# CMUT Design Equations for Optimizing Noise Figure and Source Pressure

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***Abstract*—The key parameters for the design of a capacitive micromachined ultrasonic transducer (CMUT) cell are the dimensions of the vibrating plate, i.e. plate diameter and thickness, and the gap height. These dimensions together with the magnitude of the dc bias voltage determine the transducer's center frequency, bandwidth, and transmit and receive sensitivity. For a given center frequency, choosing the plate dimensions typically requires a tradeoff between bandwidth and sensitivity. The expected magnitude of the ac excitation and dc bias voltage guides selection of the gap height. Finite element modeling and simulation of the equivalent circuit model help examine these design tradeoffs. However, analytical expressions give better design insight and aid quick evaluation of designs for given specifications. In this work, we develop expressions for transmit pressure and receive noise figure for a circular CMUT cell with bending plate and an ideal parallel-plate CMUT cell. In order to simplify the analysis, we neglect atmospheric pressure, assume a linear spring constant for the plate stiffness, and assume the dc bias voltage to be close to the pull-in voltage. These assumptions are typically valid for CMUTs similar to those designed for medical imaging.**

## I. Introduction

Almost every demonstrated CMUT operates in a pure plate regime associated with a deflection-to-thickness ratio less than 20%, i.e. the bending stiffness (out-of-plane stress) dominates both the static and dynamic behavior of the plate [1]. Therefore it is a valid approach to model the mechanical properties of clamped circular CMUT plates with an equivalent spring constant, $k_1$, and an equivalent mass, $m$, given by

$$k_1 = \frac{192\pi D}{a^2} = \frac{16\pi E t^3}{a^2(1-\nu^2)} \tag{1}$$

and

$$m = 1.84\pi a^2 \rho t, \tag{2}$$

where $D$ is the plate's flexural rigidity, $a$ is the plate radius, $E$ is Young's modulus, $t$ is the thickness of the plate, $\nu$ is the Poisson ratio and $\rho$ is the plate material density [2].

We assume the radiation resistance $R_{rad}$ dominates the damping and that it equals the plane-wave radiation impedance

$$R_b = \pi a^2 R_{rad}. \tag{3}$$

Typical medical imaging transducers elements that are approximately λ/2 in size will have a complex radiation impedance whose real part is less than (3); for more accuracy, the calculated radiation impedance for a given design could be used in place of (3).

It is convenient to think in terms of the mechanical quality factor, $Q$, and the undamped angular resonance frequency, $\omega_0$, to design the cell dimensions as these are indicative of the transducer's angular center frequency and bandwidth. We can write these design variables in terms of the CMUT cell dimensions and material properties as

$$\omega_0 = \frac{10.22}{a^2\sqrt{\rho t/D}} = \frac{2.95t}{a^2}\sqrt{\frac{E}{\rho(1-\nu^2)}} = = \frac{1.27}{a}\sqrt{\frac{R_{rad}Q}{\rho^{3/2}}\sqrt{\frac{E}{1-\nu^2}}} \tag{4}$$

and

$$Q = 5.43\frac{t^2}{a^2 R_{rad}}\sqrt{\frac{E\rho}{1-\nu^2}} = 1.84\frac{\rho t}{R_{rad}}\omega_0. \tag{5}$$

Rearranging gives the plate dimensions in terms of the design variables, i.e.

$$t = 0.543\frac{R_{rad}Q}{\rho\omega_0} \tag{6}$$

and

$$a = \frac{1.27}{\omega_0}\sqrt{\frac{QR_{rad}}{\rho^{3/2}}\sqrt{\frac{E}{1-\nu^2}}}. \tag{7}$$

To determine the gap height, it is sometimes convenient to design directly in terms of the effective gap height, i.e. the gap height minus the insulator thickness divided by the insulator's relative permittivity. For example, the gap height may be dictated by the fabrication process or by a displacement requirement. Other times, it is more convenient to think in terms of pull-in voltage, which maps to the gap height. The pull-in voltage might be dictated by the expected ac and dc operating voltages.

