Non-linear Dynamics of Flexural Disc Transducers

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Abstract : **Flexural transducers are well suited for power emission in fluids, thanks to their low mechanical impedance. Their performance is nevertheless compromised by the occurrence of non-linearities stemming from high-deflection and/or high electric field. This experimental work catalogs typical alterations of the transducers' dynamics resulting specifically from high electrical field non-linearities.**

We use depoled transducers to model efficient ones under high electrical field conditions. The depoled transducer are not subject to purely structural non-linearities as the deflection remains low but still exhibit electric-field-related non-linearities.

Results show that the response of the transducers can undergo major qualitative changes as the strength of the excitation increases : hysteresis cycles, super-harmonic excitation sharp saturation of the displacement, non periodic response of the transducer to an AC forcing tension and unstable transfer of energy between the spectral components of the displacement signal. Practical implications include transducers' design, model parameter fitting and second harmonic generation.

INTRODUCTION

Possible sources of non-linearities in piezoelectrically driven transducers are numerous [1]. Beside the non-linearities in the driving circuitry (saturation of ICs and of the transformer, distortion in ICs, ...), the three most common sources of non-linearities are geometrical effects (stress stiffening, ...), inertial effects (rotary inertia, ...) and material non-linearities (bound layers and saturation of the piezoelectric response).

The scope of this work is to catalog different types of alteration of the transducers dynamics resulting exclusively from material non-linearities, and more precisely from the non-linearities entailed by the presence of an high electric field E in the piezoelectric material. We focus in particular on the first mode of vibration of flexural transducers.

APPROACH

As a study system, we would like to enhance the E-field related material non-linearities (NL) without generating other ones. Circuit linearity can be easily insured but increasing the applied E field results in :

1) a bad repeatability caused by some steady depoling during the characterization

2) the coming into play of structural non-linearities (NL), caused by the high deflection generated by the high voltage.

The "trick" used to overcome these difficulties consists in simulating an efficient transducer subject to E-field related NL with a depoled transducer. Reducing the "linear" responsivity of the transducer to the E-field, enhances the relative strength of the non-linear effects. A crude model, for explanation purposes, consists in a 1D oscillator including purely structural and E-field related NL. X represents the deflection of the transducer's membrane and obeys to :

$$\ddot{X}+\eta.\dot{X}+\omega.X+\alpha.X^2+\beta.X^3+\ldots$$
$$=f.E+\delta.E^2+\varepsilon.E.X+\ldots.$$

with obvious notations.

When the piezoelectric is depoled, f is small and so becomes the deflection X. Consequently, the structural NL term : $\alpha.X^2+\beta.X^3+\ldots$ can be neglected but not the E-filed related term : $\delta.E^2+\varepsilon.E.X+\ldots.$ which gains relative strength compare to $f.E$. In practice, the coefficients δ , ε... are also affected by the depoling but in a less critical fashion (since NL are experimentally observed). Again, the model is only presented to justify the experimental procedure ; in real devices non analytical NL are not excluded.

SET-UP AND PROCEDURE

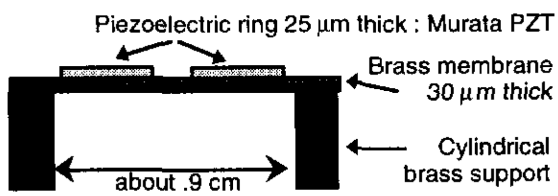

fig.#1 : transducer geometry

The piezoelectric flexural mode disc transducers used consist in a membrane, constrained along its circumference, which is driven by one overlaying piezoelectric ring.
The membrane is a 13 mm-diameter brass disc of 30 micron thickness. It is bounded with epoxy along its circumference to a 2 mm-thick brass cylinder, which reduced the effective diameter of the membrane to 9 mm. A 25-micron-thick-piezoelectric ring, made of Murata surface wave transducer is bounded (epoxy) on the membrane. Its inner and outer diameters are typically 2 and 7 mm. In fact the piezoelectric ring diameters and thicknesses vary from one transducer to the other and this results in variation in the distribution of the natural frequencies. As one consequence, internal resonances are observed on some of the transducers and not on others. The first resonance frequencies fall in the 2 kHz-20 kHz range. A repeated operation of those transducers over several months and under up to 10-20 Volts resulted in some significant depoling of the piezoelectric layer.

