APPLICATION OF SURFACE IMPEDANCE APPROACH TO ULTRASONIC WAVE PROPAGATION IN LAYERED ANISOTROPIC MEDIA

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ABSTRACT

Matrix methods have been extensively used for wave propagation problems in layered media. Direct application of these methods using partial waves or transfer matrix formulation result in numerical problems known as exponential dichotomy. In this paper, we use the concept of surface impedance tensor to tackle this problem. Given the surface impedance tensor along an initial surface, a recursive algorithm is used to calculate the surface impedance tensor for the other layers. The methodology inherently prevents exponential dichotomy. We present applications of the surface impedance approach to practical problems. As examples, we consider the anisotropy of SAW velocity for different electrical boundary conditions, plane wave reflection from immersed layered structures with general anisotropy, and internal field distributions for predicting defect detection sensitivity.

INTRODUCTION

Ultrasonic wave propagation in layered media has been investigated by many researchers and some solution methodologies are well established. A review of developments in this area, especially in the application of matrix methods to wave propagation can be found in [1]. For isotropic materials, the velocity potential method can be used to construct the boundary condition matrix and then solving for its determinant results in the modal solutions. For forced excitations, the coefficients can be found by Cramer's rule, or matrix inversion [2]. When the materials are anisotropic, the velocity potential method can not be used and equivalently the partial wave method can be substituted. If the media are piezoelectric, one should consider 8 partial waves for each layer and solve the Christoffel's equation. Then, the boundary condition matrix size becomes 8N, where N is the number of layers. For large N, this results in large boundary condition matrices, which have extremely large and small entries if the layers are thick as compared to the wavelength of ultrasonic waves. The transfer matrix formulation solves the large matrix problem by defining a transfer matrix for each layer, which has a fixed size independent of N [3]. However, the problem of thick layers can not be solved by this method and the maximum layer thickness is limited to about 15 MHz-mm in frequency-thickness product (fd). Also, when an elastic parameter of the medium continuously varies with depth, the problem is discretized to many thin layers, creating large computational load. I: addition to these exact methods, there have been some approximate approaches to the same problem, such as using Laguerre polynomials [4]. This method enjoys being non-iterative, but the accuracy is degraded especially for large fd.
In this paper, we first describe the concept of surface impedance tensor, which is a result of the state vector formulation as the transfer matrix method [5]. This tensor can be used in a recursive formula in case of multilayered structures and it is numerically robust even for large fd values. We then proceed to several practical applications and discuss how different boundary conditions and discontinuities on the field variables can be accommodated by the surface impedance approach. In particular, we compute the SAW velocity in a $ZnO/SiO_2/GaAs$ structure as an application for SAW device technology. For NDE and acoustic microscopy applications we evaluate the plane wave reflection coefficient for layered composites immersed in fluids and discuss its relevance to defect detection sensitivity.

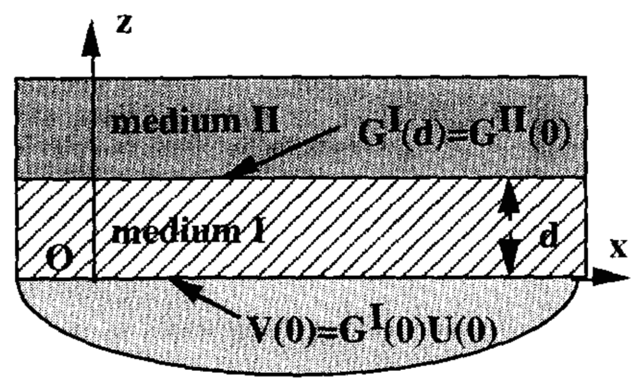

Figure 1. The geometry and coordinates for the surface impedance tensor.

THE SURFACE IMPEDANCE TENSOR

To derive the surface impedance tensor in a lossless generally anisotropic and piezoelectric medium, we consider a planar layer as shown in Fig. 1. We also aasume plane harmonic waves propagating in the x-z plane. Then, the mechanical and electrical field variables are of the form

$$\mathbf{T}_z(x,z,t) = \tilde{\mathbf{T}}_z(z)e^{j(\omega t - k_x x)}$$
$$\mathbf{v}(x,z,t) = \tilde{\mathbf{v}}(z)e^{j(\omega t - k_x x)}$$
$$\phi(x,z,t) = \tilde{\phi}(z)e^{j(\omega t - k_x x)} \quad (1)$$
$$D_z(x,z,t) = \tilde{D}_z(z)^{j(\omega t - k_x x)}$$