CMUTs operating in the MHz frequency range in water have plates deflecting by a negligible amount due to atmospheric pressure. In this case, at pull-in, the average displacement equals 15% of the gap height $g_0$, i.e. we write

$$g_{pi} = 0.85\, g_0. \tag{8}$$

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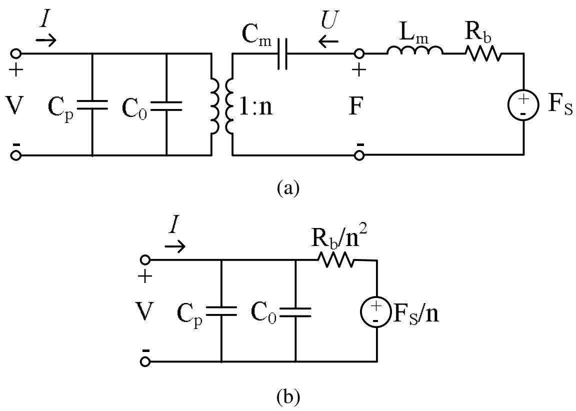


Fig. 1. Equivalent circuits used to calculate transmit and receive performance. (a) Standard CMUT equivalent circuit. (b) Simplified equivalent circuit assuming frequencies close to the undamped resonance frequency.

The pull-in voltage at zero ambient pressure is given by

$$V_{PI}|_{P_{atm}=0} = 0.39\sqrt{\frac{g_0^3 k_1}{\pi a^2 \varepsilon_0}} = \frac{1.56}{a^2}\sqrt{\frac{Et^3 g_0^3}{\varepsilon_0(1-\nu^2)}} = 0.39\sqrt{\frac{g_0^3 R_{rad} Q\omega_0}{\varepsilon_0}}. \quad (9)$$

Rearranging (9) gives the gap height in terms of $V_{\mathrm{pi}}$, i.e.

$$g_0 = 1.87\left(\frac{\varepsilon_0 V_{pi}^2}{R_{\mathrm{rad}}\, Q\, \omega_0}\right)^{1/3}. \quad (10)$$

Using (4)-(10), we can easily compute the CMUT cell dimensions that will give a desired resonance frequency, mechanical quality factor, and pull-in voltage. Since the application often dictates the transducer's desired center frequency, it is important to understand how quality factor and pull-in voltage affect the transducer's transmit and receive performance.

## II. Computing a Simplified Equivalent Circuit

The CMUT equivalent circuit (Fig. 1) is a useful tool for calculating small-signal transmit and receive behavior. For this analysis, we are interested in the CMUT's performance at the center frequency, which is often similar to its performance at the natural resonance frequency. At the series resonance frequency, the $L$ and $C$ that model the plate's mechanical impedance cancel, and the circuit simplifies to a parallel resistor and capacitor. The ratio of parasitic capacitance $C_p$ to the active capacitance $C_{pi}$ is given by $\gamma$.

By assuming the applied dc voltage $V_{dc}$ equals the pull-in voltage $V_{pi}$, we can calculate the values of the equivalent circuit in terms of the design variables.

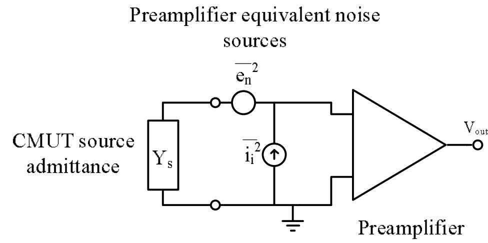


Fig. 2. Circuit used to compute the noise figure. Computation of the noise figure uses the CMUT's electrical admittance and the equivalent noise sources of the amplifier. Correlated noise sources in the amplifier are neglected.

Thus, we write for the transformer ratio at pull-in point

$$n_{pi} = 0.81\pi a^2\sqrt{\frac{\varepsilon_0 R_{rad} Q\omega_0}{g_0}} = 0.81A\sqrt{\frac{\varepsilon_0 R_{rad} Q\omega_0}{g_0}} = 0.59A\left(\frac{\varepsilon_0\,(R_{rad} Q\omega_0)^2}{V_{pi}}\right)^{1/3}, \quad (11)$$

where $A$ equals the total element area and not necessarily the area of a single cell, since the the transformer ratio scales with the number of cells. In addition, the applied dc voltage is typically between 50% and 90% of $V_{pi}$, so the aforementioned assumption overestimates the transducer's performance.

