Propagation in Finite Anisotropic Plates of Diffracting Scalar Waves

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ABSTRACT

The model developed in this paper allows to predict the propagation of Lamb waves in an anisotropic wafer. More generally it can account for the linear dispersive propagation of scalar waves in an anisotropic and finite plate and for any local alteration in the displacement component of the Lamb Wave. The validation has been done using a solid-solid contact as an emitter-receiver.

INTRODUCTION

The resolution of the wave propagation equation in an anisotropic plate, like a wafer, is rendered imprecise, in practice, by the fact that plate dimension is finite. We propose hereafter a method of resolution which introduce the border diffraction and allows to predict Lamb wave propagation in an anisotropic and dispersive wafer.

ENERGY ENHANCEMENT FACTOR

According to the Fourier theory, the *linear* wave equation can be reduced to the harmonic equation [1]. Let us name u(M) complex amplitude of the harmonic solution U(M,t), ω the angular frequency, t the time and M a point of the plate referred to an origin O of the plate by its polar coordinates (r,β). **k** will be the wave vector.

Considering a 2D scalar wave propagating in an *isotropic* medium issued from an harmonic point source located at the origin O. The wave equation reduces, in polar coordinates, to the Bessel equation :

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + k_0^2.u = 0 \qquad (1)$$

In our problem, according to K.F. Graff [2], the solution is reduced to the Hankel function $H_0^{(1)}(k_0.r)$ modulo a complex factor that to simplify we will assume equal to one. The convergence of this solution allows to use the asymptotic form:

$$H_0^{(1)}(k_0.r) \underset{\infty}{\approx} \sqrt{\frac{2}{\pi.k_0.r}}.\exp(i.k_0.r) \qquad (2)$$

Anisotropic medium. If α and β are the respective angles of the wave vector **k** and the energy velocity vector V_{Energy} at any point M, a continuous function D relates this two angles, as a consequence, any harmonic signal propagating in the β direction can be resolved in a sum of signals with phase $\mathbf{k}(\alpha_i).\mathbf{r} - \omega t$, where the α_i are the antecedents of β in the previous function.

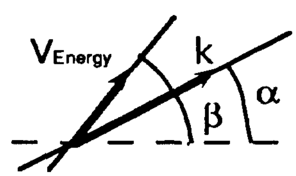

Figure 1

Formulation. Consider a point source generating wave vectors **k**(α) isotropically and the slowness curve, - locus of the extremity of the s(α)=**k**(α)/ω vectors, when the α varies. We notice that whatever the point the energy velocity vector remains always orthogonal to this curve. So the two directions are always related by:

$$\beta = \alpha - \mathrm{Arctg}\left(\frac{s'}{s}\right) \qquad (3)$$

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i) If the D function is a bijective relation between α and β, all the energy radiated in the angular sector [β–dβ, β+dβ] results of the energy of the waves whose wave vectors are pointing in the angular sector [α-dα,α+dα]. Then energy density in the β direction is proportional to dα/dβ. So, the *energy enhancement factor* of H.J. Maris [3], $E(\beta)=d\alpha/d\beta$, can be analyticaly expressed:

$$E(\beta)=\left[1+\frac{s'^2+s.s''}{s'^2+s^2}\right]^{-1} \qquad (4)$$

s' and s'' are respectively the first and second derivative of $s(\alpha)$ with respect to α.

ii) If D is not a bijective function, the different factors E_1, E_2 .. associated with the different directions α_1, α_2 .. of which β is the image can be defined and summed.

ANISOTROPIC PROPAGATION FORMULA

Using these results on anisotropy in an emitter-receiver configuration, the expected received signal v(M) is:

$$v(M)=\sqrt{E(\beta)}.f(r,\beta).\exp(i.\mathbf{k}(\alpha).\mathbf{OM}) \qquad (5)$$

with $f(r,\beta)\in\Re$ an unknown function.

i) If β has only one antecedent α through D.

$$f(r,\beta)\simeq\sqrt{\frac{2}{\pi.k_0.r}}$$

So a general solution for the signal received in M from the source O in an anisotropic medium with α is the β antecedent by D is:

$$v(M)=v(r,\beta)=\sqrt{E(\beta)}.H_0^{(1)}(\mathbf{k}(\alpha).\mathbf{OM}) \qquad (6)$$

ii) If β has several antecedents α_1, α_2 ... , the received signal v(M) is the sum of similar expressions, one for each α_i .The final result for V(M) is :