where $\mathbf{T}_z$ and $\mathbf{v}$ are the normal stress and the particle velocity vectors and ϕ and D_z are the electrical potential and displacement vectors, respectively. It can be shown that, by defining a state vector ξ, the piezoelectric field and stress equations can be written as a system of first order differential equations as,

$$\frac{d\xi}{dz} = -j\mathbf{N}\xi, \text{ where } \xi(z) = \begin{bmatrix} \tilde{\mathbf{v}} \\ \tilde{D}_z \\ \tilde{\mathbf{T}}_z \\ \phi \end{bmatrix}. \quad (2)$$

The tensor $\mathbf{N}$ is called the fundamental piezoelectricity tensor [5,6]. One can write the solution to Eq. 2 by direct exponentiation yielding

$$\xi(z) = e^{-j\mathbf{N}z}\xi(0) \quad (3)$$

where $\xi(0)$ is the initial condition at z=0. This approach leads to the transfer matrix method, and the matrix $e^{-j\mathbf{N}z}$ is called the transfer or propagator matrix. This solution is appropriate for multilayered media, since the overall solution can be obtained by multiplication of the solutions for each layer. However, numerical difficulties are encountered for large fd values, because the transfer matrix includes decaying as well as growing exponentials in the z-direction. The solution in Eq. 2 can be written in another form by using the eigenvalue-eigenvector decomposition of the transfer matrix. Then, Eq. 3 can be written as

$$\xi(z) = \begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 \\ \mathbf{L}_1 & \mathbf{L}_2 \end{bmatrix} \begin{bmatrix} \Phi_1(z) & 0_{4x4} \\ 0_{4x4} & \Phi_2(z) \end{bmatrix} \begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 \\ \mathbf{L}_1 & \mathbf{L}_2 \end{bmatrix}^{-1} \xi(0) \quad (4)$$

We note that the 8x8 eigenvector matrix on the left consists of submatrices with subscript 1 corresponding to the eigenvalues with a negative imaginary part, so that these fields propagate and decay in the +z direction. The other eigenvectors are included in the matrices with subscript 2. The diagonal matrices $\Phi_1(z)$ and $\Phi_2(z)$ have exponential entries with corresponding eigenvalues. By partitioning the state vector into two components, Eq. 4 can be expressed in a simpler form

$$\mathbf{U}(z) = \mathbf{A}_1\Phi_1(z)\mathbf{c}_1 + \mathbf{A}_2\Phi_2(z)\mathbf{c}_2$$
$$\mathbf{V}(z) = \mathbf{L}_1\Phi_1(z)\mathbf{c}_1 + \mathbf{L}_2\Phi_2(z)\mathbf{c}_2 \quad (5)$$

where

$$\xi(z) = \begin{bmatrix} \mathbf{U}(z) \\ \mathbf{V}(z) \end{bmatrix}, \ \mathbf{U}(z) = \begin{bmatrix} \tilde{\mathbf{v}} \\ \tilde{D}_z \end{bmatrix} \text{ and } \mathbf{V}(z) = \begin{bmatrix} \tilde{\mathbf{T}}_z \\ \phi \end{bmatrix}.$$

The constant vectors $\mathbf{c}_1$ and $\mathbf{c}_2$ are related to the initial conditions via Eq. 4. At this stage, the surface impedance tensor $\mathbf{G}(z)$ is defined by the relation

$$\mathbf{V}(z) = \mathbf{G}(z)\mathbf{U}(z) \quad (6)$$

The surface impedance tensor at another plane normal to z direction can be found by eliminating one of the constant vectors $\mathbf{c}_1$ and $\mathbf{c}_2$ in Eq. 5, using the initial condition on the impedance tensor at z=0 as depicted in Fig. 1. Hence, the surface impedance tensor is transferred from one surface to another. If there is an upper layer such as in Fig. 1, the surface impedance at the top of the lower layer serves as the initial condition. The surface impedance tensor is different from the characteristic impedance, it depends on the frequency and position in the medium. Since in the calculations the decaying and growing eigenvalues are separated, it inherently prevents the exponential dichotomy problems for large fd values.

