

V(z) CALCULATION FOR A FOCUSED BEAM INCIDENT ON AN INCLINED PLANE

Tsutomu Hoshimiya* and

Butrus T. Khuri-Yakub

* Tohoku Gakuin University,
Department of Applied Physics,
Tagajyo, Japan.

Stanford University,
Edward L. Ginzton Laboratory,
Stanford, California.

ABSTRACT

A material characterization with acoustic waves has been extensively developed by V(z) inversion[1]. A line-focus-beam technique[2], which uses oblique incidence of an acoustic wave into a solid sample from certain direction, is extremely useful especially in anisotropic solids. Very recently an attempt using spherical-planar pair lenses has been analyzed and demonstrated[3].

From this point of view, we present a theoretical investigation of the V(z) characteristics of a focused beam incident on an inclined solid sample in this paper. A basic formula is derived in which an acoustic beam focused with a spherical lens is incident on an sufficiently inclined sample-couplant interface plane is reflected and collected with the same acoustic lens(monostatic case) or the other lens(bistatic case), and V(z) curves are analyzed using the actual physical parameters of materials.

An angular spectrum approach for an acoustic wave incident on an inclined plane

In the present analysis, a basic scheme representing an oblique incidence of a focused acoustic wave is considered. An acoustic wave generated by a piezoelectric transducer is focused by an acoustic lens. An object solid half-space located at z is assumed to have a density and a sound velocities of ρ and v_L , and v_S , respectively.

If $z < 0$, an acoustic wave is focused inside a couplant which is isolated from a solid sample under test, whereas, on the other hand, if $z > 0$, a focal point is assumed to be embedded in a sample object, defined as same as a standard notation. A focus point of an acoustic beam is assumed to be (x, y, z) , and a scan is done over an x-y plane and a V(z) curve will be obtained at each scan coordinate (x, y) .

A sample-couplant interface plane which is inclined with an angle γ is characterized by its normal unit vector at a interface plane \hat{n} taken from solid side to liquid side defined by

$$\hat{n} = \begin{pmatrix} 0 \\ -\sin \gamma \\ -\cos \gamma \end{pmatrix} \quad (1)$$

With this rotation, incident angles into a sample are changed as the following rule;

$$\begin{aligned} \theta_x &\longrightarrow \theta'_x = \tan^{-1} \left(\frac{k'_x}{k'_z} \right) = \tan^{-1} \left(\frac{k_x}{\sqrt{k_0^2 - (k_x^2 + k_y^2)}} \right) \\ \theta_y &\longrightarrow \theta'_y = \tan^{-1} \left(\frac{k'_y}{k'_z} \right) = \tan^{-1} \left(\frac{k_y}{\sqrt{k_0^2 - (k_x^2 + k_y^2)}} \right) + \gamma \end{aligned} \quad (2)$$

where single-primed k-vectors mean those incident into a solid sample under study. And reflected wave vectors are changed by the equations

$$\begin{aligned} k_x &\longrightarrow k''_x = k_x \\ k_y &\longrightarrow k''_y = \cos 2\gamma k_y - \sin 2\gamma k_z \\ k_z &\longrightarrow k''_z = -\sin 2\gamma k_y - \cos 2\gamma k_z \end{aligned} \quad (3)$$

where double-primed wave vectors denote those of reflected waves.

After a similar calculus with the ref. [1], we have an expression for an incident field

$$A(k_x, z) = P(k_x) \frac{k_0}{k_z} \exp j(-k_z z) \quad (4)$$

where pupil function $P(k_x)$ of a lens with focal length f_0 and diameter $2r_0$ is represented by

$$P(k_x) = \text{circ} \left(\frac{k_x / k}{r_0 / f_0} \right) \quad (5)$$

where $\text{circ}(x)$ is a circular function.