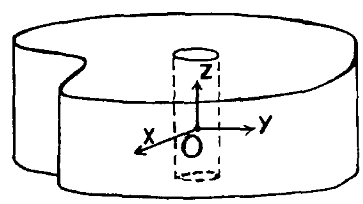
$$\mathrm{Real}\left(\sum_{\alpha_i}\sqrt{E(\beta)}.H_0^{(1)}(\mathbf{k}(\alpha_i).\mathbf{OM}).\exp(-i\omega t)\right)$$

DIFFRACTION BY THE EDGES

Focus our attention on the diffraction by the edge. During reflection, the diffraction occurs on an infinitely small element of the free edge part of the plate.

i) *Isotropic medium.* The phenomenon is similar to an *aperture* diffraction. Solutions of thewave equation (1) satisfy $u(r,\theta,z)=u(\theta,r)$ and $g(r,\theta,z)=H_0^{(1)}(k.r)$. Using the Green's formula in/on a closed cylindrical surface A (Figure 2):

fig.2



$$\iiint_V(u.\Delta g-g.\Delta u)dv=\oiint_A\left(u\frac{\partial g}{\partial n}-g\frac{\partial u}{\partial n}\right)da$$

leads to

$$u(0)=\frac{1}{4i}\int_C\left(\frac{\partial u}{\partial n_{in}}H_0^{(1)}(k.r)-\cos(\theta_r).k.u.H_1^{(1)}(k.r)\right)dl \qquad (8)$$

From a Physical point of view, u(0) is the signal diffracted in O and the sommation can be achieved using solely the knowledge of the u(M) signal values on the contour. By a change in coordinates, O can represent any of the points on the wafer where the signal amplitude is wanted to be known. This leads to the new genereral expression:

$$u(0) = \frac{k}{2}\int_C \left(H_0^{(1)}(k.r').H_0^{(1)}(k.r).\left(\frac{\cos(\theta_i)+\cos(\theta_r)}{2}\right)\right) dl \qquad (10)$$

and with the asymptotic expressions of the Hankel functions :

$$u(0) = \frac{2}{i\pi}\int_C \frac{e^{i.k.r'}}{\sqrt{r'}} \frac{e^{i.k.r}}{\sqrt{r}} \left(\frac{\cos\theta_i + \cos\theta_r}{2}\right) dl \qquad (11)$$

Now we can use the diffraction formula (10) to predict the diffracting *reflection* of a signal, joined to the properties of an aperture-edge.

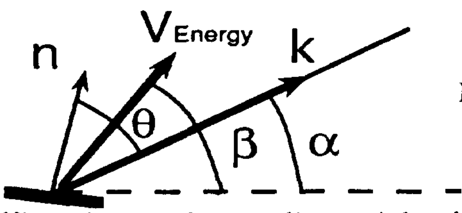


Figure 5

ii) *Anisotropic medium.* Adapting (10), the signal received in M from a point source O is:

$$v(M) = v(r,\beta) = \sqrt{E(\beta)}\,.H_0^{(1)}(\mathbf{k}(\alpha).\mathbf{OM})$$

For any unitary vector **n**:

$$\partial v/\partial n \simeq -\sqrt{E(\beta)}\,k(\alpha).\cos(\theta).H_1^{(1)}(\mathbf{k}(\alpha).\mathbf{OM})$$

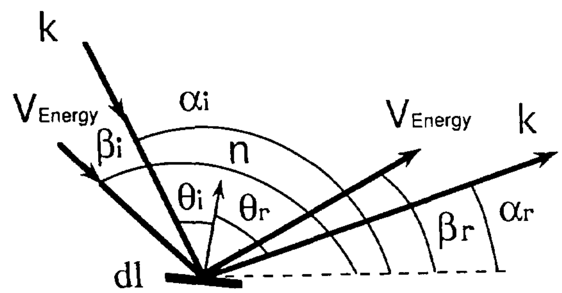


Figure 4

And with the symbols of Figure 4,we propose the following new diffraction formula for an anisotropic medium. :

$$u(0) = \frac{1}{2}\int_C \sqrt{E(\beta_i)}\,.H_0^{(1)}(\mathbf{k}(\alpha_i).\mathbf{r}').\sqrt{E(\beta_r)}\,.H_0^{(1)}(\mathbf{k}(\alpha_r).\mathbf{r}).\left(\frac{k(\alpha_i).\cos(\theta_r)+k(\alpha_r).\cos(\theta_r)}{2}\right) dl \qquad (12)$$

EXPERIMENTAL VALIDATION

The experimental arrangement consists of a PZT-5H 230 kHz 40% bandwidth transducer, bounded at the top of a quartz pin. The other end of the pin, rounded to a radius of 100 μm, is going to be in direct contact with the wafer.This shape garanties a sufficient coupling coefficient. We evalute changes in the squared amplitude of the received signal when the receiver rod describe a circle around the emitter. Bursts formed with 3 to 7 cycles have been used as excitation signal.Pressure applied to the contact pin-wafer may changes the coupling coefficient and consequently the amplitude of the signals. To dismiss this effect, a second fixed emitter is used to make a calibration of pressure.

