos \theta) f_1 (1 - f_3) \\ & + R(E_1, -E_3, \cos \theta) (1 - f_1) (1 - f_3) \\ & - R(-E_1, E_3, \cos \theta) f_1 f_3 \end{aligned}$$





# Neutrino Transport

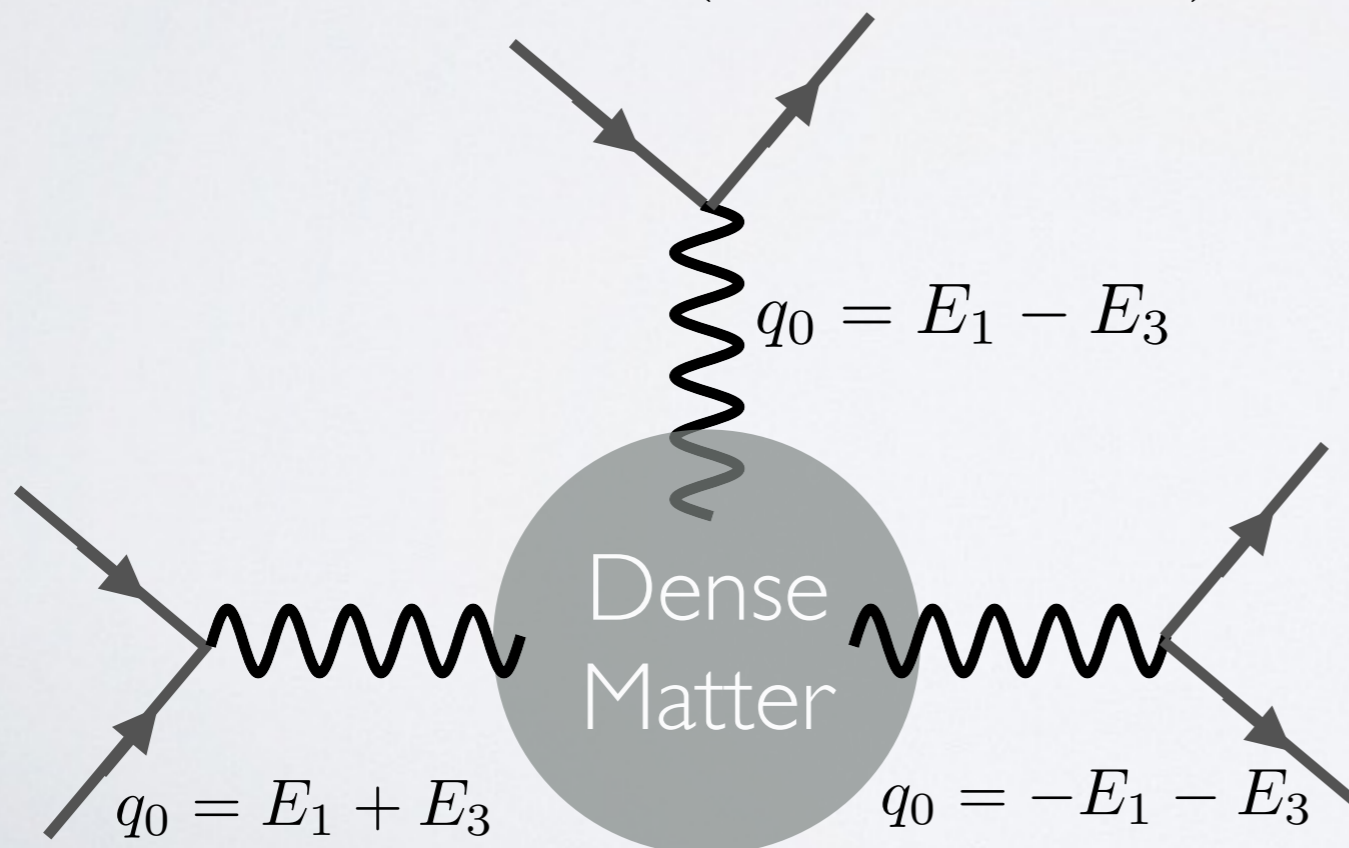
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$$\frac{\partial f(E_1)}{\partial t} = \int \frac{d^3 k_3}{(2\pi)^3} R(E_1, E_3, \cos \theta) f_3 (1 - f_1) \longrightarrow \text{scattering-in}$$

$$- R(E_3, E_1, \cos \theta) f_1 (1 - f_3) \longrightarrow \text{scattering-out}$$

$$+ R(E_1, -E_3, \cos \theta) (1 - f_1)(1 - f_3) \longrightarrow \text{pair-production}$$

$$- R(-E_1, E_3, \cos \theta) f_1 f_3 \longrightarrow \text{pair-annihilation}$$



# Neutrino Transport

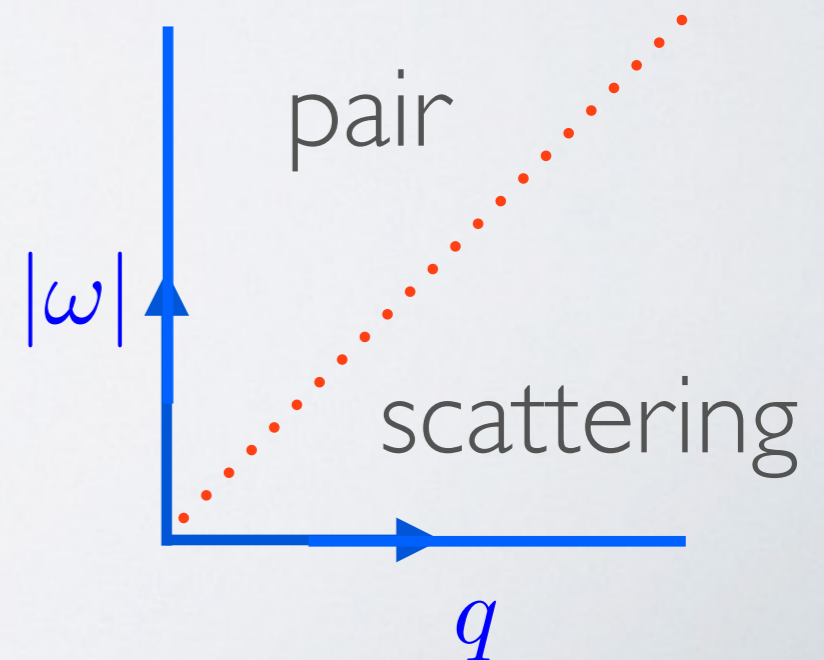
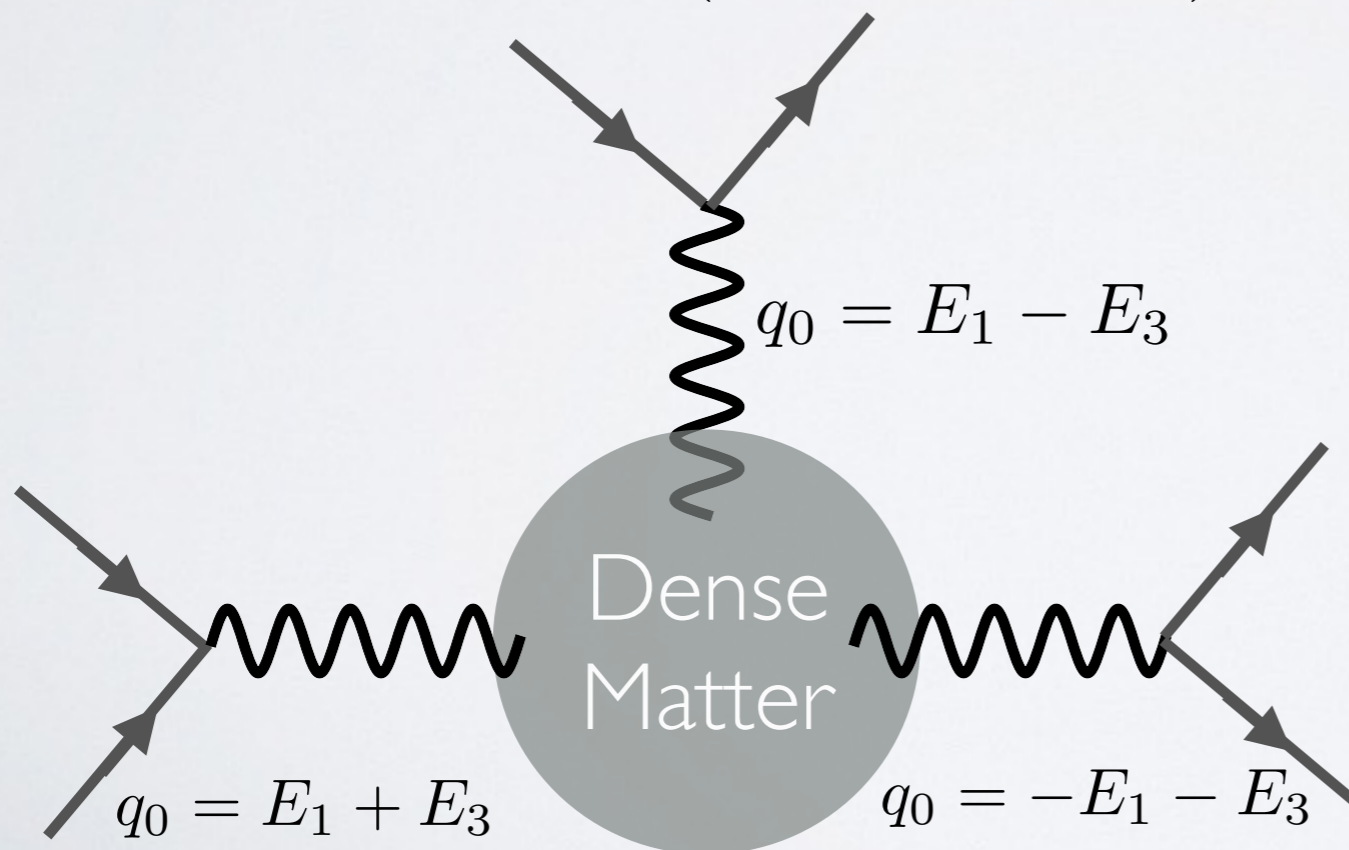
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# Neutrino Cross Sections

Differential Scattering/Absorption Rate:

response function of the medium

$$R(E_1, E_3, \cos \theta) = G_F^2 L(E_1, E_3, \cos \theta) \times S_{[\rho, Y_e, T]}(q_0, q)$$

neutrino/lepton kinematic factor

- Neutral and charged current reactions contribute.

$$\mathcal{L}_{int}^{cc} = \frac{G_F}{\sqrt{2}} l_\mu j_W^\mu \quad \text{for} \quad \nu_l + B_2 \rightarrow l + B_4$$
$$\mathcal{L}_{int}^{nc} = \frac{G_F}{\sqrt{2}} l_\mu^\nu j_Z^\mu \quad \text{for} \quad \nu_l + B_2 \rightarrow \nu_l + B_4$$

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$$l_\mu^\nu = \bar{\psi}_\nu \gamma_\mu (1 - \gamma_5) \psi_\nu, \quad j_Z^\mu = \bar{\psi}_4 \gamma^\mu (c_V - c_A \gamma_5) \psi_2$$