For convenience, we partition the surface impedance tensor as

$$\mathbf{G} = \begin{bmatrix} \mathbf{G_m} & \mathbf{g}_{\mathbf{T}D_z} \\ \mathbf{g}_{\phi\mathbf{v}}^T & g_{\phi D_z} \end{bmatrix} \quad (7)$$

where $\mathbf{G_m}$ is a 3x3 matrix representing the pure mechanical part of $\mathbf{G}$, $\mathbf{g}_{\mathbf{T}D_z}$ and $\mathbf{g}_{\phi\mathbf{v}}^T$ are 3x1 vectors for the electromechanical coupling between the field quantities in the subscripts. The electrical potential and the normal electric displacement vector are related by the scalar $g_{\phi D_z}$.

APPLICATIONS

SAW Devices

We consider a ZnO piezoelectric film on a {001}-cut GaAs halfspace, which has a thin SiO_2 passivation film. This is a common structure for SAW devices. In the specific example, the frequency is chosen as 225 MHz, in order to make a direct comparison with the work in [7]. In Fig. 2, we plot the SAW velocity variation as a function of propagation angle from the <100> direction on the {001} surface of GaAs halfspace for 8μm ZnO film. We apply different electrical boundary conditions at the top surface, which is used to calculate the fractional change in phase velocity and predict the efficiency of SAW excitation. For a short circuited top

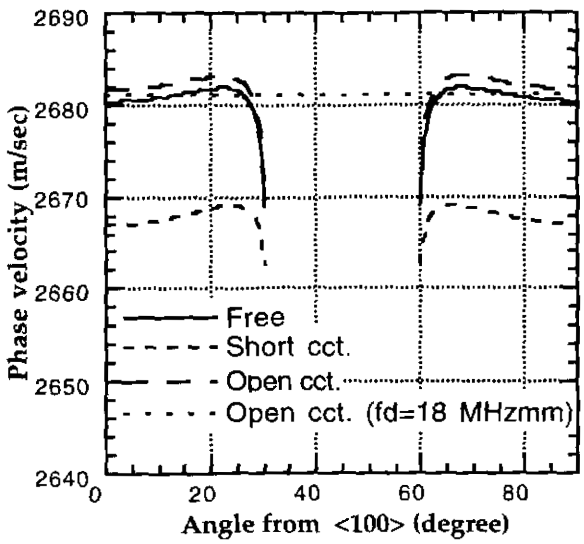

Figure 2. The SAW velocity variation with angle for different electrical boundary conditions on ZnO.

boundary condition on ZnO, the following conditions are imposed on the surface impedance tensor:

$$\mathbf{G}_{SiO_2}(0) = \mathbf{G}_{GaAs} \qquad (8a)$$

$$\mathbf{G}_{ZnO}(0) = \mathbf{G}_{SiO_2}(d_{SiO_2}) \qquad (8b)$$

$$\det(\mathbf{G}_{ZnO}(d_{ZnO})) = 0 \qquad (8c)$$

where d_{SiO_2} and d_{ZnO} are the thickness of the corresponding layers. The first two parts of Eq. 8 satisfy the continuity relations, whereas Eq. 8c imposes a free and short circuit interface with non zero field components. For an ideal open circuit condition ($D_z(d_{ZnO}) = 0$), the definition of the surface impedance is changed and a relation similar to 8c is obtained [8]. For a free electrical condition where the electrical fields are continuous, the condition 8c takes the form

$$g^{v}_{\phi D_z} + \mathbf{g}^{T}_{\phi \mathbf{v}} \mathbf{G}^{-1}_{\mathbf{m}} \mathbf{g}_{\mathbf{T} D_z} - g_{\phi D_z} = 0. \qquad (9)$$

In Eq. 9, $g^{v}_{\phi D_z} = (-k_x \varepsilon_0)^{-1}$ ($k_x > 0$) is a quantity specific to vacuum assuming that the halfspace is bounded by vacuum and the electric potential field decays for z > 0. The $(.)^{-1}$ superscript denotes matrix inversion. We observe that for ZnO, the difference between the ideally open and free electrical conditions is not negligible, since the relative permittivity of ZnO is comparable to unity. In Fig. 3, we also plot the SAW velocity for 80 μm thick ZnO layer corresponding to fd=18 MHz-mm, which is beyond the limit of transfer matrix methods. The velocity is constant due to the transverse isotropy of ZnO around the Z axis.

