Abstract—Transient Lamb waves induced by finite piezoelectric transducers have been widely used in ultrasonic touch systems, where a tactile object is localized through its interaction with the Lamb waves. However, this problem has not been looked in-depth theoretically, which is essential for development of robust systems. The two-dimensional theory often fails to provide a true picture as the diffraction effects are of great significance in these technologies. Reduced order models such as the plate theories, on the other hand, can support diffraction. However, they fail to support mid to higher spectrum of the Lamb modes. Moreover, many introduce errors in the range of the supported modes. Hence, for many problems, the design confidence level is only achieved upon a three-dimensional theory. We present the mathematical modeling of piezoelectrically induced elastic waves propagating through homogeneous plates subjected to a tactile object. We use a finite element method to solve the model problem. The model is validated against the experimental data.

Keywords: ultrasonic touchscreen; ultrasound; numerical modeling; Lamb waves; piezoelectric transducers; elastic waves

I. INTRODUCTION

During the last decade, there have been emerging ultrasound/acoustic-based tactile sensing technologies. These involve interaction of a tactile object such as a human finger with a solid surface in either passive [1]–[3] or active [4]–[6] forms. Plates are strongly confined along the thickness. This allows Lamb waves to propagate inside the plate. Depending on the frequency of the operation and the thickness of the plate, various Lamb modes, from the low frequency flexural waves [1] to the higher ones such as surface waves [4], can be used to localize the human finger or any tactile object. Sensing is commonly done by piezoelectric transducers mounted at the edges of the plate.

Lamb waves are multi-mode, dispersive, and have a very complicated nature. Hence, a proper design of the piezoelectric transducers, in order to excite the desired modes, can be very challenging and often standard analytical design methodologies fail to give the actual three-dimensional behavior. This may lead to a poor design and performance. Development of robust localization algorithms often requires a selective excitation of the Lamb waves. It is, thus, important to understand/analyze/model the multi-modal behavior of piezoelectric resonators and the induced waves propagating through the plate. This calls for in-detail study of the forward physics of the system, which has been mainly overlooked in the literature. We present the mathematical modeling and full three-dimensional finite element analysis of the active Lamb wave based ultrasound touch sensing. This involves interaction of the electronics with the piezoelectric transducers and the waves propagating inside the plate, and also the interaction between the waves and the touch object.

In this study, the following notations are adopted: \( i, j, k \) represent the spatial coordinates. Throughout this paper, the summation convention is used for \( i, j, k \) unless otherwise specified. \( (\cdot)_i \) and \( (\cdot)_{ij} \) indicate the components of vector and tensor fields whereas \( (\cdot),i \) and \( (\cdot),ij \) represent the partial derivatives with respect to Euclidean coordinates.

II. MATHEMATICAL MODEL

A. Problem statement

The problem of interest is multi-physics in its nature involving acoustic, elastic, and piezoelectric waves. The domain of the problem is decomposed into subdomains, each of which governed by a different physics. We first explain the underlying physics in each subdomain, and then couple them by enforcing compatible interface conditions. We consider two different scenarios: (1) Uniform plate, where the waves excited by the piezoelectric transducers propagate inside the plate with the stress-free boundary conditions both at the top and bottom faces and (2) Plate with a touch object, where an acoustic cylindrical object is placed in contact with the top face of the plate. The electronic circuit is also coupled with the model. This makes it possible to have the inputs/outputs of the model as the actual voltages of a pulse generator/digitizer.

The geometrical configuration of the problem is shown in Fig. 1. Let \( \Omega \) be the domain of the problem, which is bounded and connected, with Lipschitz boundaries \( \partial \Omega \). Let \( \Omega_{pzi} \) represent the piezoelectric regions indexed by \( i \), \( \Omega_{s} \) the elastic domain, and \( \Omega_{f} \) the touch object domain. In other words,
\( \Omega = \Omega_f \cup \Omega_s \cup_{i=1}^{N_{r_s}} \Omega_{pzi}, \) where \( N_{r_s} \) indicates the number of piezoelectric regions. Also, let \( \partial \Omega_{A,B} \) indicate the interfaces between the subdomains, i.e., \( \partial \Omega_{A,B} = \partial \Omega_A \cap \partial \Omega_B, \) with \( A,B = s,f,pzi. \) Furthermore, \( \partial D^1 \Omega_{pzi} = \partial \Omega_s \setminus \{ \partial \Omega_{s,f} \cup_{i=1}^{N_{r_s}} \partial \Omega_{s,pzi} \} \) and \( \partial D^2 \Omega_{pzi} = \partial \Omega_{pzi} \setminus \partial \Omega_{s,pzi}. \)