The parallel resistance and capacitance at pull-in point are given by

$$R_{pi} = 1.54\frac{g_0}{\varepsilon_0 A Q\omega_0} = \frac{2.88}{A}\left(\frac{V_{pi}}{\varepsilon_0\sqrt{R_{rad}}Q^2\omega_0^2}\right)^{2/3} \quad (12)$$

and

$$C_{pi} = 1.22\frac{\varepsilon_0 A}{g_0} = 0.65A\left(\frac{\varepsilon_0^2 Q R_{rad}\omega_0}{V_{pi}^2}\right)^{1/3}. \quad (13)$$

Note that the resistance and reactance are related by a simple term.

## III. Computing Receive Noise Figure

The noise figure of the amplifier relative to the resistive component of the CMUT's electrical impedance is a good indication of receiver performance. For a given amplifier equivalent voltage and current noise, we can optimize the transducer design for a low noise figure. Using classic two-port noise figure analysis (Fig. 2) [3] and the simplified equivalent circuit we can compute the noise figure.

We first compute the admittance of the CMUT equivalent circuit, i.e.

$$Y_s = 0.65\frac{\varepsilon_0 A\omega_0}{g_0}\left(Q + j1.872\left(\gamma+1\right)\right) = \\ = 0.347A\left(\frac{\varepsilon_0\omega_0^2\sqrt{QR_{rad}}}{V_{pi}}\right)^{2/3}\left(Q + j1.872\left(\gamma+1\right)\right). \tag{14}$$

Next we compute the current noise

$$i_s^2 = 2.6\frac{\varepsilon_0 AkTQ\omega_0}{g_0} = 1.39AkT\left(\frac{\varepsilon_0\sqrt{R_{rad}}Q^2\omega_0^2}{V_{pi}}\right)^{2/3} \tag{15}$$

and voltage noise

$$e_s^2 = 6.16kT\frac{g_0}{\varepsilon_0 AQ\omega_0}\frac{1}{1+\frac{3.5(\gamma+1)^2}{Q^2}} = \\ = 11.52\frac{kT}{A}\left(\frac{V_{pi}}{\varepsilon_0\sqrt{R_{rad}}Q^2\omega_0^2}\right)^{2/3}\frac{1}{1+\frac{3.5(\gamma+1)^2}{Q^2}} \tag{16}$$

of the parallel resistor. Then the noise figure is computed as follows:

$$F = 1 + \frac{\overline{\left|i_n + Y_s e_n\right|^2}}{i_s^2} = 1 + \frac{\overline{i_n^2} + \left|Y_c + Y_s\right|^2\overline{e_n^2}}{i_s^2}; \tag{17}$$

$$F = 1 + \frac{\overline{i_n^2}}{i_s^2} + \frac{\left|Y_s\right|^2\overline{e_n^2}}{i_s^2} = \\ = 1 + \frac{R_{pi}}{4kT}\overline{i_n^2} + \frac{1}{4kTR_{pi}}\left(1+\frac{3.5(\gamma+1)^2}{Q^2}\right)\overline{e_n^2}; \tag{18}$$

$$F = 1 + 0.385\frac{g_0}{kT\varepsilon_0\omega_0 QA}\overline{i_n^2} + \\ 0.162\frac{\varepsilon_0\omega_0 A}{g_0 kT}\left(Q+\frac{3.5(\gamma+1)^2}{Q}\right)\overline{e_n^2} = \\ = 1 + 0.72\frac{1}{AkT}\left(\frac{V_{pi}}{\varepsilon_0\sqrt{R_{rad}}Q^2\omega_0^2}\right)^{2/3}\overline{i_n^2} + \\ 0.0868\frac{A}{kT}\left(\frac{\varepsilon_0 Q^2\omega_0^2\sqrt{R_{rad}}}{V_{pi}}\right)^{2/3}\times \\ \left(1+\frac{3.5(\gamma+1)^2}{Q^2}\right)\overline{e_n^2}. \tag{19}$$

## IV. Total Source Pressure

In order to compute total transmit pressure in Pascals, $T_{tot}$, we assume an ac excitation voltage equal to a fixed percentage (20%) of the pull-in voltage. The total source pressure thus equals $V_{pi}/5$ times the transmit sensitivity, i.e. we write

$$T_{tot} \approx 0.59\left(\frac{\varepsilon_0(R_{rad}Q\omega_0)^2}{V_{pi}}\right)^{1/3}\frac{V_{pi}}{5}. \tag{20}$$