Experimental data were taken on a 15 cm wafer for various distance emitter-receiver. A comparison with theoritical results are presented (see Figure). Results are quite accurate.We have also checked than a 3% anisotropy of the amplitude mode in Si involves a factor 2 in the energy anisotropy.

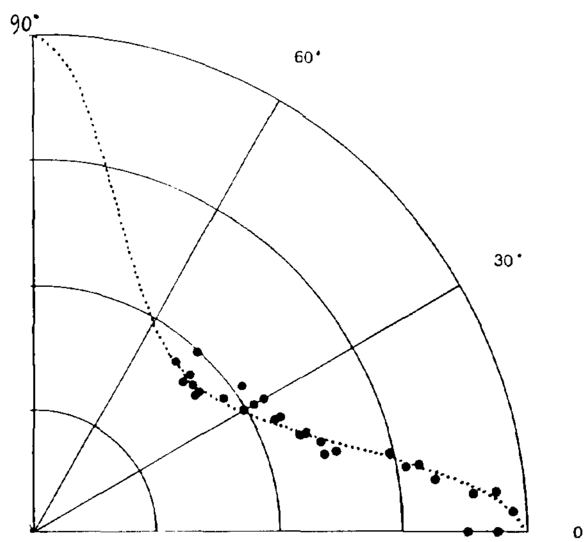
ALGORITHM FOR A SIMULATION

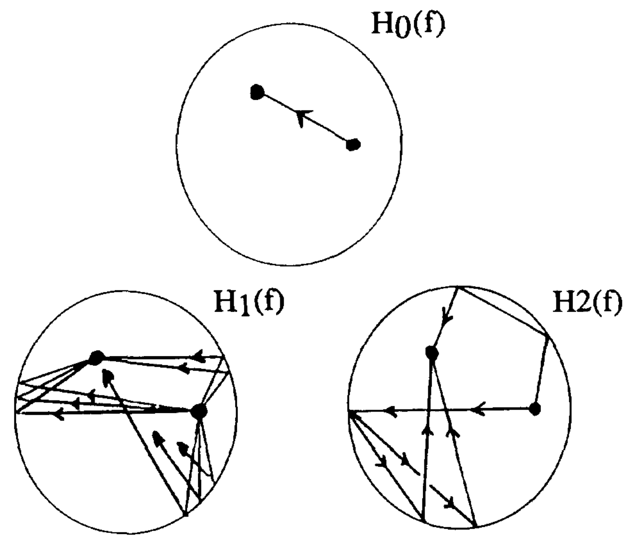
A numerical simulation has been achieved to predict the transmission of any signal through the plate, between a point shaped-emitter to a point shaped-receiver. The corresponding transfert function H(f) has been computed. H(f) was decomposed in its different contributions: the direct signal $H_0(f)$, the first reflection signal $H_1(f)$ the multiple reflection signals H2(f), H3(f)...

CONSEQUENCE

Cutting the integration of equation (12) into different parts according to the edge we can roughly divide H(f) into terms accounting for the propagation into specific regions of the plate. The received signal can be seen as a superposition of pieces of signal corresponding mainly to one region.

Suppose an increase of the temperature in a region of the plate. The wave velocity decreases introducing a delay, mostly on the signal contribution coming from this region. A track of the delay in the signal of each piece can be done from an experimentally received observation and allows detection of the local variations: temperature, thickness ... The simulated data could be used to do some kind of adapted filtering of an experimental signal.





CONCLUSION

We have developped theoritical calculation for diffraction in a 2D isotropic medium and found an explicit expression of the enhancement factor in 2D as a fonction of the wave vector angular dependence.

We present an approximation for diffraction in an anisotropic media which allows simple implementations for simulation purpose and gave very satisfactory qualitative results.

We realized an experimental measurement of the energy enhanced factor for Lamb wave in Silicium coupling ω with the plate for excitation and detection

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