**B. Linear elastic plate**

Wave propagation in general three-dimensional linear elastic media is governed by the linearized momentum balance law and a linear constitutive relation between the stress and strain. The governing equations are as follows.

\[
\rho \ddot{u}_i = \sigma_{ij,j} + f_i, \quad (1) \\
\sigma_{ij} = C_{ijkl} \varepsilon_{kl}, \quad (2) \\
\varepsilon_{kl} = \frac{1}{2} (u_{k,i} + u_{l,k}), \quad (3)
\]

in \( \Omega_s \times (0,T) \), with \( u, \varepsilon, \) and \( \sigma \) being the displacement, strain, and stress fields, respectively. The initial conditions are set to zero, i.e., \( u_i(x,t=0) = 0, \dot{u}_i(x,t=0) = 0 \), \( C \) is the fourth-order elasticity tensor with the major and minor symmetries. We also assume that \( C \) is uniformly positive definite. \( \rho \) is the mass density. The boundary conditions are as follows.

\[
\sigma_{ij} n_i = 0, \quad \text{on } \partial D^1 \Omega_s, \quad (4)
\]

\( n \) indicates the normal vector to the boundary.

**C. Piezoelectric transducers**

In piezoelectric media, one should also consider the divergence free electric displacement field, which is written in terms of the electric displacement \( D \) as below.

\[
\rho \ddot{u}_i = \sigma_{ij,j} + f_i, \quad (5) \\
D_{i} = \varepsilon_{i,jk} E_{jk}, \quad (6)
\]

in \( \Omega_{pzi} \times (0,T) \), with the stress free boundary conditions (4) on \( \partial D^1 \Omega_{pzi}. \) This is known as the quasi-static approximation to the coupled wave theory [7], which assumes the electric field is irrotational, hence can be written as the gradient of a scalar field \( \phi \) (the electric potential). The linearized constitutive relation for a general piezoelectric medium is given as

\[
\sigma_{ij} = C_{ijkl} \varepsilon_{kl} - \delta_{ij} E_k, \quad (7) \\
D_i = \varepsilon_{ijk} E_{jk} + \varepsilon_{i,k} E_k, \quad (8)
\]

where \( \varepsilon \) and \( \varepsilon \) are, respectively, the third order piezoelectric coupling tensor and the second order premitivity tensor. \( E \) is the electric field and \( E_k = -\phi_{,k} \). The electric boundary conditions are

\[
\phi = 0, \quad \text{on } \partial D^1 \Omega_{pzi}, \quad (9a) \\
\phi = V_{pz}(t), \quad \text{on } \partial D^2 \Omega_{pzi}, \quad (9b)
\]

over the metalized surfaces and over the non-metalized ones.

\[
D_i n_i = 0, \quad \text{on } \partial \Omega_{pzi} \setminus \{ \partial D^1 \Omega_{pzi} \cup \partial D^2 \Omega_{pzi} \}, \quad (10)
\]

where \( \partial D^1 \Omega_{pzi} \) and \( \partial D^2 \Omega_{pzi} \) indicate the piezoelectric terminal surfaces, i.e., the ground and signal electrodes, respectively.

**D. Linear acoustic touch object**

The touch object is assumed to behave as an ideal fluid governed by the linear acoustic theory, i.e.,

\[
\frac{1}{c_f^2} \ddot{p} - p_{,ii} = 0, \quad \text{in } \Omega_f \times (0,T], \quad (11)
\]

with

\[
p = 0, \quad \text{on } \partial D^1 \Omega_f, \quad (12)
\]

and the radiation condition along the length of the cylinder (i.e., absorbing boundary condition on \( \partial D^2 \Omega_f \)). \( c_f \) indicates the speed of sound. This is a classical model widely used in medical ultrasound in order to model ultrasound wave propagation in human tissue.

**E. Electronic circuit**

The boundary conditions (9a,b) can only reflect the open/closed circuit operating conditions. These can be improved through coupling circuit components in order to broaden the range of the operating conditions, which in turn provides more experimentally oriented simulations. Here, we consider a simple circuit composed of a resistor and a voltage source in series with the piezoelectric terminals 1, 2, at which

\[
V_{pz}(t) = V_{sc}(t) + RI(t), \quad \text{on } \partial D^2 \Omega_{pzi}, \quad (13)
\]

where \( V_{sc}(t) \) is the voltage source, \( R \) is the resistor, and \( I(t) \) is the net current. When the piezoelectric domains are simulated in the receive mode (i.e., the direct piezoelectric effect), the voltage source \( V_{sc}(t) \) is set to zero.