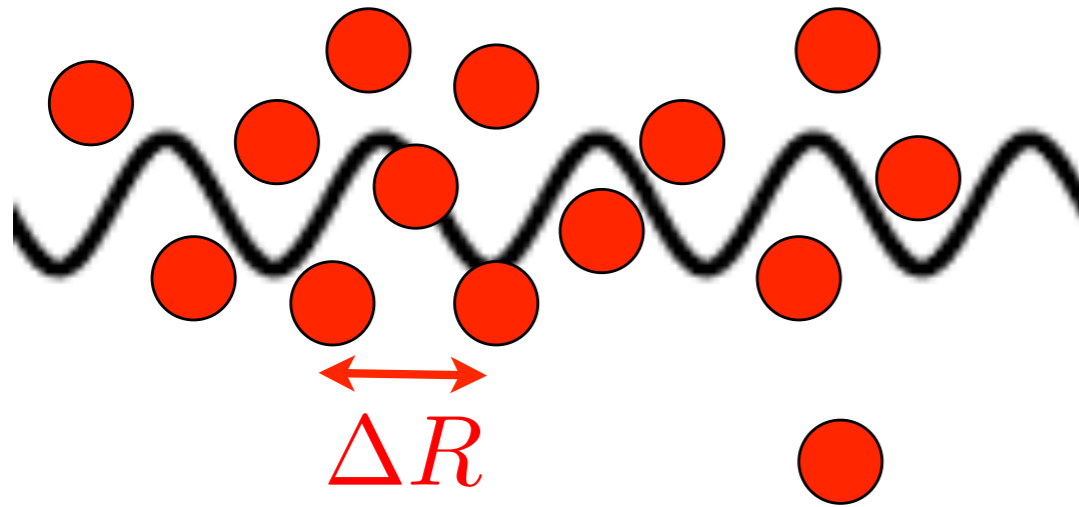
$$c_V^n = -1.0, \quad c_A^n = -1.26(-1.1)$$

$$c_V^p = 0.07, \quad c_A^p = 1.26(1.4)$$

$$c_V^e = 1.92, \quad c_A^e = 1. \quad [\nu_e]$$

$$c_V^e = -0.08, \quad c_A^e = -1. \quad [\nu_X]$$

# Neutrino Interactions at High Density



At small  $q_0$  and  $q$  the neutrino cannot resolve a single nucleon.

Sawyer (1975, 1989)

Iwamoto & Pethick (1982)

Horowitz & Wherberger (1991)

Burrows & Sawyer (1999)

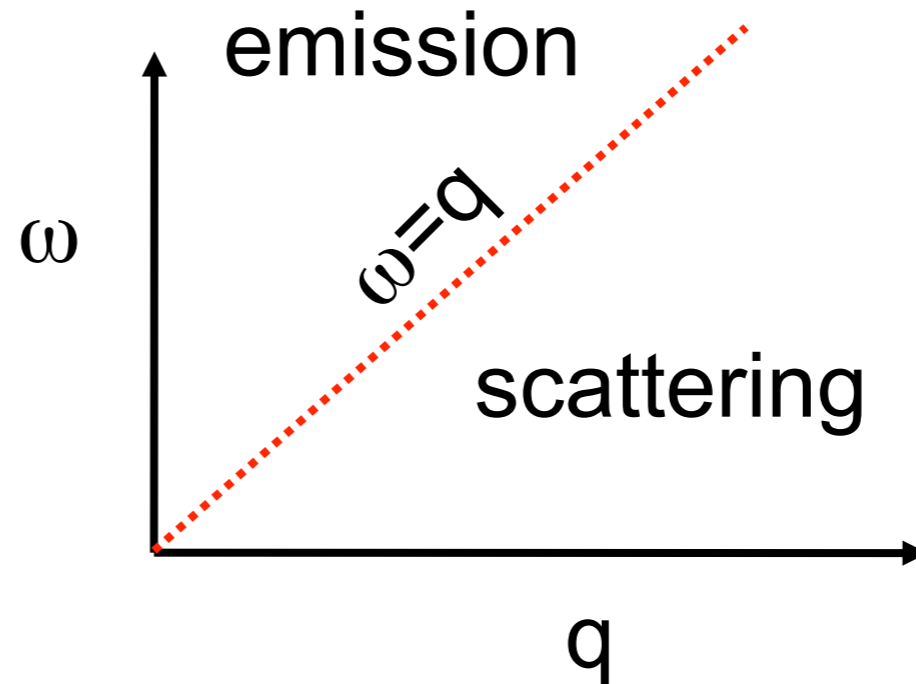
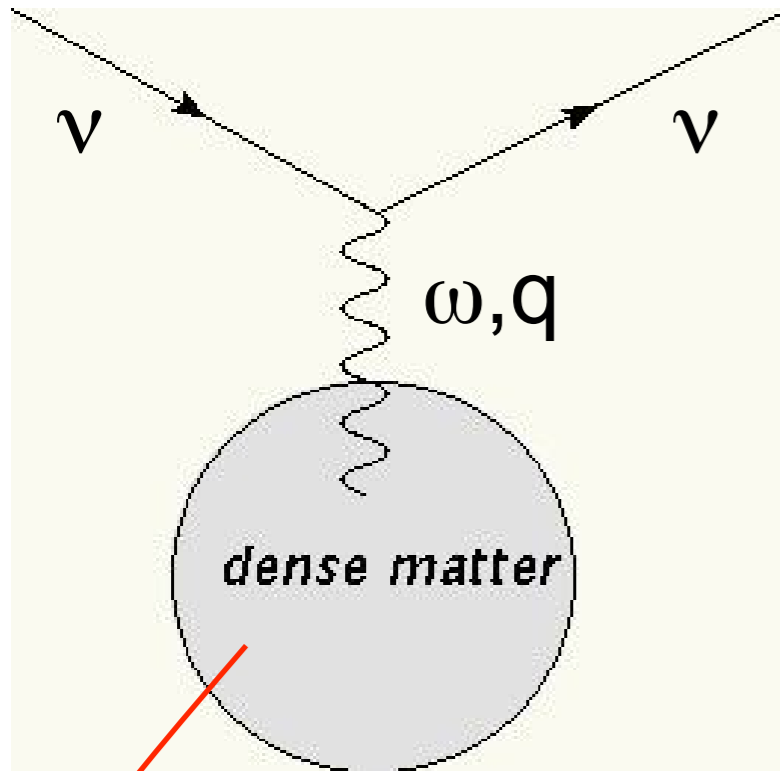
Reddy, et al. (1998, 1999, 2000, 2001, 2002)

- Neutrinos “see” more than one particle in the medium.
- Nature of spatial and temporal correlations between nuclei, nucleons and electrons affect the scattering rate.
- Nucleon dispersion relation is altered. Energy shifts are important.

$$E_i(k) = \sqrt{k^2 + M^{*2}} + U_i \equiv K(k) + U_i$$

- Phase transitions to quark matter, matter with strangeness can greatly alter the neutrino mean free path.

# Neutrino Rates



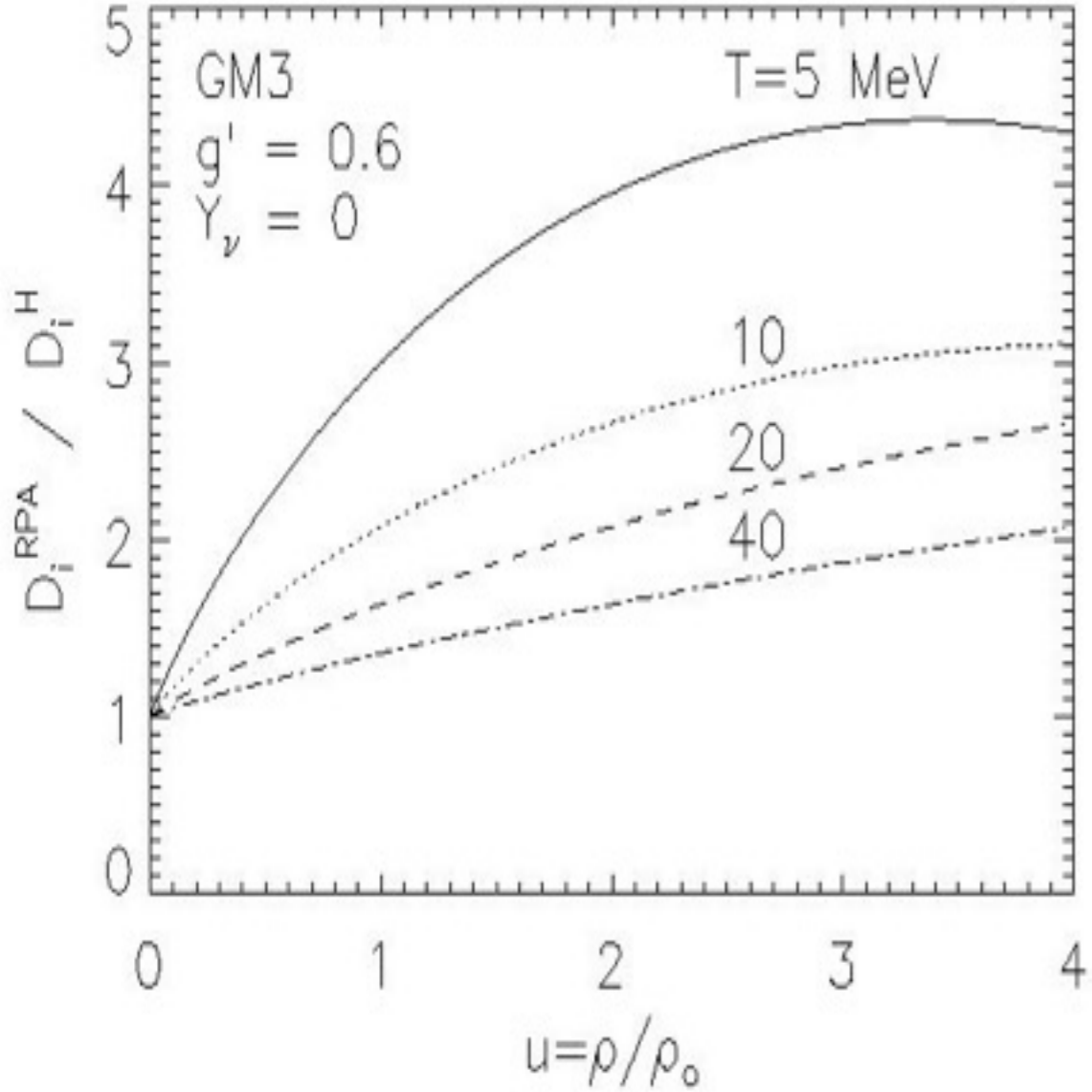
$$\frac{d^2\sigma}{V d\cos\theta dE'} \approx G_F^2 \frac{E}{E'} \text{Im} \left[ L_{\mu\nu}(k, k+q) \Pi^{\mu\nu}(q) \right]$$