Reflected wave with a wave vector \mathbf{k} is therefore given by a product of incident wave $A(\mathbf{k}; z)$ and Fresnel reflectivity $R(k_x, k_y)$ as

$$B(\bar{\mathbf{k}}; z) = R(k_x, k_y) A(\bar{\mathbf{k}}'; z) \quad (6)$$

Detector response V(z) given by Kino-Auld's reciprocity theorem is given with Parseval's theorem as the following form

$$V(z) = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A(\bar{\mathbf{k}}; z) R(k'_x, k'_y) (k_x \sin \gamma + k_z \cos \gamma) B(k''_x, k''_y; z) dk'_x dk'_y}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (k_x \sin \gamma + k_z \cos \gamma) |A(k_x, k_y; z)|^2 dk_x dk_y} \quad (7)$$

where single-primed and double-primed k-vectors are those incident into a solid sample under study and those of reflected waves, respectively. If we concentrate our attention to a relative change in V(z; γ) with angle γ , we can renormalize

V(z) curves by replacing V(z; γ) with its normal incident value V(z; 0). A renormalized $\hat{V}(z)$ which is proportional to the numerator in eq.(7) is rewritten as

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P_1(k_x, k_y) \frac{k_0}{k_z} R(\tan^{-1}(k_x/k_z), \tan^{-1}(k_y/k_z) + \gamma) \exp(-2jk_z z) dk_x dk_y \quad (8).$$

The expression of V(z) in eq. (8) is proportional to the integral of the product of an original (circular) pupil function $P_1(k_x, k_y)$, its image caused by reflection of an inclined plane located at F+z (F=focal length) $P_2(k_x, k_y)$, the Fresnel reflection coefficient of the sample $R(k_x, k_y)$, and a propagation factor $\exp[-2j(k^2 - (k_x^2 + k_y^2))^{1/2} z]$ over the entire k_x and k_y space.

A pupil function of a transmitter characterized by $P_1(k_x, k_y)$ will vanish when

$$k_x^2 + k_y^2 > k_0 \cdot \frac{r_0}{f_0} \quad (9).$$

On the other hand, the pupil function of the receiver, the location of which is rotated around x-axis by angle 2γ , $P_2(k_x, k_y)$ vanishes when

$$k_x'^2 + k_y'^2 > k_0 \cdot \frac{r_0}{f_0} \quad (10).$$

The physical meaning of the Eq. (8) is an overlap integral of pupil functions of an acoustic lens, which acts as both a transmitter and a receiver, over entire k_x and k_y space.

The right hand side of both Eq.s (9) or (10) is a maximum value of transverse k-vectors k_T .

Methods of calculation.

To calculate a V(z) curve for an inclined sample, we extended a theory of V(z) inversion. In the present analysis, a V(z) expression based on an Auld-Kino's reciprocity theorem in a wave-number space is directly calculated due to lacking in an axial-symmetry of the system.

To analyze this problem with an angular spectrum approach, we made MATLAB and Mathematica programs to accomplish an 2-dimensional integration over entire k-space.

The following data shown are V(z) curves for an ideal reflector(R=1) to show the effect of misalignment of transmitter and receiver acoustic lenses.

In order to calculate V(z) curves for an inclined anisotropic sample, we must combine analysis described above with a calculation of the reflectivity of acoustic waves in an anisotropic sample.

A Christoffel equations characterizing the cubic sample are coupled with boundary conditions of the water-cubic crystal interface. A third-order algebraic equation or eigenvalue equation derived from the Christoffel equations results in a velocity and polarization vectors as eigenvalues and eigenvectors, respectively[4-6].

Particle displacements(or velocities) and stress tensor components continuity conditions are coupled and results in inhomogeneous linear algebraic equations. The amplitude variables R, L, S₁, and S₂, which correspond to reflected longitudinal wave, refracted longitudinal wave, and refracted shear waves, respectively, are determined with an incident unity amplitude longitudinal wave. The reflectivity for an longitudinal wave R is calculated by the Kramer's method for an algebraic 4 by 4 matrix equations derived from boundary conditions on velocity and normal stress components.

A FORTRAN program to solve an algebraic third order equation and a MATLAB program to determine eigenvalue problems are developed. Snell's law naturally derived from boundary conditions is used to check the availability of the solutions of the eigenvalue problem.

Results and discussions

In calculating V(z), we first considered an overlap in the integral expression in eq. (8), when perfect reflector(R(theta)=1) is considered. The Fig. 1 shows an overlap area of pupil functions of transmitter and receiver acoustic lenses for an inclined plane solid sample. As expected, overlap region decreases linearly at the beginning and then shows a nonlinear deviation from linear shifting of spherical lens response in k-space.