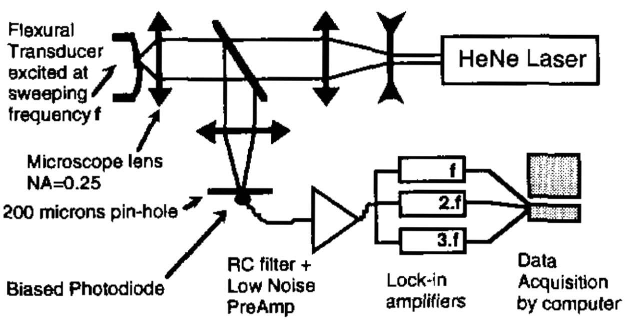

fig.2 : displacement measurement

With a confocal microscope, we measure the displacement of the transducer as a function of frequency. The schematics of the optical set-up is reproduced in fig.#2. Basically, a laser beam is first expanded and then focused by a microscope lens on the surface of the transducer. The reflected light goes back through the microscope lens, and a second lens focuses it on a pinhole. A biased photodiode collects the light right behind the pinhole. When the transducer moves, the reflected signal is no longer focused on the pinhole and indeed the photodiode receives less light. For displacement measurements, a slight off-set of the initial position on the membrane and a proper choice of the microscope lens numerical aperture is required to have a quasi-linear response of the photodiode to displacement. Sub-micron resolution are easily achievable. A R-C filters collects the signal from the photodiode and its AC components are amplified with a low noise voltage pre-amplifier. The output is then directed to three lock-in amplifiers which extract the fundamental frequency and the two next harmonics of the signal. The frequency driving the transducer are quasistatically swept up and down while a computer collects the lock-in amplifiers' outputs. This is repeated for magnitudes of the driving voltage ranging from 0V to about 40V. The spot on the transducer's surface where the displacement is measured is about 45 degrees off the diameter passing through the electrical connection of the upper electrode. This insures sensitivity to the non-axisymmetric modes (they induce little displacement at the center of the membrane and are oriented by the symmetry-breaking electrode connection). Measurements at the center of the membrane are occasionally necessary to discriminate between the vibration of a mode and the harmonic vibration of a lower mode.

RESULTS

Depending on the fabrication and aging variability among the 5 transducers tested, different qualitative behaviors are observed. The following results consist in a catalog of the typical non-linear behaviors observed.

Resonance frequency drift and Hysteresis: fig.3,4
Those two effects are widely described in the literature (see [2] for references). The bending right or left of the resonance curves becomes more pronounced as the applied voltage increases. At some point, the curve gets multi-valued and hysteresis cycles start to develop. Some slight saturation of the maximum displacement with respect to voltage is attributed to some nonlinear damping.

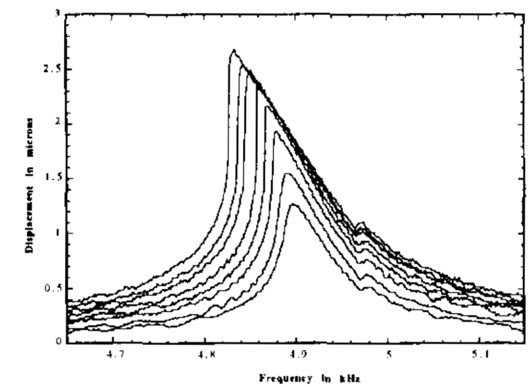

fig.#3 : Resonance frequency drift

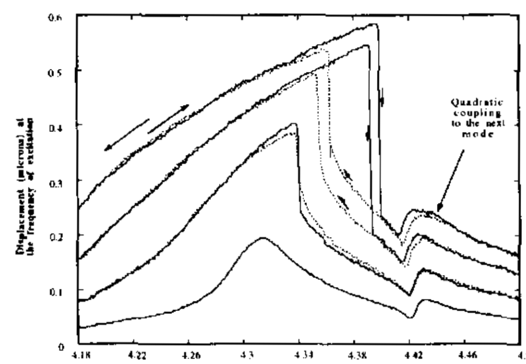

fig.#4 : Hysteresis

Saturation : fig.#5,6

The displacement at the frequency of the excitation, as it is measured by the first lock-in amplifier, shows a sharp saturation whose level is independent of the driving voltage and the direction of the frequency sweep During this saturated regime, the output of the second lock-in amplifier, "locked" at the harmonic 2*f, is about four times higher than before the saturation occured(not plotted). Note that the hysteresis cycle mentioned above is still present.