$$L_{\mu\nu}(k, k+q) = \text{Tr} [ l_\mu(k) l_\nu(k+q) ]$$

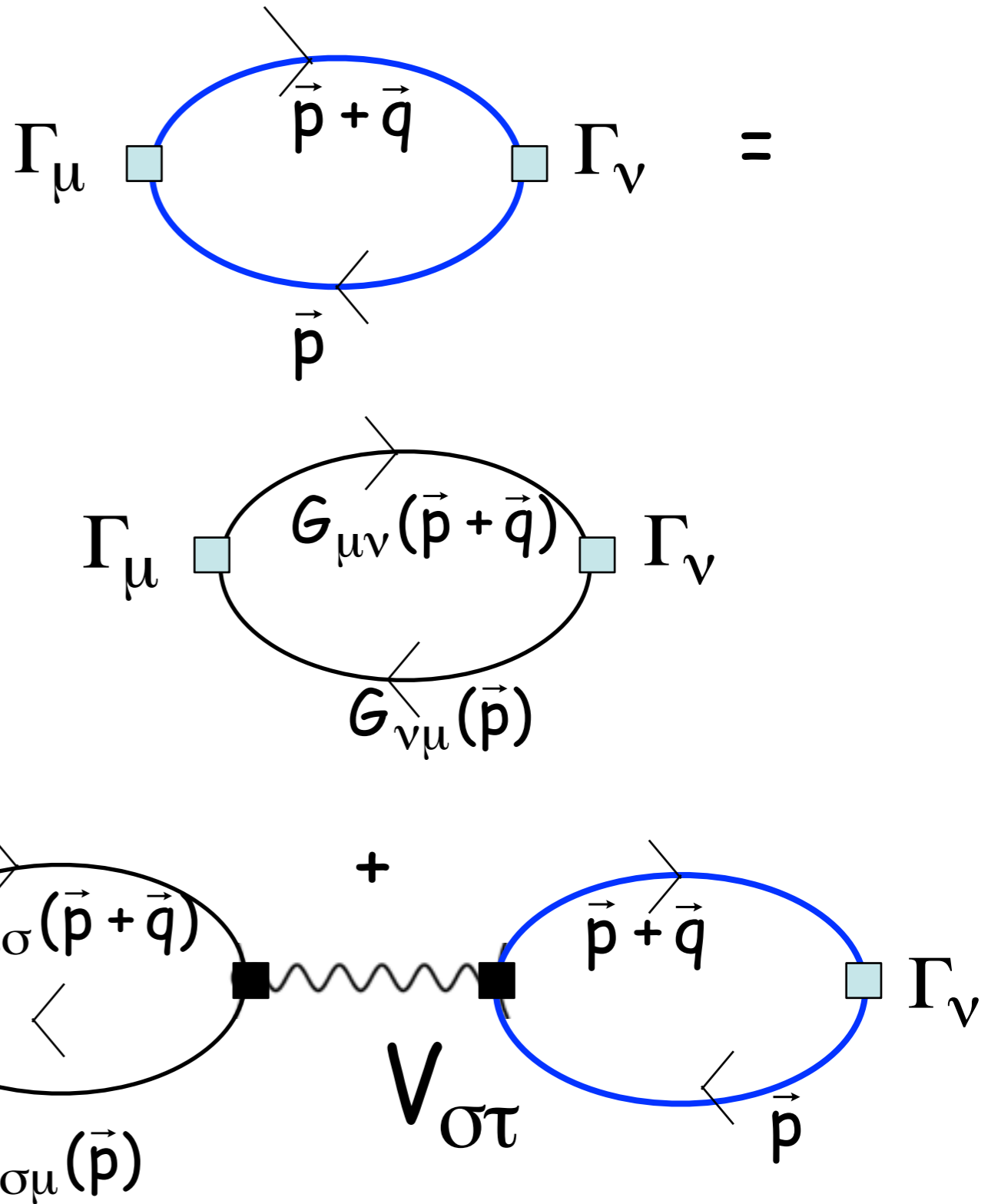
$$\Pi^{\mu\nu}(\omega, q) = \int \frac{d^4 p}{(2\pi)^4} \text{Tr} [ j^\mu(p) j^\nu(p+q) ]$$

Dynamic structure factor:  
spectrum of density, spin and current fluctuations.

# Neutrino Diffusion Enhanced by Nuclear Correlations



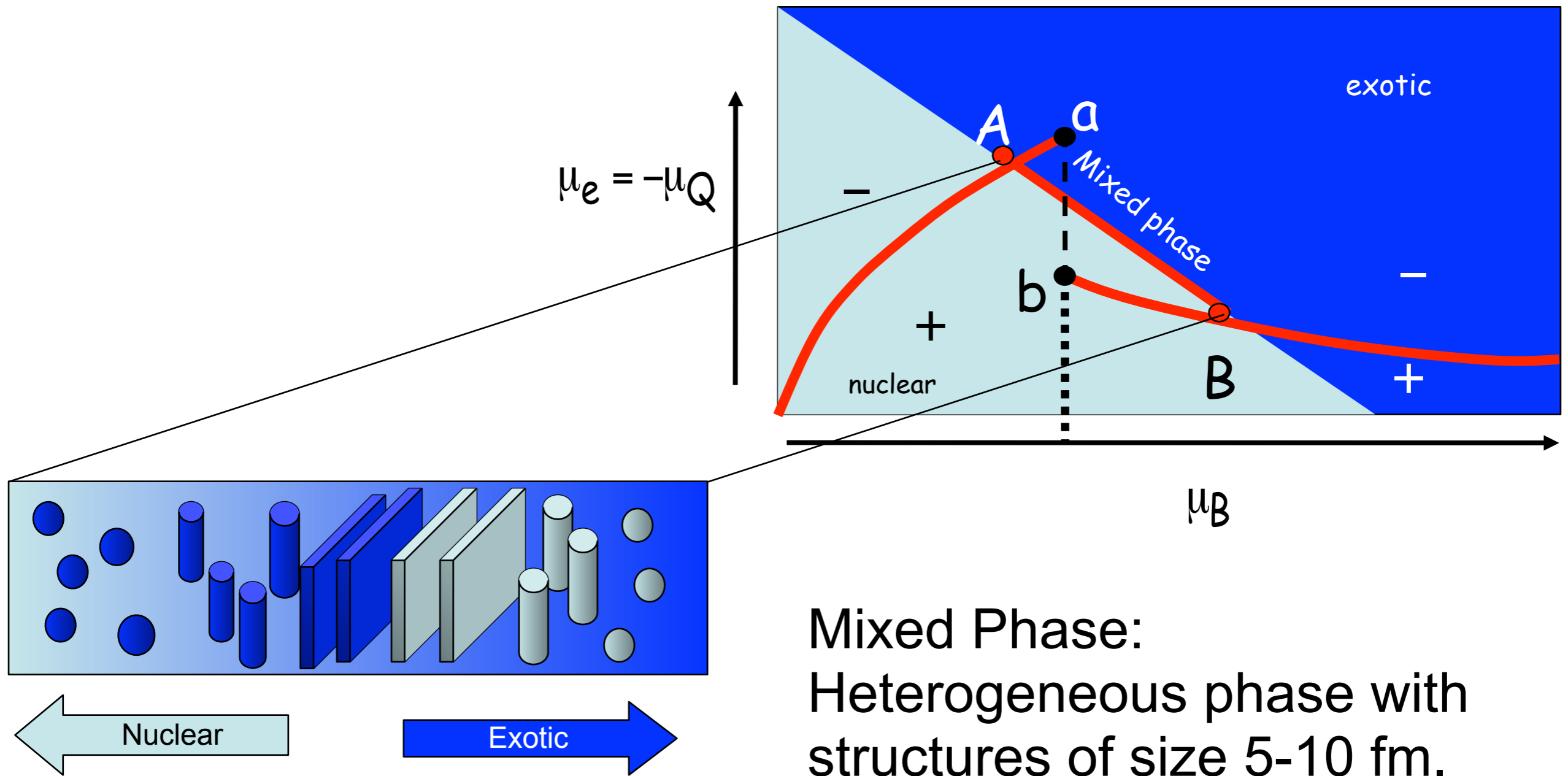
$$\Pi^{\mu\nu}(\omega, q) =$$



Horowitz & Wehrberger, (1991)  
 Burrows & Sawyer, (1999)  
 Reddy, Pons, Prakash, Lattimer, (1998,1999)

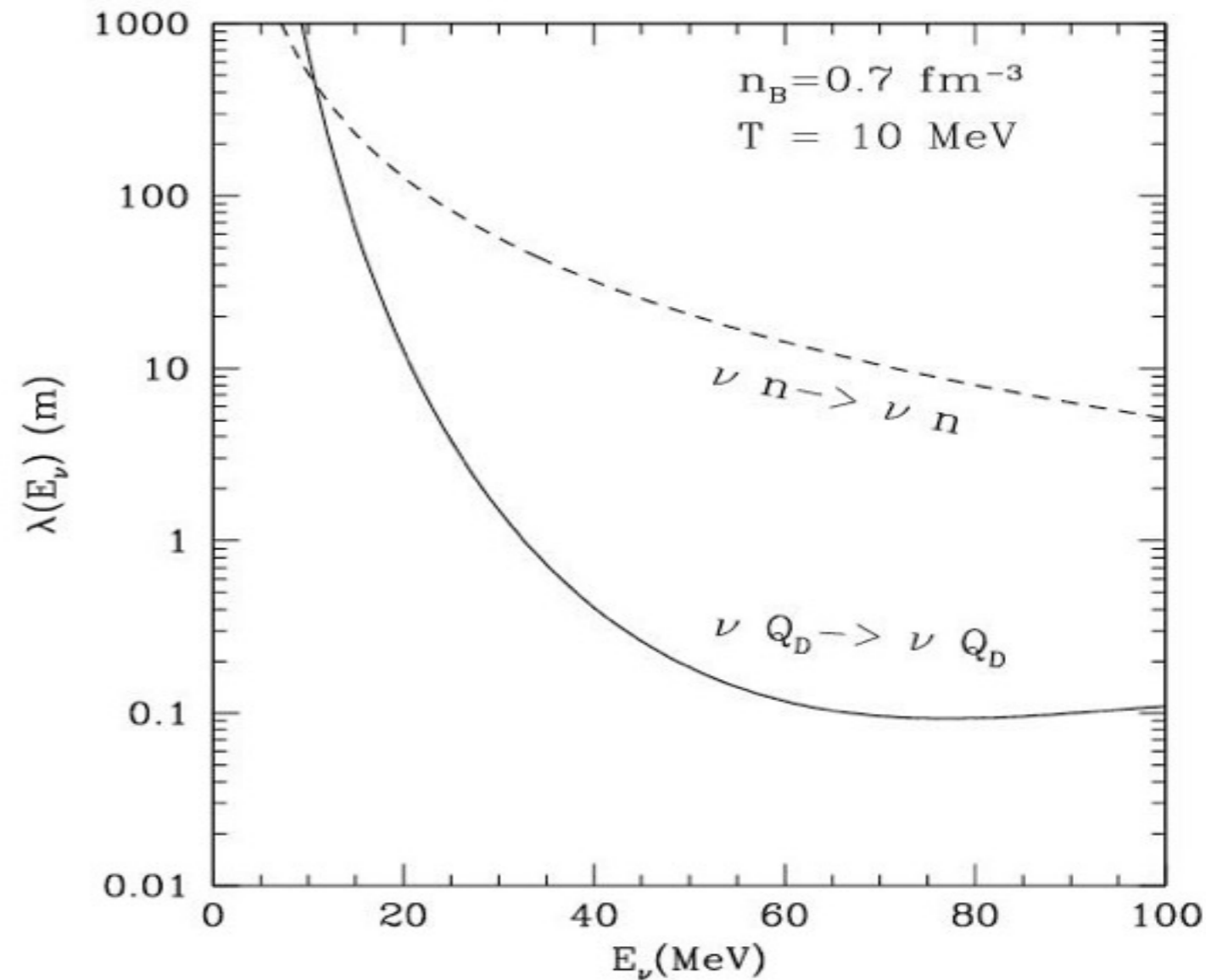
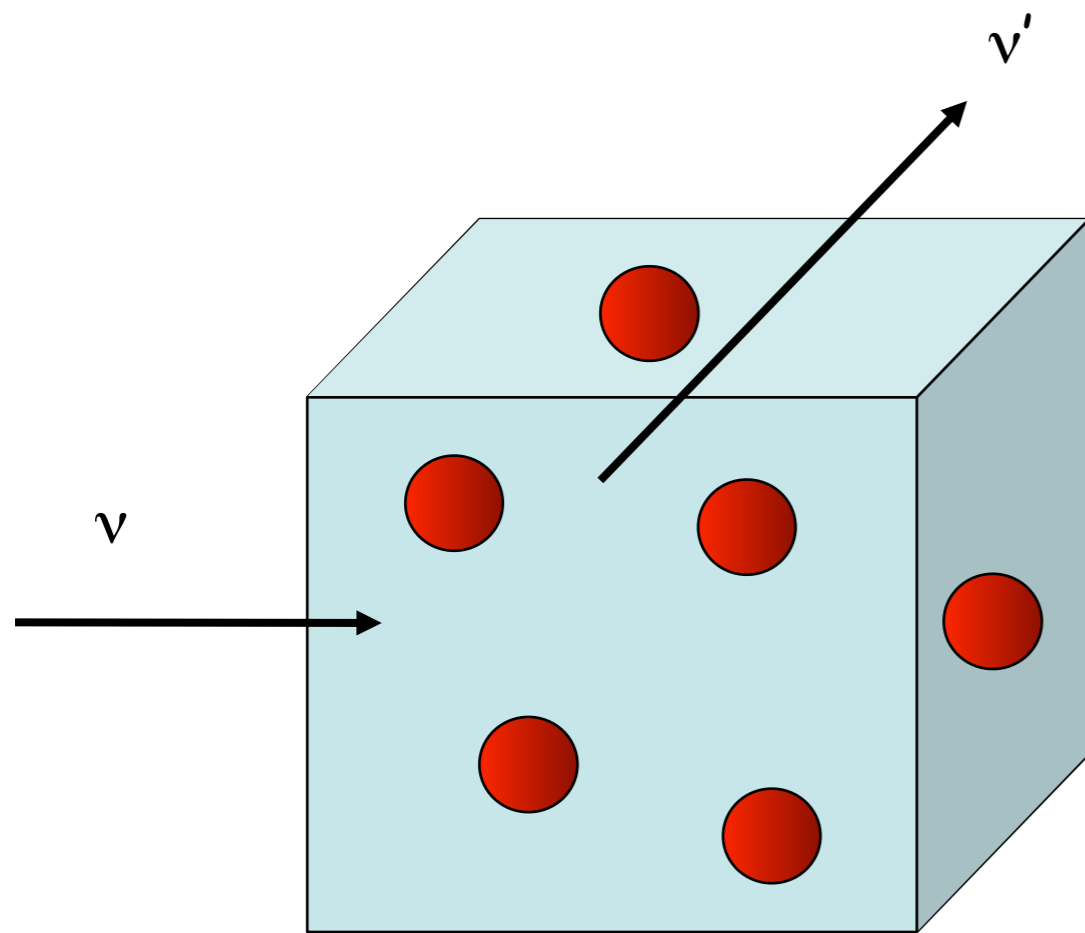