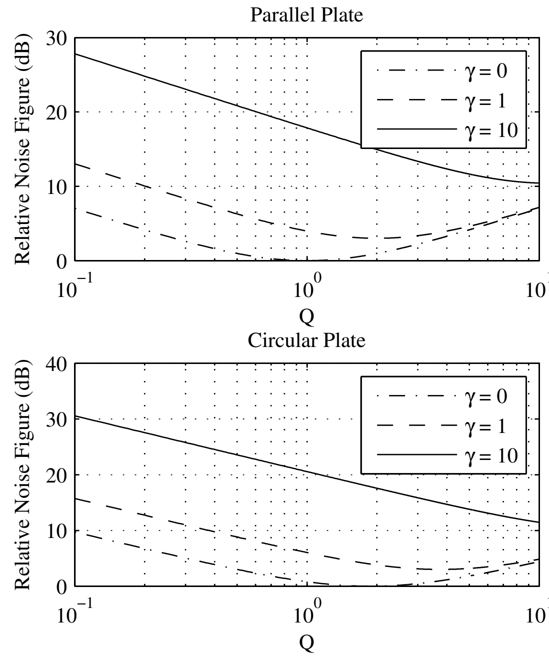


Fig. 3. Contribution of voltage noise to the noise figure as a function of mechanical $Q$ and parasitic capacitance $C_p$. An optimal value of $Q$ minimizes the voltage noise contribution to noise figure. The optimum value depends on the amount of parasitic capacitance $C_p$.

## V. Some Design Guidelines

From the expressions for noise figure, (19), and total source pressure, (20), we can derive some guidelines for choosing $Q$ and the pull-in voltage $V_{pi}$ for a given resonance frequency. Increasing $Q$ results in a stiffer equivalent spring constant, and, thus, a smaller gap for the same pull-in voltage. The resulting increased electric field in the gap increases sensitivity, and, thus, total output pressure. Furthermore, the smaller gap results in a lower electrical input impedance that is less sensitive to current noise. However, increasing $Q$ typically comes at the expense of bandwidth.

For minimizing the voltage noise contribution to noise figure, (19) shows that an optimum value of $Q$ exists; the optimal value depends on the parasitic capacitance given by $\gamma$ (Fig. 3).

Since we assume the dc and ac voltages are proportional to the pull-in voltage, increasing the pull-in voltage requires larger ac and dc voltages. Increasing the pull-in voltage effectively increases the gap, resulting in less transmit sensitivity. However, (20) shows that the increased ac voltage compensates for the larger gap resulting in a net increase in total output pressure. The larger gap results in a higher electrical impedance, which increases the current-noise contribution to the noise figure and reduces the voltage-noise contribution.

Finally, (19) shows that a larger transducer, which has lower electrical impedance, is more sensitive to voltage noise and less sensitive to current noise.

## VI. Analysis of Ideal Parallel-Plate CMUT Cell

Repeating the noise-figure and source pressure analysis for an ideal parallel-plate capacitor shows the extent that we can improve the CMUTs performance with a piston-like plate (e.g. [4]).

For a parallel-plate capacitor, pull-in occurs when the deflection equals 1/3 of the gap. The gap at pull-in thus equals

$$g_{pi} = \frac{2}{3} g_0. \tag{21}$$

Pull-in voltage maps to a larger gap for the parallel-plate transducer and we can write the following equations:

$$g_0 = \frac{3}{2} \left( \frac{\varepsilon_0 V_{pi}^2}{R_{rad} Q \omega_0} \right)^{1/3}; \tag{22}$$

$$\begin{aligned} V_{pi} &= \sqrt{\frac{8}{27} \frac{g_0^3 k_1}{A \varepsilon_0}} = 0.54 \sqrt{\frac{g_0^3 k_1}{A \varepsilon_0}} = \\ &= 0.54 \sqrt{\frac{g_0^3 R_{rad} Q \omega_0}{\varepsilon_0}}; \end{aligned} \tag{23}$$

