A noncontacting technique for measuring surface tension of liquids

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We have developed a novel instrument for measuring the surface tension of liquids. It utilizes ultrasonic and optical methods to excite and detect a capillary wave packet. A focused ultrasonic transducer is used to generate a mound on the surface of a liquid. Capillary waves propagate radially outward from the mound, and a noncontacting confocal optical microscope measures the amplitude of the wave packet at any distance away from the excitation point. We invert the measured wave amplitude to obtain the dispersion relation of the capillary waves and hence the surface tension.

Capillary waves are surface waves which propagate at the liquid surfaces. In the ocean, these are the ripples with wavelength less than 1.7 cm generated by the wind interaction with water. The restoring force for capillary waves is the surface tension which acts to minimize the surface area of the liquid. Therefore, capillary waves in the open sea are of interest for being an indicator of surface tension and viscoelastic properties of the surface and the liquid body.

In this paper, we present a novel instrument for generation and detection of capillary waves in order to measure *in situ* the surface tension of liquids. Current techniques of surface tension measurement such as the Wilhelmy plate[1] method are laboratory measurement techniques that require extreme care to make a reliable measurement, and many of them fail in measuring the surface tension of the liquids in an open environment like the ocean surface and in the presence of surfactants. The instrument described here is noncontacting and gives reliable and repeatable measurements within 0.3% error.

In order to generate capillary waves, we employ a focused F3 ultrasonic transducer from Precision Acoustic Devices, Fremont, CA with 48-mm focal length operating at 3 MHz. Excitation is achieved with a short tone burst at the transducer resonant frequency. The transducer is located in the water and its focal plane is at the water surface. The focal spot size is approximately 1.5 mm and the depth of focus is 30 mm. When the transducer is excited with an electrical tone burst, acoustic waves are focused at the water surface by the acoustic lens. The radiation pressure[2] associated with the acoustic waves lifts a portion of water surface whose width is comparable to the spot size of the beam. This perturbation creates a Gaussian-like water mound[3] at approximately 100-μm height. As the water surface relaxes to its equilibrium position, capillary waves are generated.

We detect the capillary wave motion at a particular position as a function of time. In order to achieve this we employ a confocal optical microscope[4] specially designed for this purpose. The structure of the microscope along with the overall system are shown in Fig. 1. The microscope operation relies on coupling the light reflected from the object plane to a photodetector through a small pinhole. A photodiode with a couple of kHz bandwidth will suffice as the photodetector. As the object moves in and out of the focus of the objective lens, the light intensity detected by the photodetector behind the pinhole varies, resulting in the response called $V(z)$. We operate the microscope in the small signal regime since the height variation of the water surface along the z axis is at most hundreds of micrometers while the full width half maximum of the $V(z)$ is several millimeters wide. This enables us to correlate linearly the signal from the amplifier output to surface height variations.

A complete theory of water waves[5] requires solving the Navier–Stokes equation. If the fluid is assumed to be incompressible and the amplitude-to-wavelength ratio of the waves are very small, for an inviscid fluid, the Navier–Stokes equation simplifies to

$$\frac{\partial \mathbf{v}}{\partial t} = \mathbf{F} - \frac{1}{\rho}\nabla p, \tag{1}$$

where $\mathbf{v}$ is the velocity field, $\mathbf{F}$, is an external force such as gravity or surface tension, and p is the pressure field. Including the effect of gravity and surface tension, the plane-wave solution of Eq. (1) with proper boundary conditions gives rise to the well-known Kelvin's dispersion equation[6]

$$\omega^2 = [gk + (\tau/\rho)k^3]\tanh(kh), \tag{2}$$

where ω is the angular frequency, g is the gravitational acceleration, k is the wave number, ρ is the density of the fluid, τ is the surface tension, and h is the depth of fluid. The minimum phase velocity ω/k occurs at a wavelength of 1.714 cm in clean water. For wavelengths smaller than 1.714 cm, the phase velocity of the surface waves increases with decreasing wavelength and the dominating restoring force in this capillary regime is the surface tension. On the other hand, for wavelengths greater than 1.714 cm, the phase velocity increases with increasing wavelength and the dominant restoring force is the gravitational force. A full dispersion relation including the effects of the finite viscosity of the liquid and the elastic films at the liquid surface is derived in the paper by Lucassen *et al.*[5]