# First-Order Phase Transitions in the PNS Core



Mixed Phase:  
Heterogeneous phase with  
structures of size 5-10 fm.

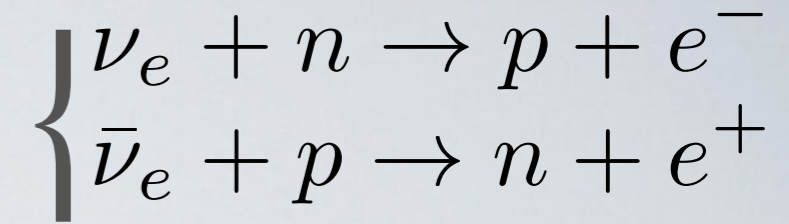
# Neutrino Mean Free Path in a Mixed Phase



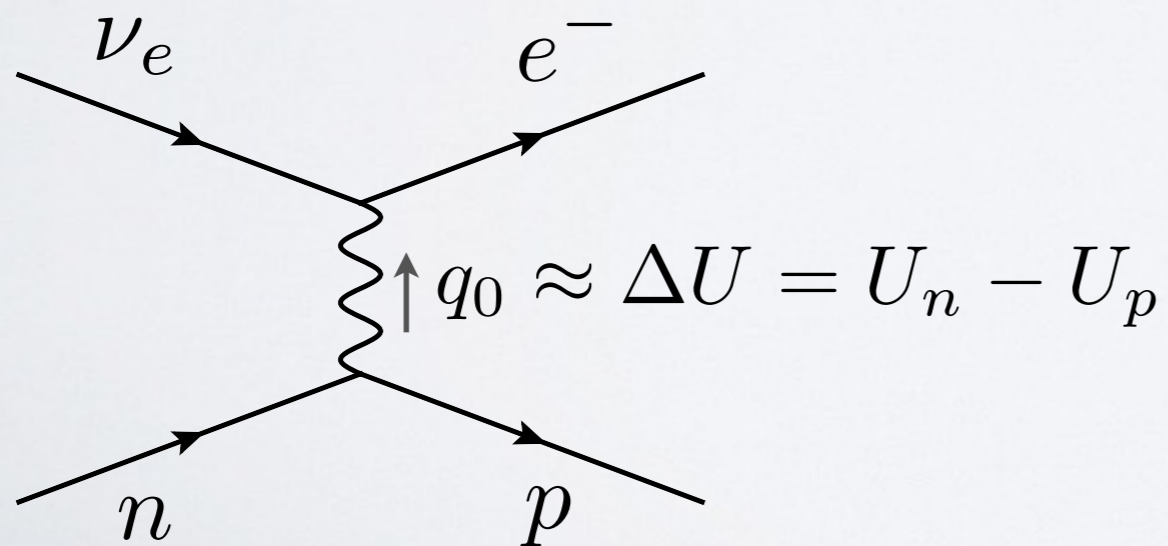
Scattering from quark droplets in a quark-hadron transition.

$$\frac{d\sigma}{d\cos(\theta)} = N_D \frac{G_F^2}{16\pi} S_q Q_W^2 E_\nu^2 (1 + \cos(\theta))$$

# Charged Current Reactions

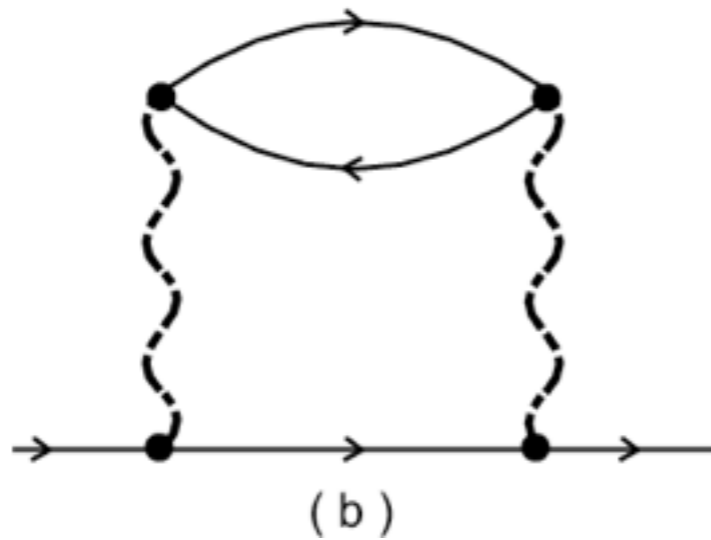


- Determine the electron neutrino spectra and deleptonization times.
- Final state electron blocking is strong for electron neutrino absorption reaction.
- Asymmetry between mean field energy between neutrons and protons alters the kinematics.



Reddy, Prakash & Lattimer (1998)  
Roberts (2012)  
Martinez-Pinedo et al. (2012)  
Roberts & Reddy (2012)

# SINGLE PARTICLE ENERGY SHIFT & DAMPING



$$E_n(p) \approx m_n + \frac{p^2}{2m_n^*} + U_n + i \Gamma_n$$

$$E_p(p+q) \approx m_p + \frac{(p+q)^2}{2m_n^*} + U_p + i \Gamma_p$$

Energy Transfer in the Charged Current Process:

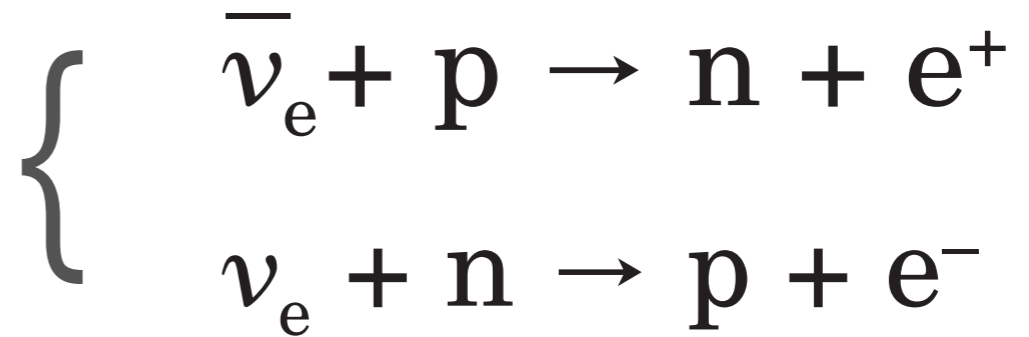
$$q_0 = E_n(p) - E_p(p+q) \simeq \frac{pq}{2m_n^*} + (m_n - m_p) + (U_n - U_p)$$

$$\simeq 0 \quad \simeq 1.3 \text{ MeV}$$

$$\Delta U = U_n - U_p \approx 40 \frac{n_n - n_p}{n_0} \text{ MeV}$$

# $Y_e$ in the Neutrino Driven Wind

Is set by the charged current reactions in two regions.



$$Y_e^{\text{NDW}} \approx \frac{\dot{N}_{\nu_e} \langle \sigma_{\nu_e} \rangle}{\dot{N}_{\bar{\nu}_e} \langle \sigma_{\bar{\nu}_e} \rangle + \dot{N}_{\nu_e} \langle \sigma_{\nu_e} \rangle}$$