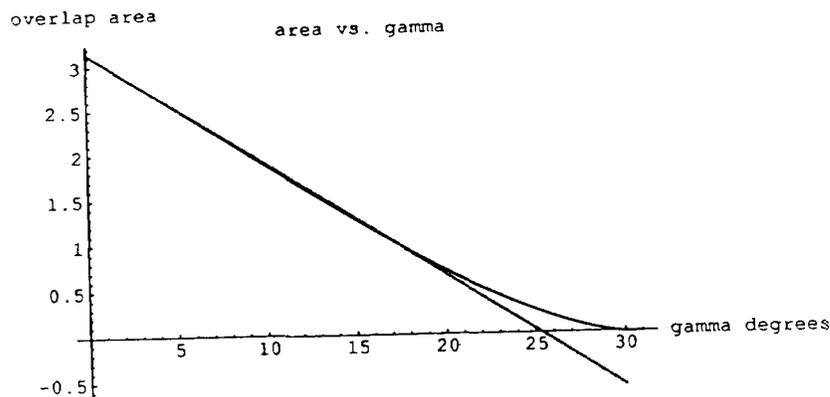


Fig. 1 Dependence of the overlap area of a transmitter pupil function $P_1(k_x, k_y)$ and a receiver pupil function $P_2(k_x, k_y)$ on an inclination angle gamma.

In the present paper, three samples are considered. Namely, perfect reflector, an isotropic material (Si_3N_4), and an anisotropic sample (z-cut Si).

1) Perfect reflector

In Fig. 2, $V(z)$ curves calculated for a perfect reflector with inclination angles $\gamma = 4, 8, 16, \& 24$ degrees are shown simultaneously.

Throughout this work, we considered an acoustic lens with an acceptance angle of 60 degree, namely, a parameter $\sin(\alpha) = k_r/k_0$ used in an angular spectrum approach being equal to $1/2$.

$V(z)$ curves seems to decrease rather linearly up to 16 degree, however, a drastic change will appear at 24 degree. This will be interpreted that a ray reflected with an angle of $2 \times \gamma$ exceeds an acceptance angle of 30 degree shows a drastic change not only in amplitude but also in smearing effect in interference pattern.

2) Isotropic material

In Fig. 3, $V(z)$ curves with inclination angles $\gamma = 4, 8, 16, \& 24$ degrees for an isotropic material Si_3N_4 are similarly calculated with a formula of reflectance of acoustic wave at liquid-isotropic solid interface shown in a standard textbook. (A reflectance of Si_3N_4 is calculated using its material constants.)

3) Anisotropic material

In order to calculate $V(z)$ curves for an inclined anisotropic sample, we must combine analysis described above with a calculation of the reflectivity of acoustic waves in an anisotropic sample.

An eigenvalue equations characterizing the cubic sample are derived from the Christoffel equations and result in a velocities and polarization vectors as eigenvalues and eigenvectors, respectively.

Boundary conditions of the water-cubic crystal interface results in 4 by 4 matrix equations. The amplitude variables $R, L, S_1,$ and S_2 , which correspond to reflected longitudinal wave, refracted longitudinal wave, and refracted shear waves, respectively, are determined by solving this equations with respect to an incident unity amplitude longitudinal wave. The reflectivity for an longitudinal wave R is calculated by the Kramer's method.

We have solved this by numerical procedure. Reflectivities for a longitudinal wave as a function of incidence angle θ are calculated for changing an incidence oriented with angle from the $[100]$ axis of Si, and the corresponding $V(z)$ curves are being calculated and those results will be published elsewhere.

Conclusion.

In this paper, we have presented an angular spectrum study for the $V(z)$ characteristics of a focused beam incident on an inclined solid sample. $V(z)$ curves are calculated and discussed using the actual physical parameters of samples. Our calculations are for a "monostatic" arrangement, in which the transmitter and receiver transducers are identical. This scheme has size and cost advantages as compared to the "bistatic" arrangement.

Our results indicate that $V(z)$ is proportional to the integration of the product of an original (circular) pupil function $P_1(k_x, k_y)$, its image caused by reflection of an inclined plane located at $F+z$ (F =focal length) $P_2(k_x, k_y)$, the Fresnel reflection coefficient of the sample $R(k_x, k_y)$, and propagation factor $\exp[-2j(k^2 - (k_x^2 + k_y^2))^{1/2} \cdot z]$ over the entire k_x and k_y space.

The results indicate that if the inclination angle is chosen properly, it is possible to measure the anisotropy of the sample with a spatial resolution corresponding to the focal spot of the beam in water.