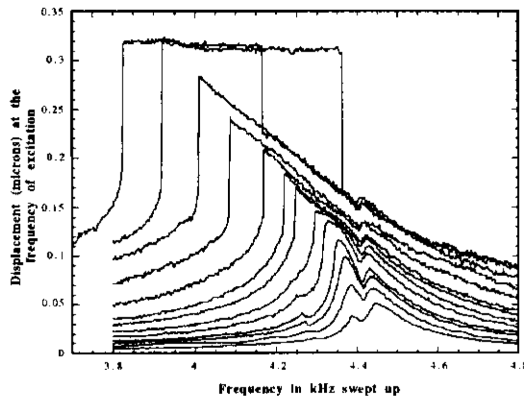

fig.#5 : Saturation, frequency swept up.

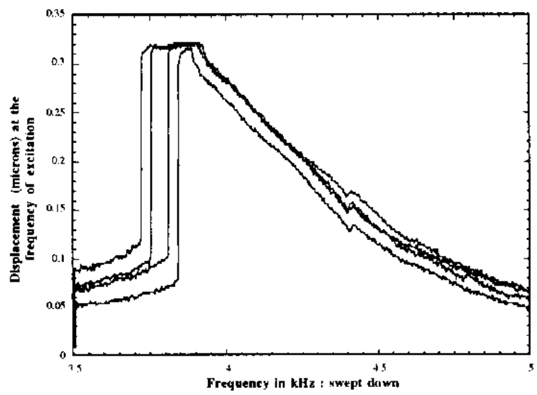

fig.#6 : Saturation, frequency swept down

Instability : fig.#7 @ f~4 kHz

Some transducers presented a unstable response of the first lock-in amplifier output. The displacement at the excitation frequency "f" drops as the 2*f displacement signal sharply raises. Note that a direct comparison of the magnitude of those two signals is not straightforward since it depends on the location of the laser spot on the membrane but the values clearly demonstrate a extremely efficient second harmonic generation.

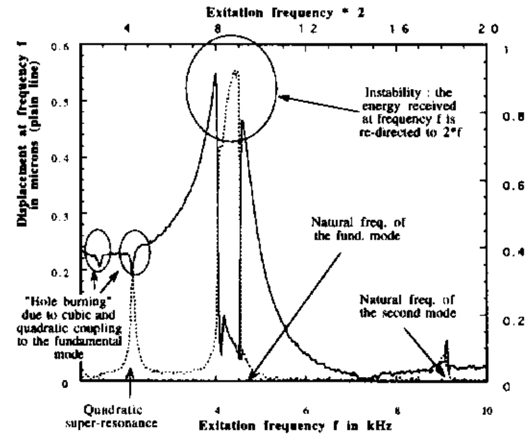

fig.#7 : Instability & Super-harmonic generation

Super-harmonic excitation : fig.#7 (and **fig.4** @ f~4.42 kHz)

Super-harmonic excitation consists in exciting a mode by applying some forcing at a fraction of its natural frequency. For example, quadratic non-linearities will insure coupling of a driving signal at $f_0/2$ with a mode resonant at f_0 (and at the next order, $f_0/4$ and f_0 would be coupled). One can note the "holes burning" or "dips" in the response of the first lock-in amplifier (plain line) and the energy transfer into the second harmonic signal (dashed line). This result suggests that in conjunction with an analytical modeling, super-harmonic excitation allows a separate quantification of the quadratic and cubic terms and can be used for model parameter fitting.

Non-periodic response

Due to coupling between two vibration modes, the response can suddenly get non-periodic. This transition is subject to hysteresis with respect to voltage and frequency. In this non-periodic regime, the lock-in outputs, which doesn't have a straightforward interpretation in this case, show erratic wiggles.