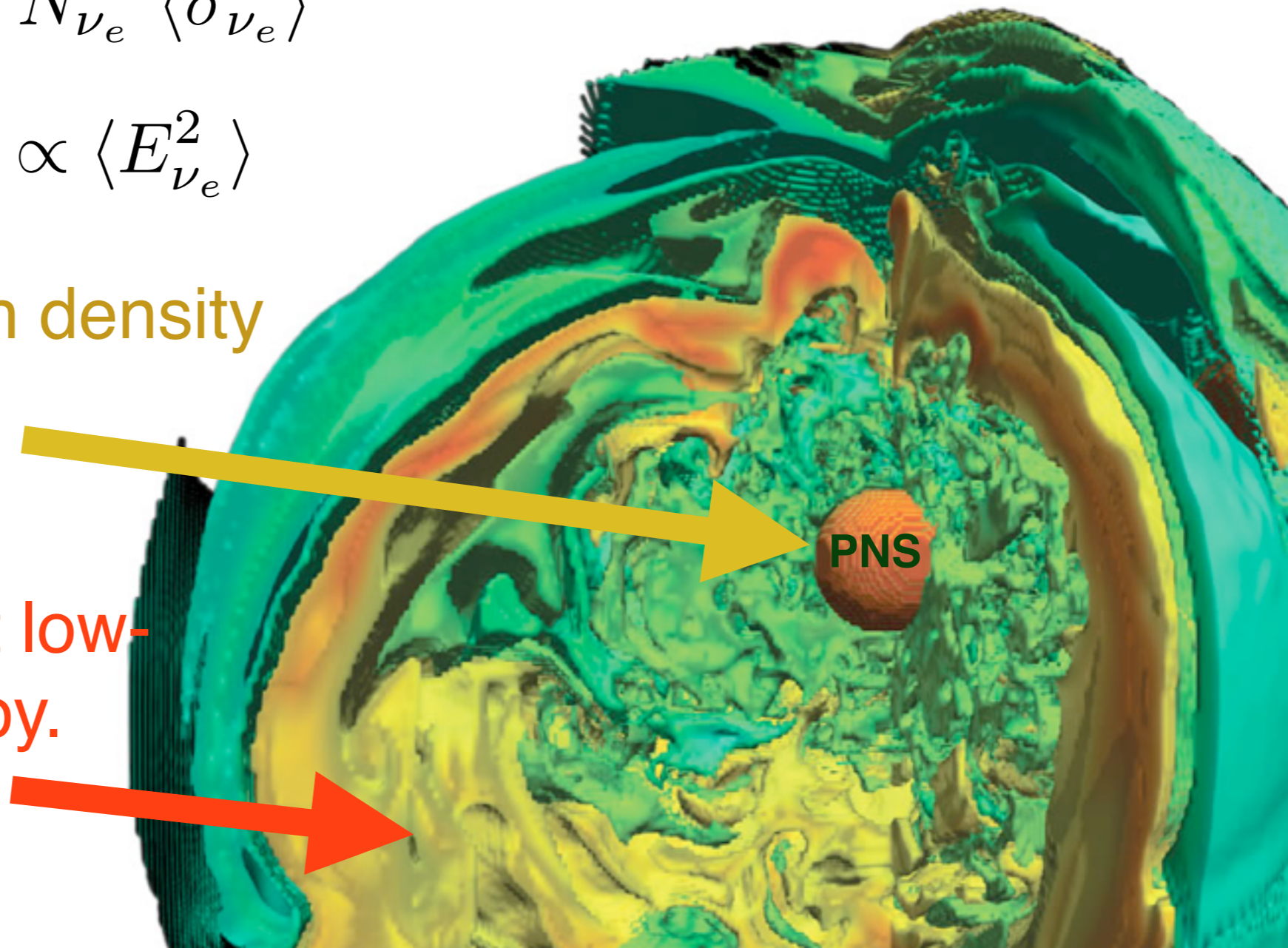
$$\langle \sigma_{\bar{\nu}_e} \rangle \propto \langle E_{\bar{\nu}_e}^2 \rangle \quad \langle \sigma_{\nu_e} \rangle \propto \langle E_{\nu_e}^2 \rangle$$

Neutrino-sphere at high density and moderate entropy.

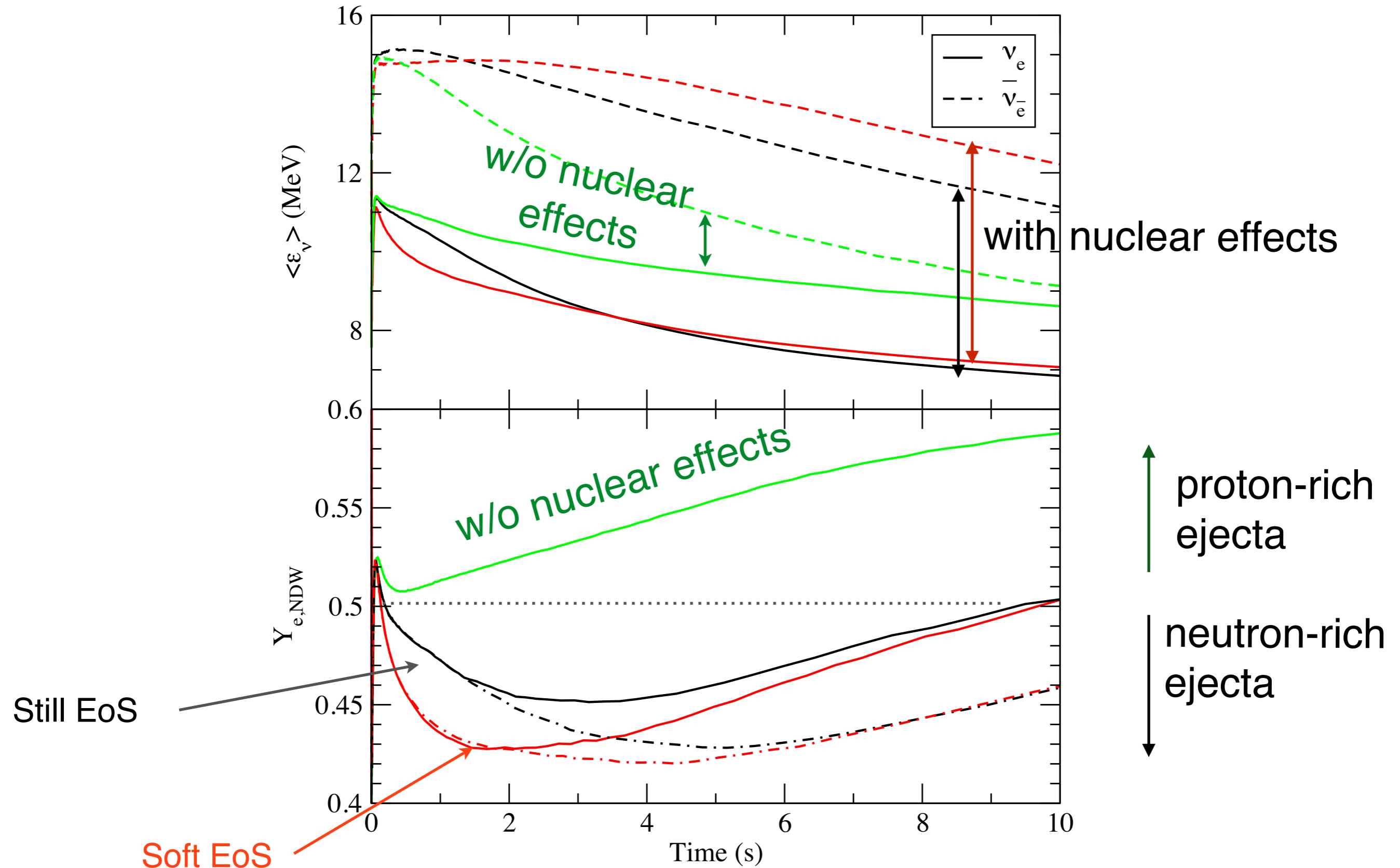
$R \sim 10\text{-}20 \text{ km}$

Neutrino driven wind at low-density and high entropy.