$$\begin{aligned} n_{pi} &= 1.225 A \sqrt{\frac{\varepsilon_0 Q R_{rad} \omega_0}{g_0}} = \\ &= A \left( \frac{\varepsilon_0 (R_{rad} Q \omega_0)^2}{V_{pi}} \right)^{1/3}; \end{aligned} \tag{24}$$

$$R_{pi} = \frac{0.67}{A} \frac{g_0}{\varepsilon_0 Q \omega_0} = \frac{1}{A} \left( \frac{V_{pi}}{\varepsilon_0 \sqrt{R_{rad}} Q^2 \omega_0^2} \right)^{2/3}; \tag{25}$$

$$C_{pi} = \frac{3}{2} \frac{\varepsilon_0 A}{g_0} = A \left( \frac{\varepsilon_0^2 Q R_{rad} \omega_0}{V_{pi}^2} \right)^{1/3}; \tag{26}$$

$$Y_s = 1.49 \frac{\varepsilon_0 \omega_0 A}{g_0} \left( Q + j \left( 1 + \gamma \right) \right); \tag{27}$$

$$i_s^2 = 6 \frac{k T \varepsilon_0 Q \omega_0 A}{g_0} = 4 A k T \left( \frac{\varepsilon_0 \sqrt{R_{rad}} Q^2 \omega_0^2}{V_{pi}} \right)^{2/3}; \tag{28}$$

$$\begin{aligned} F &= 1 + 0.167 \frac{g_0}{k T \varepsilon_0 Q \omega_0 A} \overline{i_n^2} + \\ &\quad 0.375 \frac{\varepsilon_0 \omega_0 A}{g_0 k T} \left( Q + \frac{(1+\gamma)^2}{Q} \right) \overline{e_n^2} = \\ &= 0.25 \frac{1}{AkT} \left( \frac{V_{pi}}{\varepsilon_0 \sqrt{R_{rad}} Q^2 \omega_0^2} \right)^{2/3} \overline{i_n^2} + \\ &\quad 0.25 \frac{A}{kT} \left( \frac{\varepsilon_0 \sqrt{R_{rad}} Q^2 \omega_0^2}{V_{pi}} \right)^{2/3} \left( 1 + \frac{(1+\gamma)^2}{Q^2} \right) \overline{e_n^2}; \end{aligned} \tag{29}$$

and

$$T_{tot} \approx \left( \frac{\varepsilon_0 (R_{rad} Q \omega_0)^2}{V_{pi}} \right)^{1/3} \frac{V_{pi}}{5}. \tag{30}$$

TABLE I
Example Design Parameters

| | |
|---|---|
| $f_0$ | 10 MHz |
| $Q$ | 0.3 |
| $V_{pi}$ | 100 V |
| $e_n$ | 5.8 nV/vHz |
| $i_n$ | 10 fA/vHz |
| $\gamma$ | 25% |
| $A$ | 250 m × 250 m |
| $R_{rad}$ | 1.5 MRayls |

TABLE II
Resulting Cell Geometry and Performance

| | Circular Cell | Parallel Plate |
|---|---|---|
| $t$ | 1.8 $\mu$m | - |
| $a$ | 26 $\mu$m | 26 $\mu$m |
| $g_0$ | 270 nm | 220 nm |
| $R_{pi}$ | 40 kΩ | 14 kΩ |
| $C_{pi}$ | 2.5 pF | 3.77 pF |
| $F$ | 6.1 dB | 5.6 dB |
| $Tx$ | 488 kPa | 827 kPa |

## VII. Evaluating a Typical Design

Table I gives some design variables for a typical 5-MHz 2D CMUT array element. Choosing a higher resonance frequency than the desired center frequency compensates for the frequency shift caused by damping. A $Q$ value of 0.3 is a compromise between wide bandwidth and good sensitivity. A pull-in voltage of 100 V is appropriate for pulse-voltages on the order of 20 V, like those used in [5]. Finally, the given amplifier noise parameters are typical for a low-power amplifier. Table II gives the resulting designs and their estimated performance. Compared to the circular cell, the parallel-plate cell produces about 70% more pressure and has a 0.5-dB lower noise figure.

## VIII. Conclusion

Designing in terms of $Q$, $f_0$, and $V_{pi}$ enables quick estimation of cell dimensions and electronics requirements. Furthermore, it allows a direct comparison between different CMUT structures such as a bending plate circular cell and an ideal parallel-plate cell.

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