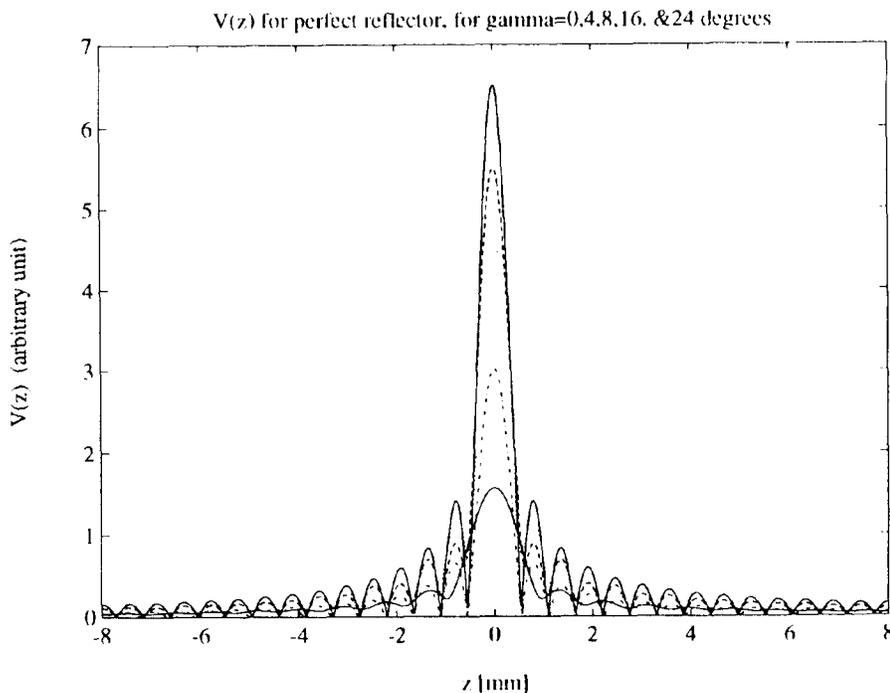


Fig. 2 $V(z)$ curves for perfect reflector, calculated for an inclination angle $\gamma = 0, 4, 8, 16, 24$ degrees.

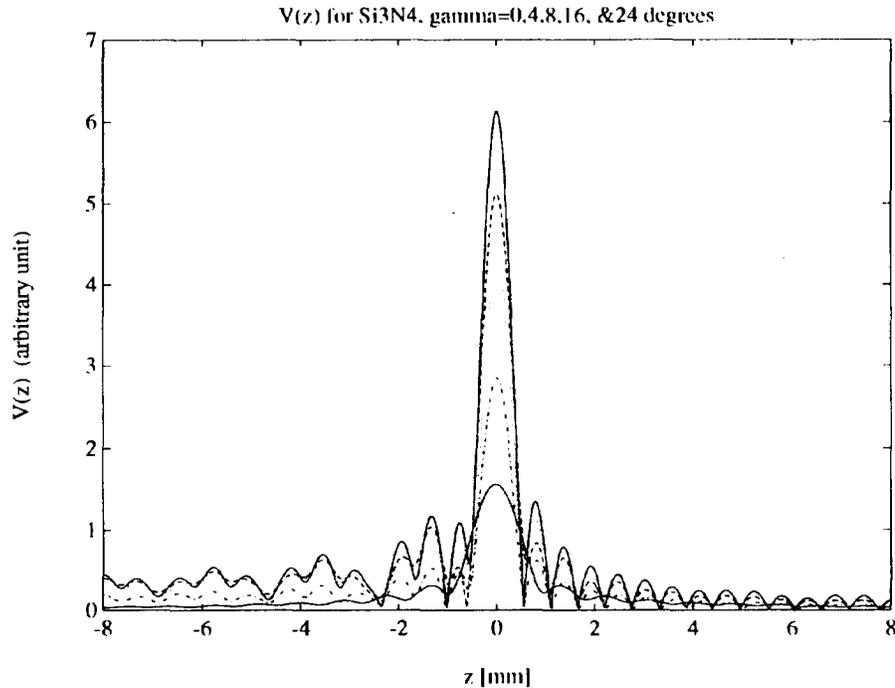


Fig.3 $V(z)$ curves for an isotropic solid Si_3N_4 , calculated for an inclination angle $\gamma=0, 4, 8, 16, 24$ degrees.

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