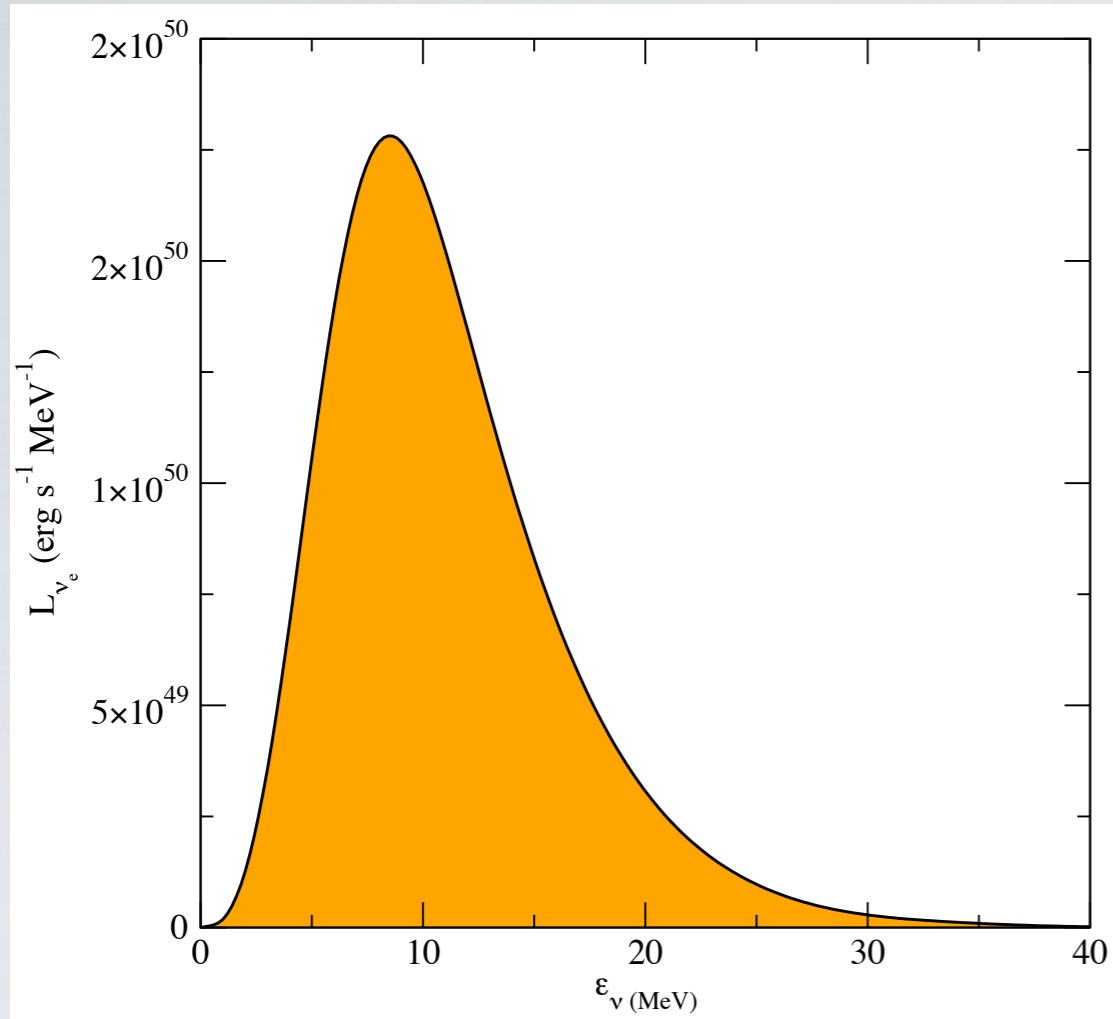
$R \sim 10^3\text{-}10^4 \text{ km}$



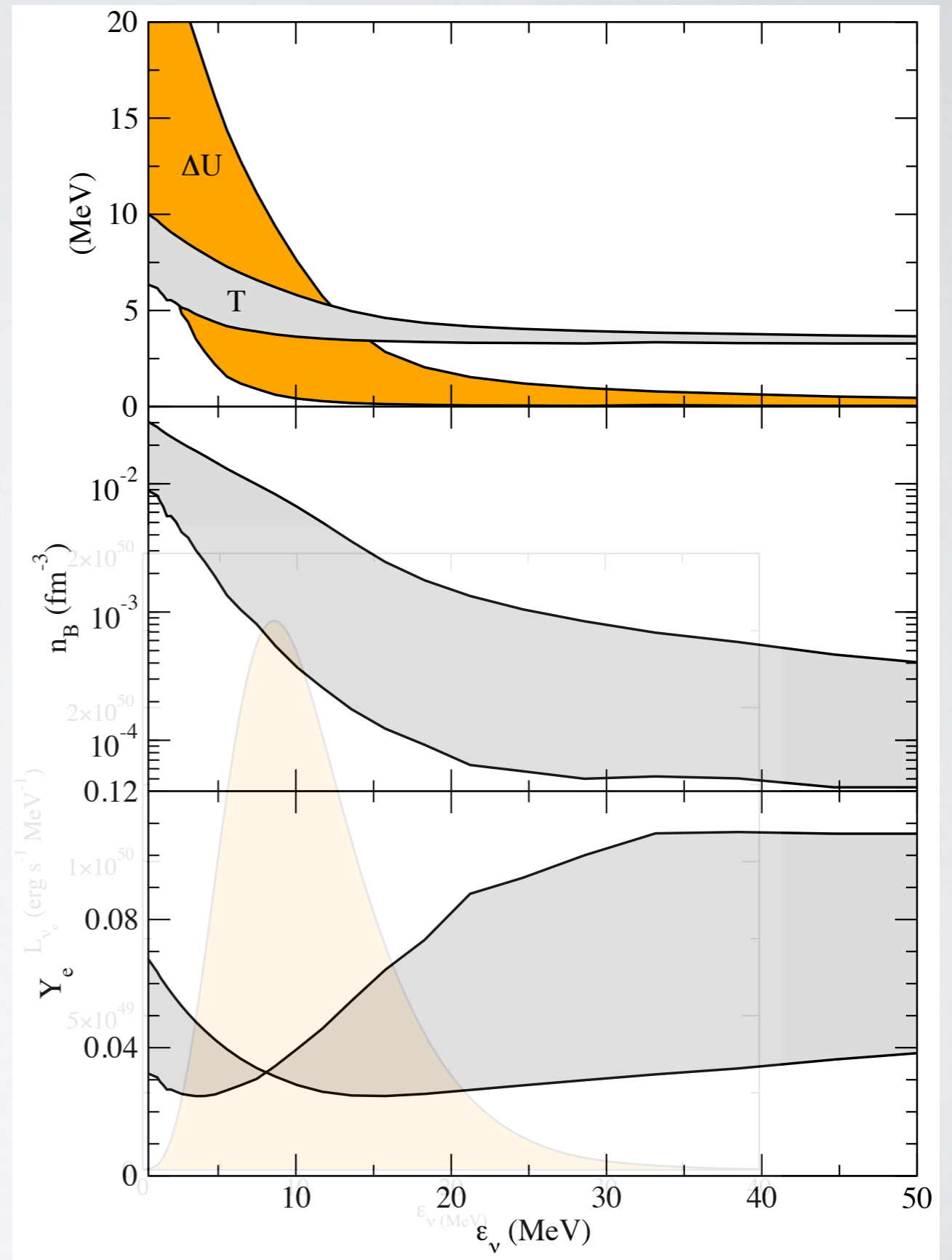
# Spectra & Nucleosynthesis



# Spectra at late times

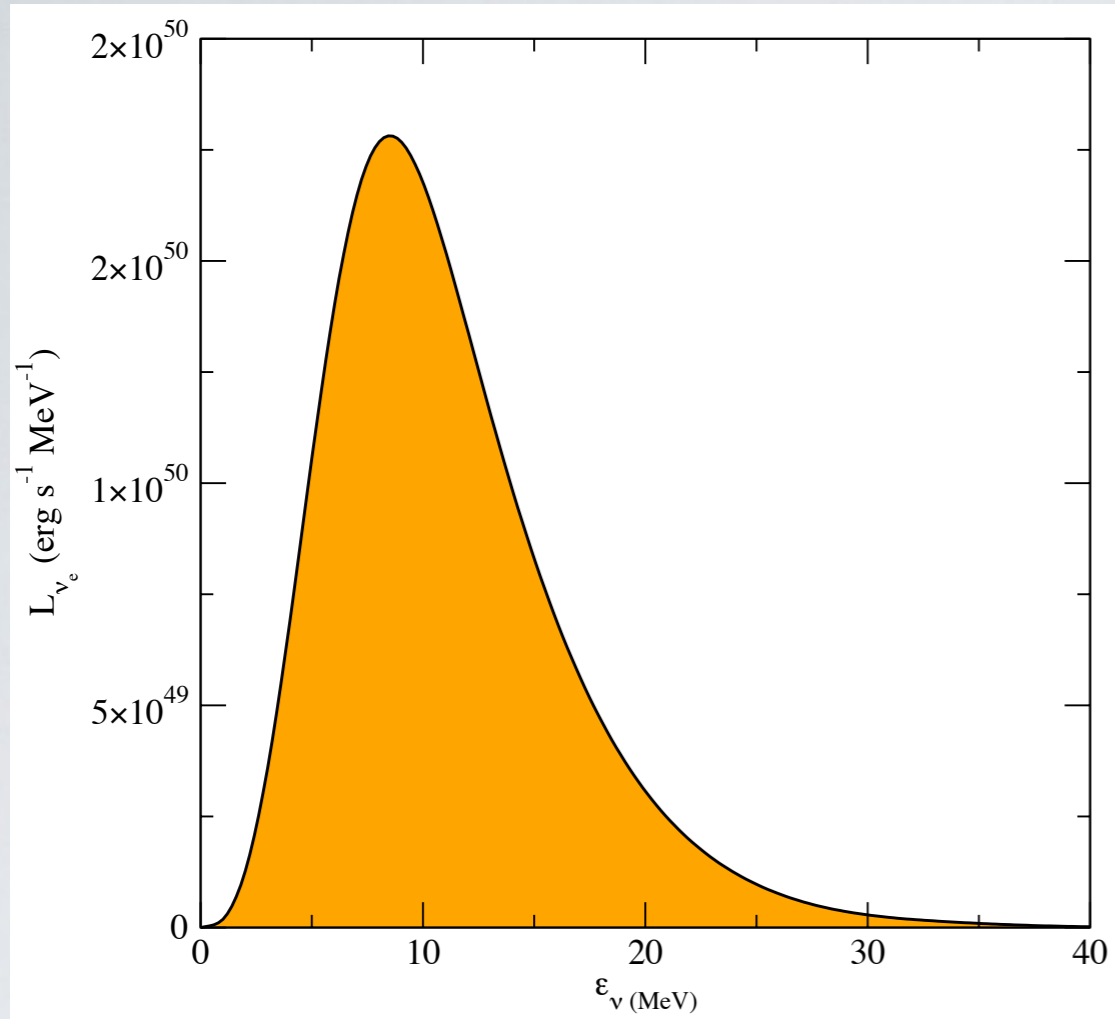


- Decoupling occurs at relatively high density.
- Spectra influenced by energy shifts and nuclear correlations.

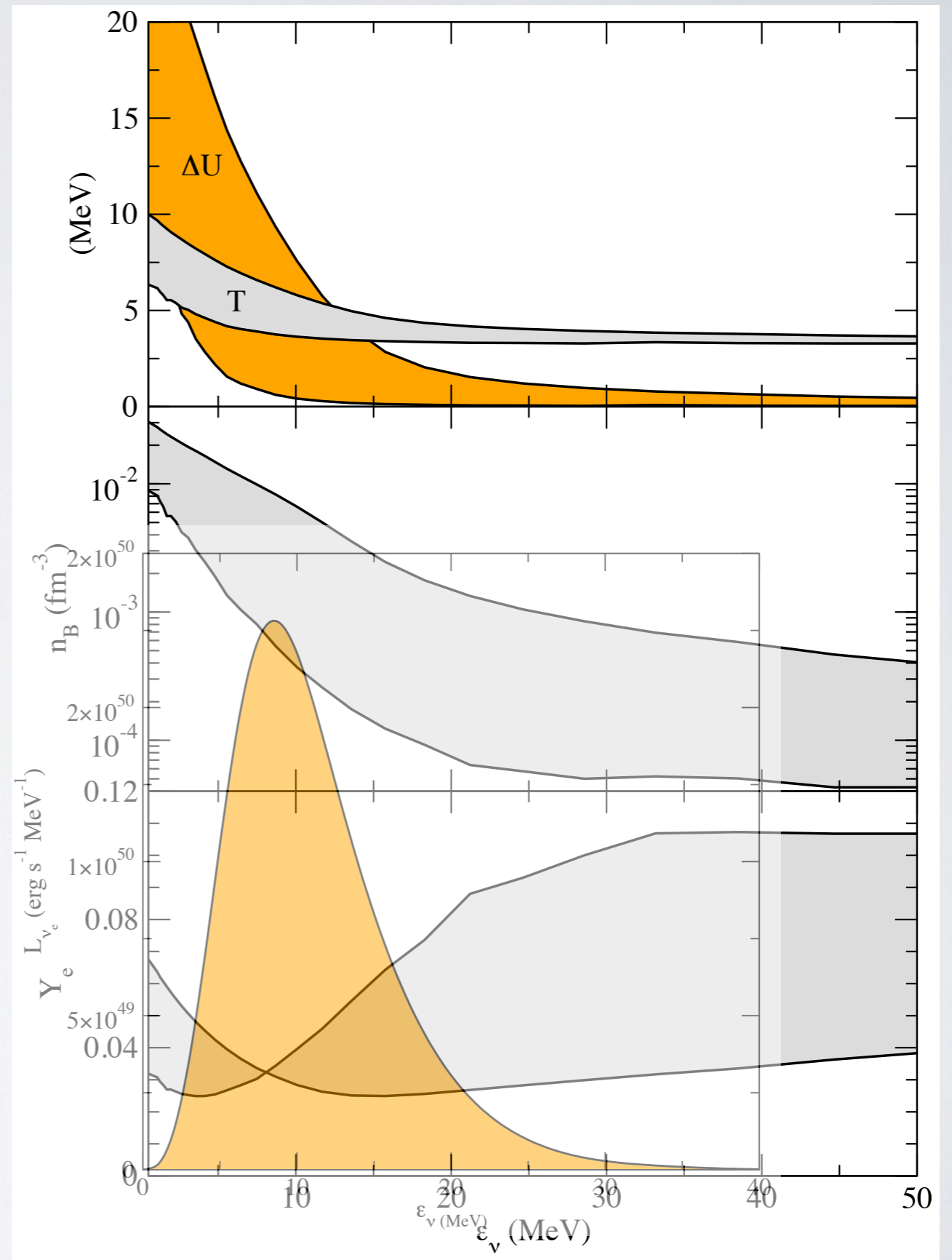


Figures from PNS simulations by Roberts (2012)

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Figures from PNS simulations by Roberts (2012)