**F. Interface conditions**

The interface conditions between the piezoelectric and elastic domains are the continuity of the displacement and traction fields, that is to write

\[
\sigma_{ij} n_i = \sigma_{ij}^{pzi} n_i, \quad u_i^{pzi} = u_i^{s,pzi}, \quad \text{on } \partial \Omega_{s,pzi}. \quad (14)
\]

The interface conditions between the elastic and acoustic domains are the continuity of the normal acceleration and traction, i.e.,

\[
\sigma_{ij}^{s} n_i = -\rho_f n_j, \quad \ddot{u}_i^{s} n_i = -p_{,k} n_k / \rho_f, \quad \text{on } \partial \Omega_{s,f}. \quad (15)
\]

where \( \rho_f \) indicates the mass density of the acoustic domain.

**III. IMPLEMENTATION**

The presented model was implemented using COMSOL Multiphysics 4.3a (2013) finite elements simulation software. The system of ordinary differential equations arose from the finite element discretization was integrated using the Alpha method, with the damping factor 0.75 to damp out the higher order spurious modes. This leads to an implicit integration, for which a linear system needs to be solved at each timestep. As the system is large (over two millions degrees of freedom) and not symmetric, we used the GMRES method with a proper
preconditioning to solve these equations. For the prototype simulations, quadratic elements with 10 points per minimum wavelength in each subdomain were applied, with CFL = 0.25.

**IV. EXPERIMENTAL VALIDATION**

The model is validated against the experiment as shown in Fig. 2. The transmitter and receiver are bonded to the edges of a plate that are 3 in apart. The transducers are 1.6 mm × 1 mm × 0.83 mm PZT-5H polarized toward the edge at which they are bonded to the plate. The receivers is terminated with 50 Ω. The transmitter is excited with a single cycle sine wave at 800 kHz. A good indication of an accurate model is how well the physical dispersion can be captured without introducing the numerical dispersion error, for which the first few zero-crossings can be analyzed. The measurement is in a good agreement with the numerical result. However, there exist some discrepancies. Some of the possible sources include the uncertainties in the electronic loading, plate dimensions, and piezoelectric material properties and dimensions.

**V. NUMERICAL EXAMPLE**

As an example, we considered an active touch system driven at a low MHz regime. This problem is more delicate compared to the very high or low frequency ones, as the dispersion and existence of several modes of propagation are the predominant factors in determining the results. In this regime, the reduced order models fail to give a good indication as to what happens once a tactile object is placed in contact with the plate. The means 

For the S0 transducers (Fig. 3) and the A0 ones (Fig. 4). As can be seen, the A0 transducer is more sensitive to the touch object as the perturbations appear stronger. Quantitatively, this can be seen through calculating the leaked energy into the touch object. However, the S0 transducer provides a more efficient coupling, hence stronger wave energy propagation though the plate. Furthermore, the S0 waves travel faster, hence they potentially collect more information as they bounce off the edges more times in a given period of time. This can be useful if the information buried in the reflections is used in the inversion process, in a ray-chaotic sense. Snapshots of propagation induced by the S0 transducer are shown in Fig. 5a,b and the A0 one in Fig. 6a,b. Snapshots of propagation for both polarizations in presence of the finger are shown in Fig. 5,6c.

**VI. SUMMARY**

We have developed a fully coupled three-dimensional model in order to study piezoelectrically induced Lamb wave propagation in plates in contact with a tactile object. This is essential for qualitative and quantitative evaluations of active ultrasound-based touch technologies, which in turn leads to improvements in design of the ultrasonic touch systems and development of robust localization/imaging algorithms. The
Fig. 5: Lamb waves induced by the S0 transducer. The colormap scale is in dB and visualizes the amplitude of the displacement vector field. (a) and (b) show the propagation at two time instants. (c) illustrates the Lamb waves in presence of an absorbing touch object.

Fig. 6: Lamb waves induced by the A0 transducer. The colormap scale is in dB and visualizes the amplitude of the displacement vector field. (a) and (b) show the propagation at two time instants. (c) illustrates the Lamb waves in presence of an absorbing touch object.

Fig. 4: Output voltage for different configurations of the A0 excitation. The solid blue and dashed red curves indicate the output signal without and with the touch object, respectively. The results are normalized with respect to the maximum amplitude of the input.

which together assist with the inversion process.

ACKNOWLEDGMENT

This work was funded by Intel Corporation.

REFERENCES