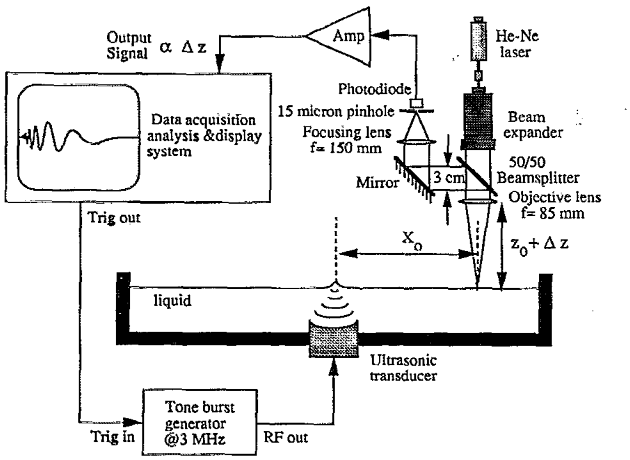

FIG. 1. Schematic of the capillary wave generation and detection system.

The numerical evaluation of the surface tension by this broadband excitation technique is based on measuring the wave profile at a fixed distance as a function of time.[7] If $g(x_0,t)$ is the wave profile acquired by the confocal optical microscope, taking the fast Fourier transform (FFT) of $g(x_0,t)$ yields

$$\mathrm{FFT}[g(x_0,t)] = G(\omega)\exp[j\Phi(\omega)], \quad (3)$$

$G(\omega)$ and $\Phi(\omega)$ being the amplitude and the phase, respectively, of $g(x_0,t)$ in the temporal frequency domain and x_0 is the distance between excitation and the detection points. $\Phi(\omega)$ is related to spatial frequency k by

$$\Phi(\omega) = kx_0, \quad (4)$$

which, by using Kelvin's dispersion equation for deep water approximation [$\tanh(kh) \approx 1$], gives the final equation for calculating surface tension:

$$\tau = \frac{x_0^3\rho\omega^2}{[\Phi(\omega)+\Phi_0]^3} - \frac{x^2\rho g}{[\Phi(\omega)+\Phi_0]^2}, \quad (5)$$

where Φ_0 is the phase offset added to compensate for the initial phase error. For a pure liquid with low viscosity such as water, the relative error in surface tension calculation based on Eq. (5) will be less than 0.3%. On the other hand, in the presence of elastic films at the surface, the relative error will be much higher due to omission of tangential stresses arising from surface tension gradients. The relative error can be as high as 6% for an elastic film on the water surface with a surface dilational modulus of 10 mN/m. This error can be remedied by doing two measurements at different distances for the purpose of calculating the spatial damping coefficient and afterwards solving the full dispersion equation given in Ref. 5. Unfortunately, it is computationally involved much more than Eq. (5) but nevertheless yields more information about the surface viscoelastic properties.

We performed several experiments with different samples to test and compare the performance of our technique against the Wilhelmy plate technique. The surface tension

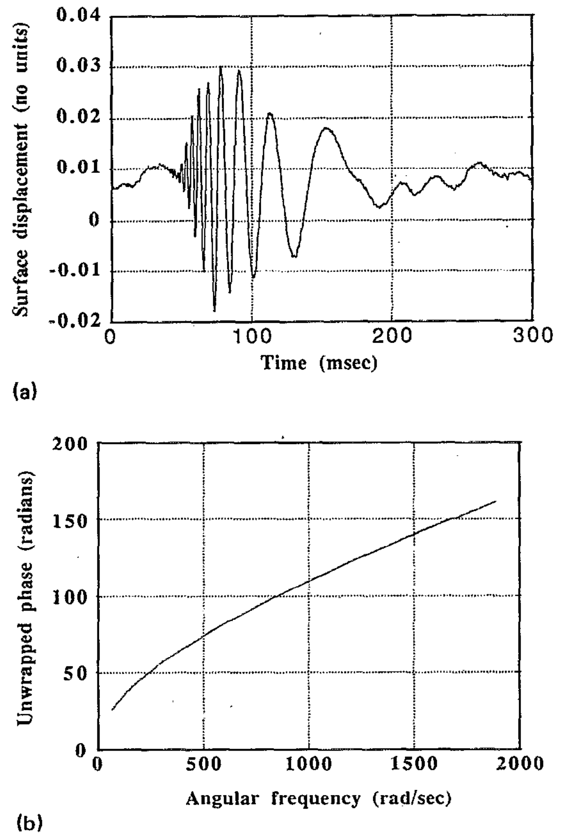

FIG. 2. (a) A typical capillary waveform as a function of time for clean water sample. The signal is detected by the confocal optical microscope at 40 mm away from the excitation point. (b) The unwrapped phase of the FFT of the signal shown in (a) as a function of frequency.