# PNS evolution: Role of EoS.

Heat transport : Neutrino diffusion + convection

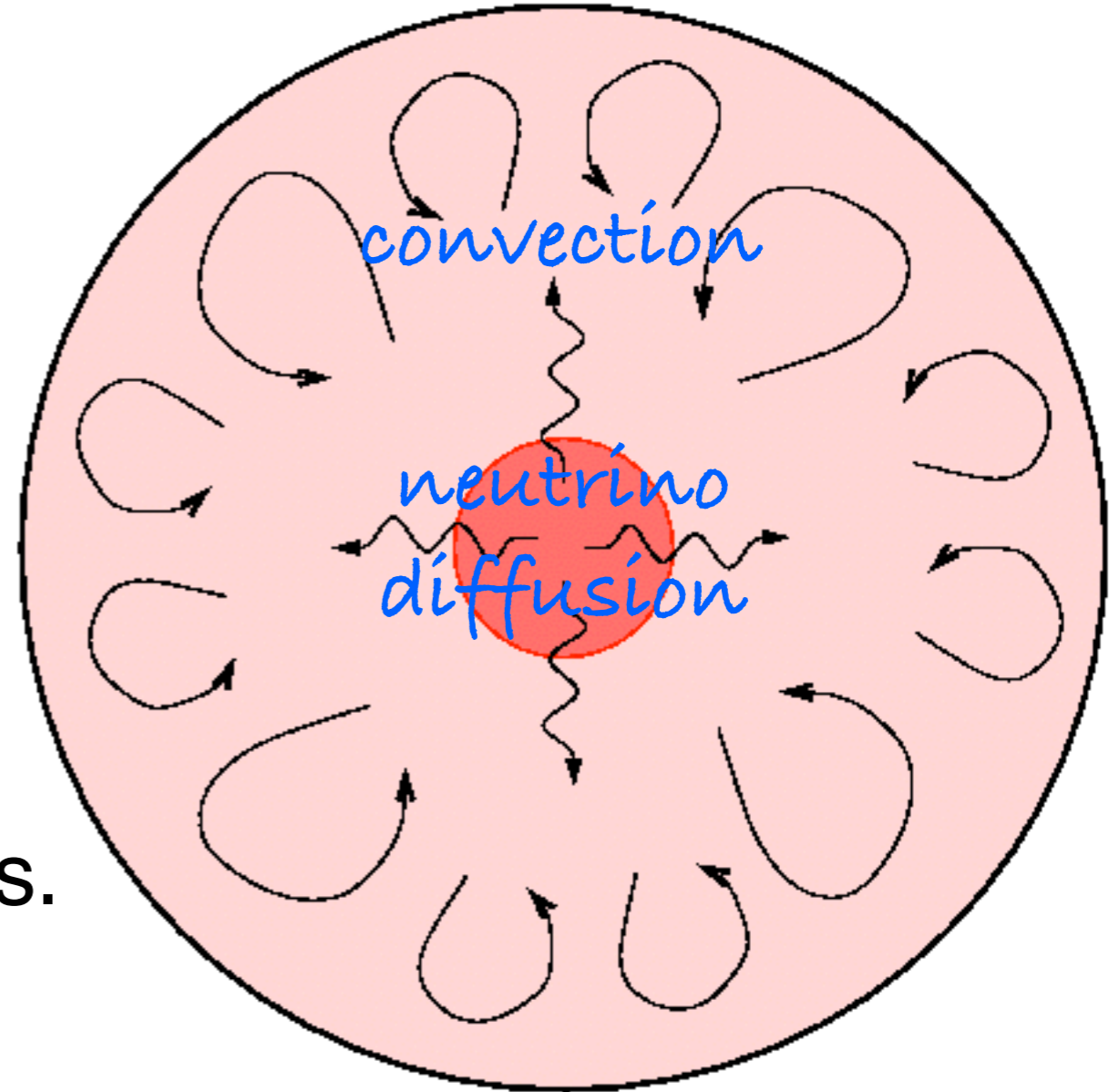
Diffusion:

$$\tau_{\text{diff}} \simeq \frac{R^2}{c \lambda_\nu} \approx 3 - 5 \text{ s}$$

Convection:

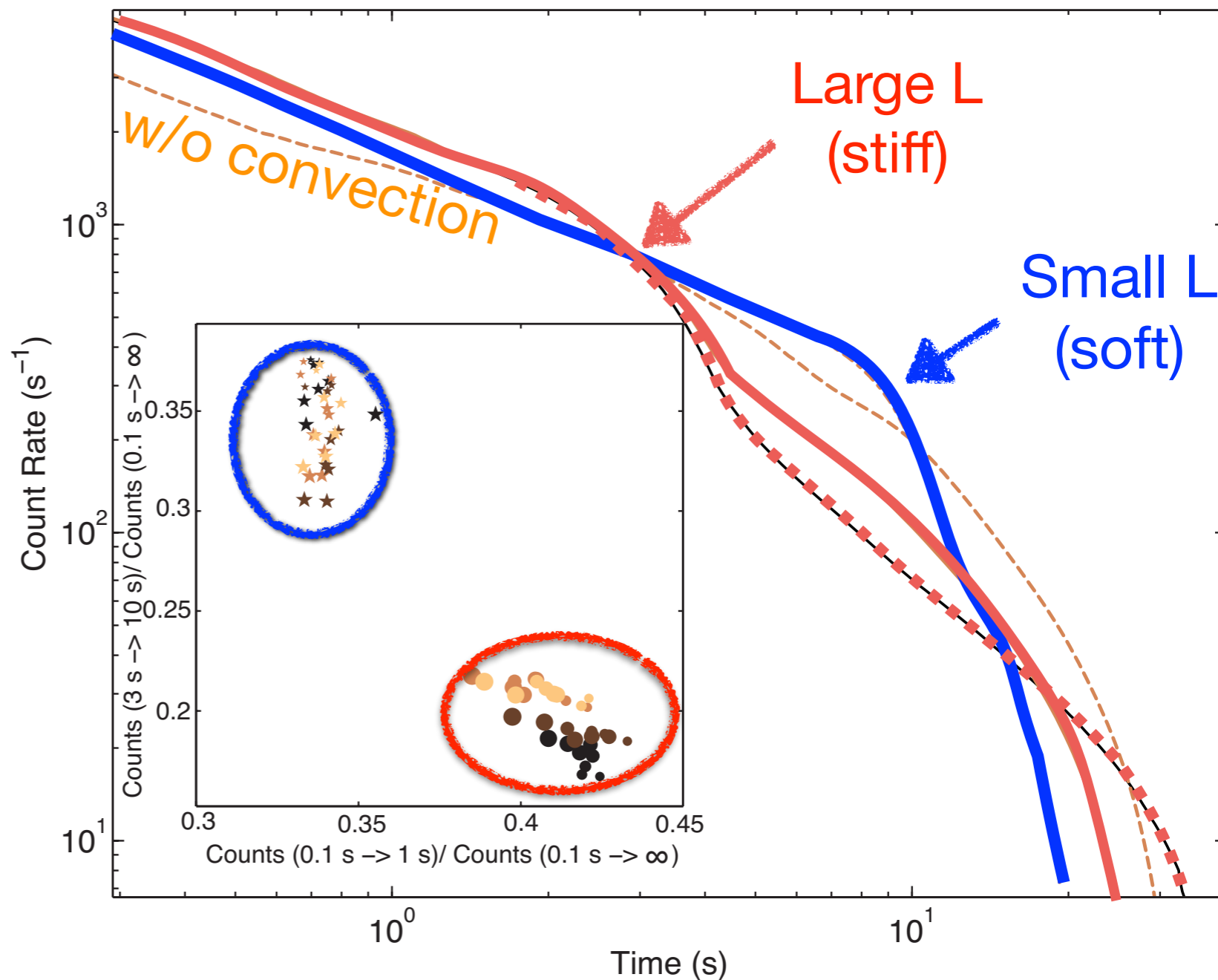
Convection is driven by composition and entropy gradients.

The buoyancy of matter depends on nuclear equation of state.



# Observable signatures of convective transport

- Neutrino flux is enhanced during convection.
- There is break in the light curve (when convection ends).
- Fraction of events between 3-10 s provides good discrimination.

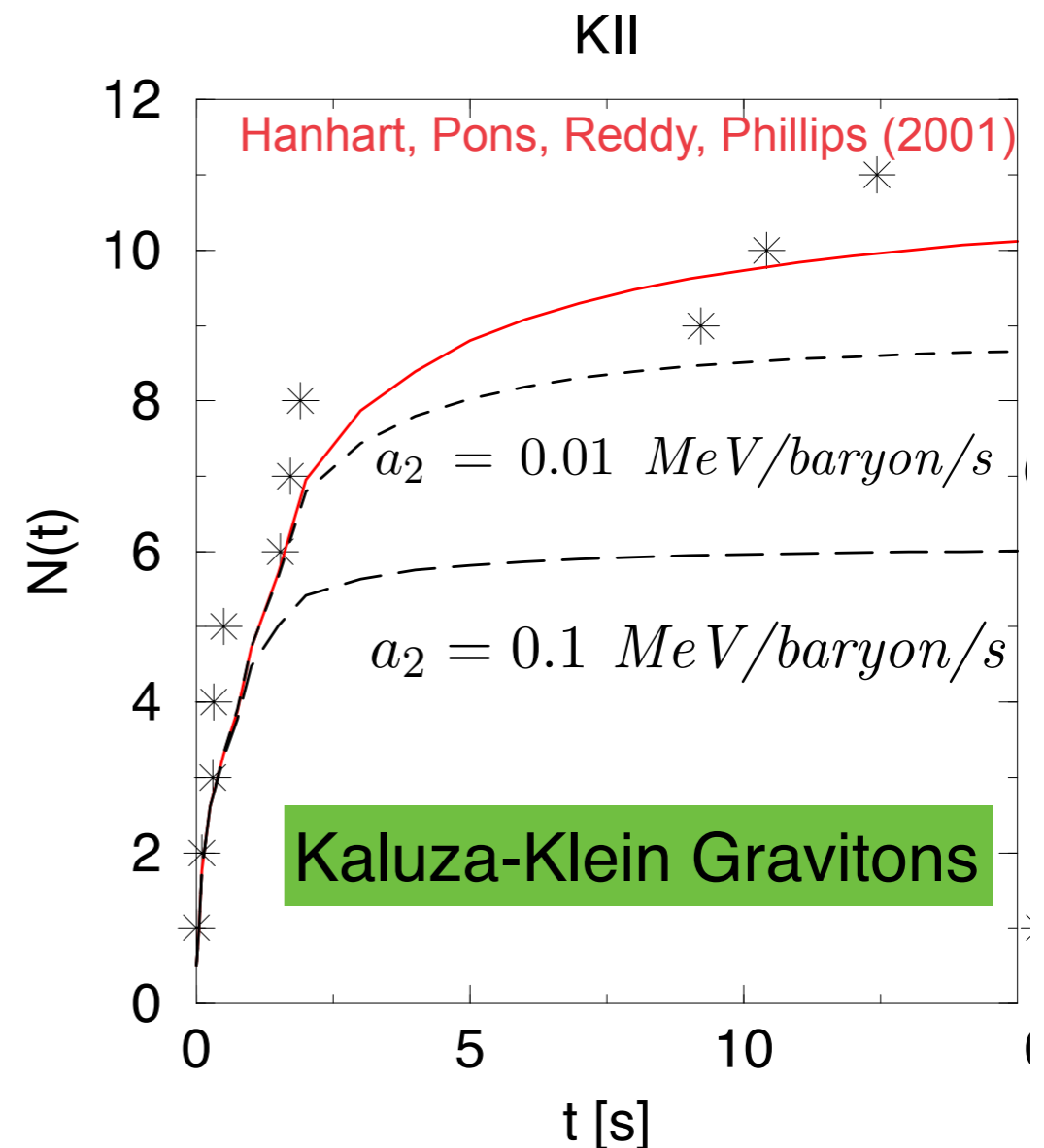
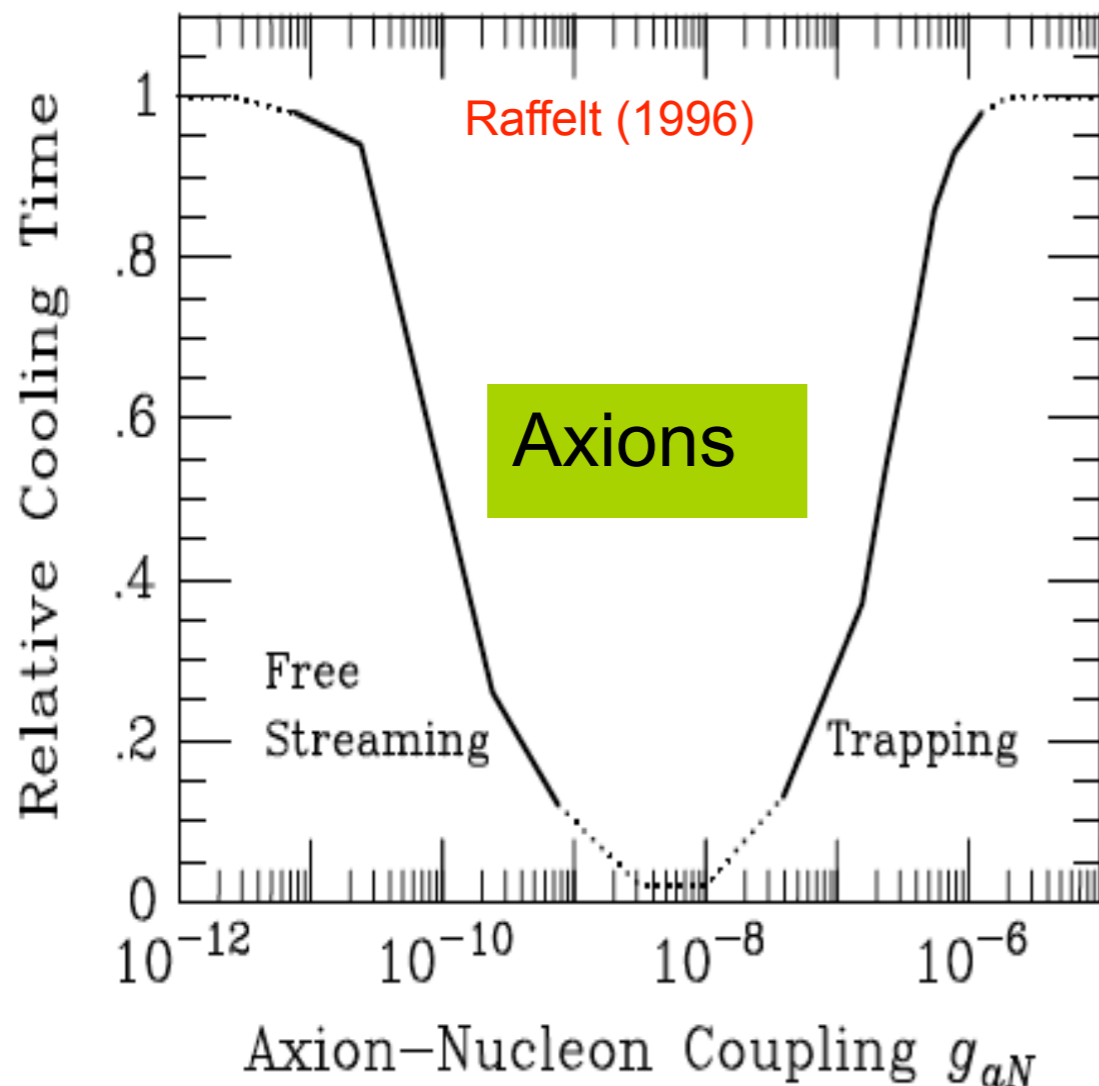