In fig.#8 the two modes of interest result from a peak doubling/splitting of the fundamental vibration mode. As the driving voltage increases, a larger and larger window of frequency is subject to the non-periodic regime. On a spectrum analyzer, the non-periodic signal appears to be a superposition a large number of modes/harmonics (not plotted). Note that a system of two slightly different oscillators, non-linearly coupled, will also present a some peak doubling.

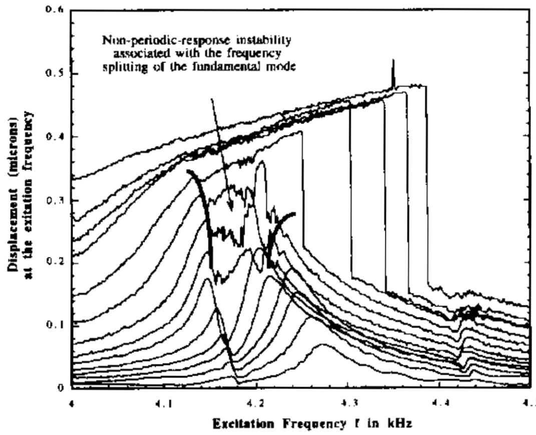

fig.#8 : Non-periodic response due to mode doubling

In fig.#9, the coupled modes are the fundamental and the first non-axisymmetric mode of vibration, which is resonant at about two times the resonance frequency of the fundamental mode. A mean to prevent the occurrence of this regime consists in preventing the fundamental mode to be nearly commensurable with the first other modes, by a proper choice of the geometrical dimensions or by some post-design tuning of the device. In fig.#9, the frequency is swept down.

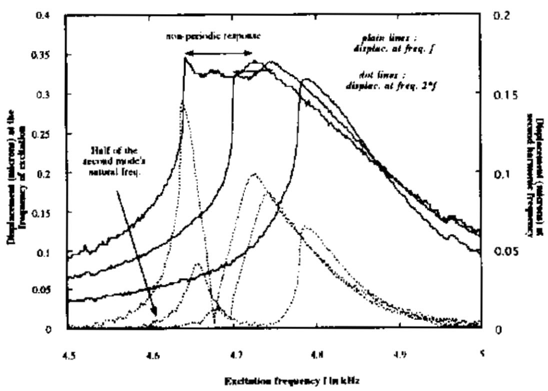

fig.#9 : Non-periodic response due to coupling between the fundamental and the first antisymmetric mode

SUMMARY

The process of piezoelectric depoling, occurring in transducer over time, enhances the relative strength of the sources of material non-linearities. Using this fact, a set of flexural mode transducer exhibiting electric-field-related non-linearities in low deflection operation was realized and characterized. Measurement of the membrane deflection versus the driving voltage permitted to identify typical non-linear alterations of the transducer dynamics including : hysteresis cycles, sharp saturation of the displacement, non periodic responses to an AC forcing tension and unstable transfer of energy between the spectral components of the displacement signal.

CONCLUSION

The practical conclusions arising from those measurements are :
- piezoelectric depoling enhances the relative strength of non-linearities.
- super-harmonic excitation may allow a separate fitting of the quadratic vs. cubic non-linear parameters of a model.
- E-field related non-linearities can induce an extremely efficient second harmonic generation.
- a near commensurability of the mode of interest with another mode can cause a non-periodic response. Commensurability of the natural frequencies should indeed be considered when power transducers are designed.
- peak doubling in a non-linear system can cause a non-periodic response.

More generally, this study points at effects indicating that a transducer operation in subject to a non-linear dynamics. Basic numerical simulations, not reported here, show that in the context of structural non-linearities, some of the same effects also arise : resonance frequency drift, hysteresis cycling, super-harmonic excitation and non-periodic response of systems presenting some commensurability between their natural frequencies [3]. This suggests that the other effects described here may also result from other sources of non-linearities.

REFERENCES

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[3] A.H.NAYFEH, L.D. ZAVODNEY - Transactions of the ASME, Vol.**55,** Sept. (**1988**)p.706-710 - Experimental observation of amplitude- and phase-modulated response to two internally coupled oscillators to a harmonic excitation