measurements that are done include clean water measurements, and those with soluble and insoluble surfactants. As a soluble surfactant, increasing concentrations of SDS (sodium dodecyl sulfate) has been used to lower the surface tension. Among the insoluble surfactants, tricresyl phosphate, butyl stearate, oleyl alcohol, and sorbitan monooleate are the ones that are being used. These materials are carefully spread on the clean water surface to form monolayers. A typically capillary wave signal for clean water sample as obtained by the confocal optical microscope is shown in Fig. 2(a). By taking FFT of this signal we calculate the phase and then unwrap it. The unwrapped phase for the signal in Fig. 2(a) is shown in Fig. 2(b). Applying Eq. (5) to the unwrapped phase for every frequency component will yield independently calculated surface tension values for every frequency component in the frequency band of measurement. At this point, Φ_0 is iteratively calculated to yield the lowest standard deviation for the surface tension values calculated. This procedure compensates for the initial phase errors. After the addition of the phase bias Φ_0, the average of the recalculated surface tension values in the frequency band will be the final surface tension value. As it is shown in Fig. 3, the results obtained from the capillary wave technique are in very good agreement with the results from Wilhelmy slide technique. The repeatability of our technique especially in the case surfactants is much better than the Wilhelmy plate technique. Our technique also enables us to measure damp-

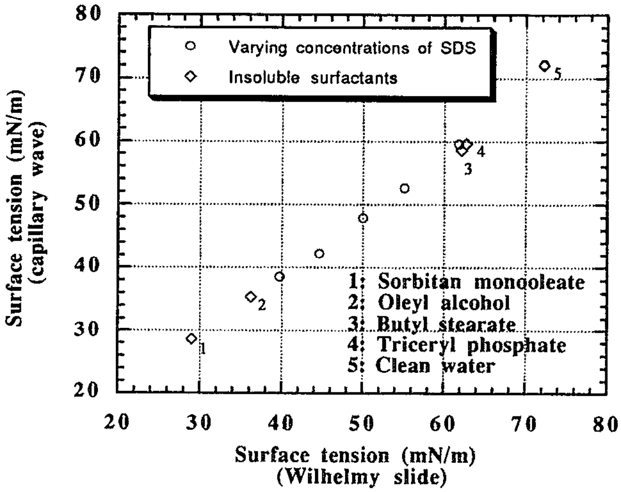

FIG. 3. Measured values of surface tension with different types of surfactants comparing the results of our technique with Wilhelmy plate technique.

ing coefficient of water waves in broad range of frequencies from 10 up to 300 Hz within a single measurement. Damping coefficient measurements will be presented in a separate paper in the future.

In conclusion, we have developed a novel technique for measuring surface tension. It is a noncontacting, rugged technique capable of performing the measurements in harsh environments like the ocean. With this technique, the whole measurement can easily be automated and with slight modifications it can be made capable of measuring the damping coefficient of the surface waves as a function of frequency.

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[1] J. F. Padday, Surf. Colloid Sci. **1**, 124 (1969).
[2] B. T. Chu and R. E. Apfel, J. Acoust. Soc. Am. **72**, 1673 (1982).
[3] S. A. Elrod, B. Hadimioglu, B. T. Khuri-Yakub, E. G. Rawson, E. Richley, C. F. Quate, N. N. Mansour, and T. S. Lundgren, J. Appl. Phys. **65**, 3441 (1989).
[4] G. Q. Xiao, T. R. Corle, and G. S. Kino, Appl. Phys. Lett. **53**, 716 (1988).
[5] E. H. Lucassen-Reynders and J. Lucassen, Adv. Colloid Sci. **2**, 347 (1969).
[6] A. L. Fetter and J. D. Walecka, *Theoretical Mechanics of Particles and Continua*, (McGraw-Hill, New York, 1980), Chap. 10.
[7] B. T. Khuri-Yakub, P. A. Reinholdtsen, C. -H. Chou, J. R. Vesecky, and C. C. Teague, Appl. Phys. Lett. **52**, 1571 (1988).