Count rate in Super-Kamiokande for galactic supernova at 10 kpc.

# Hypothetical Weakly Interacting Particles

Since neutrinos are trapped for  $\sim 20$  s a new particle that couples more weakly to matter can radiate away the binding energy.

When this energy loss  $> 10^{19}$  ergs/g/s it will shorten the neutrino time scale by a factor of 2.

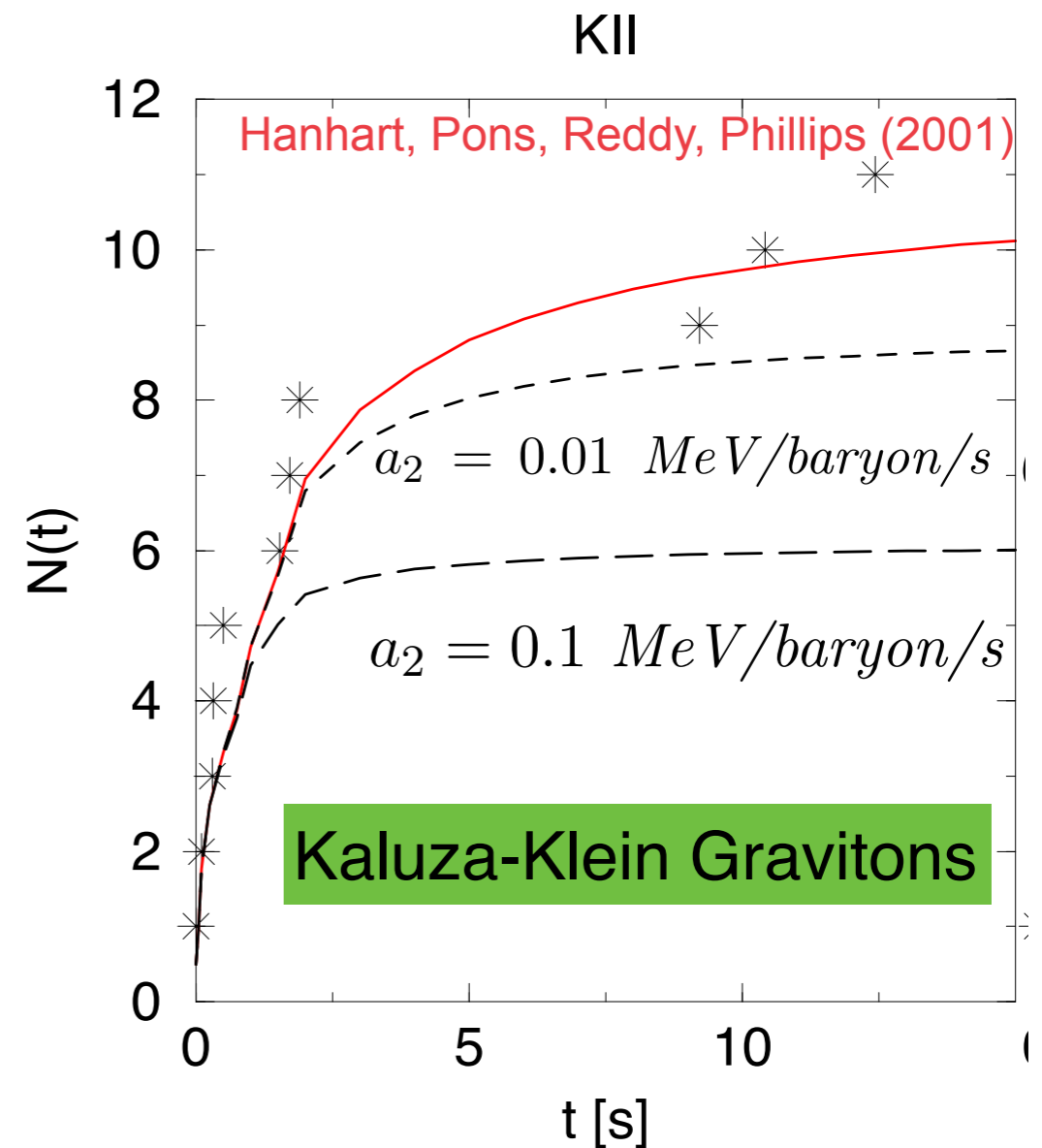
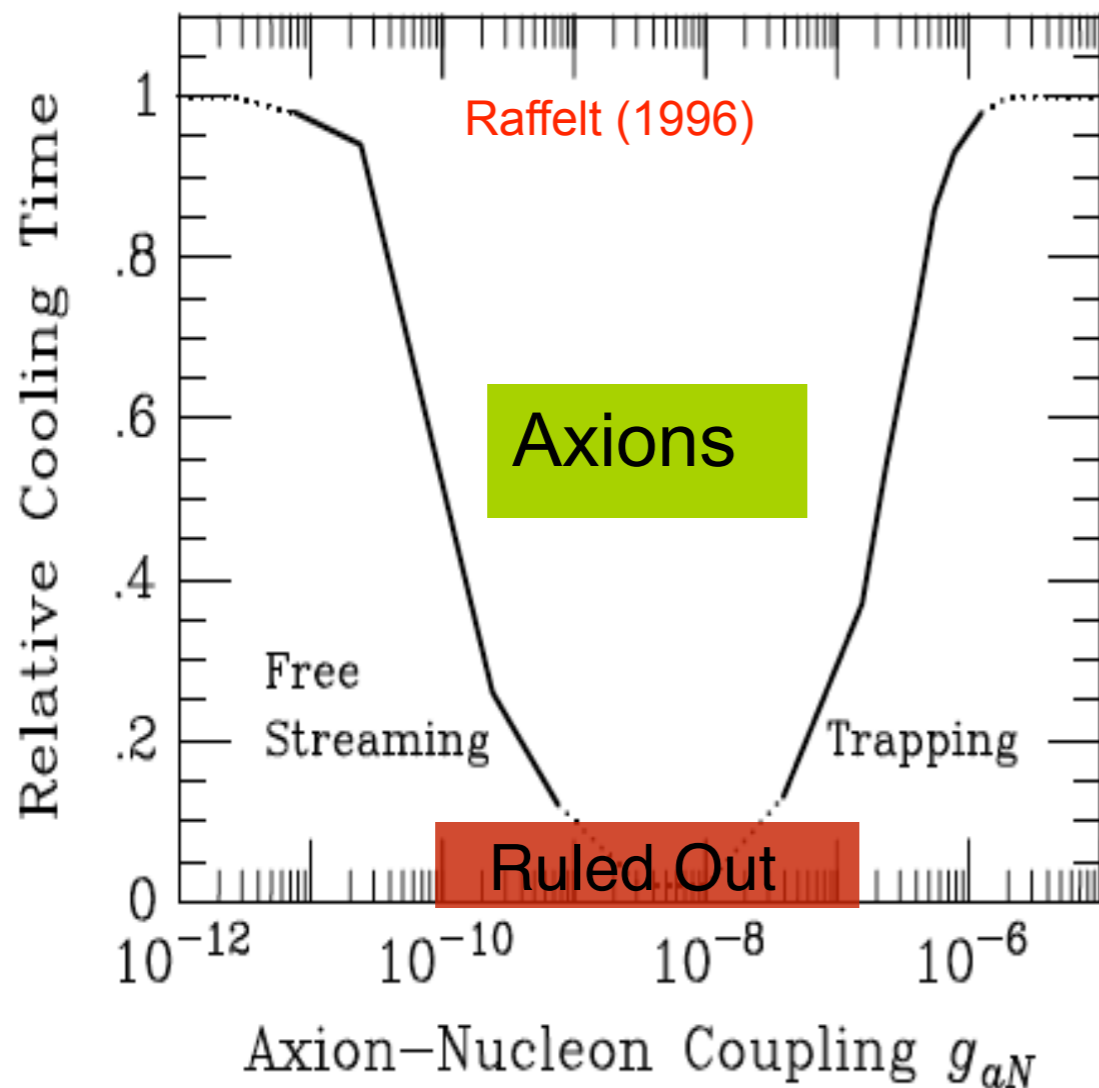


$$\frac{dE}{dt} = a_n \left( \frac{n_B}{n_0} \right) \left( \frac{T}{10 \text{ MeV}} \right)^{p_n} \chi(X_n, X_p) \text{ MeV/baryon/s}$$

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# SN neutrinos can reveal early evolution of a neutron star