<u>NDE of layered composites</u>

Solid samples are immersed in coupling fluids for acoustic microscopy and many NDE applications [9,10]. The reflection of acoustic plane waves is then used for material characterization and defect detection. Using the surface impedance approach, the reflection coefficient can be expressed in a compact manner as

$$R(\omega,\theta,\varphi) = \frac{Z_s - Z_f}{Z_s + Z_f} \qquad (10)$$

where the quantities Z_f and Z_s are the normal impedances of the fluid and the solid defined as

$$Z_f = -\frac{\rho v_l}{\cos(\theta)} \text{ and } Z_s = -\frac{1}{\mathbf{G}^{-1}(3,3)}.$$

Here ρ and v_l are the density and speed of sound in the fluid and **G** is the surface impedance tensor of the solid at the fluid-solid interface. For an immersed plate, the initial condition on the surface impedance tensor is defined by the fluid as

$$\mathbf{G}_{\mathbf{mf}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & Z_f \end{bmatrix}. \qquad (11)$$

One can obtain the reflection coefficients for immersed plates starting with Eq. 11 as the initial condition and imposing the continuity conditions as in Eq. 8 at each interface. Finally, defining Z_s, the reflection coefficient can be evaluated as in Eq. 10. A "kissing bond" is often used as a defect at an interface [10]. This defect can easily be incorporated in the impedance tensor, by changing the continuity condition to

$$\mathbf{G}^{\mathrm{II}}_{\mathbf{ms}}(0) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \mathbf{G}^{\mathrm{II}}_{\mathbf{ms}}(3,3) \end{bmatrix} \text{ and}$$

$$\mathbf{G}^{\mathrm{II}}_{\mathbf{ms}}(3,3) = \frac{1}{\mathbf{G}_{\mathbf{m}}^{\mathrm{I}^{-1}}(3,3)}. \qquad (12)$$

So, the effect of "kissing bond" is a change in the initial condition of the surface impedance for the upper medium denoted by $(.)^{\mathrm{II}}$.

Consider a three layered composite each layer being orthotropic with elastic parameters as in [10]. The layers are rotated by 45°/135°/45° from their X-axis to numerically resemble monoclinic material on its symmetry axis [11].

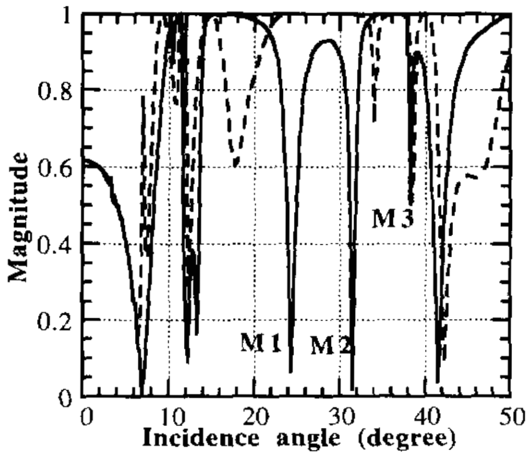

Figure 3. The magnitude of reflection coefficient for an fd=5MHz-mm composite plate in water. Solid line: good bond, dashed line: "kissing bond" at the lower interface.

The layer thicknesses are 0.25mm, 0.5mm, 0.25 mm, respectively. In Fig. 3, we plot the magnitue of the reflection coefficient at f=5 MHz with solid line assuming a good bond at each interface. A dip in the reflection coefficient is an indication of Lamb wave excitation in the plate. We label three of these modes as M1, M2 and M3. When the continuity conditions are satisfied, all three modes are excited. We also plot the reflection coefficient for a "kissing bond" at one of the interfaces with dashed lines. The modes M1 and M2 are not excited in the defective plate, whereas mode M3 is only slightly perturbed. So, for detection of interface defects in this composite plate, modes M1 and M2 should be used.

This result can be predicted by investigating the internal field distribution of these modes. The surface impedance approach allows field calculations via a simple integration method as discussed in [5,8]. The internal field distribution for modes M1 and M2 in a free plate shows that ultrasonic energy flow is concentrated at the interfaces [8]. In contrast, mode M3 carries energy mostly in the layers. So, the efficacy of Lamb wave modes for detecting interface defects can be estimated by their energy flow distribution in the plate and the appropriate modes can be selected accordingly.

CONCLUSION

The surface impedance approach is a general method for wave propagation problems in multilayered anisotropic and pirezoelectric materials. Avoiding the numerical problems suffered by the conventional methods, it enables a robust tool to be used in many different applications. Using the surface impedance approach, the internal fields of forced and free vibrations in layered materials can be found and used in determining mode selection for defect detection. Possible applications to lossy media and non-planar geometries are under investigation.

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