CMUT Design Equations for Optimizing Noise Figure and Source Pressure

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***Abstract*—The key parameters for the design of a CMUT cell are the plate size (e.g. the plate diameter and thickness) and the gap size. These dimensions together with the magnitude of the dc bias voltage determine the transducer's center frequency, bandwidth, and transmit and receive sensitivity. For a given center frequency, choosing the plate size typically requires a tradeoff between bandwidth and sensitivity. The expected magnitude of the ac excitation and dc bias voltage guides selection of the gap size. Finite element modeling and simulation of the equivalent circuit model help examine these design tradeoffs. However, analytical expressions give better design insight and aid quick evaluation of designs for given specifications. In this work, we develop expressions for transmit pressure and receive noise figure for a conventional circular CMUT cell and an ideal parallel-plate CMUT cell. To simplify the analysis, we neglect atmospheric pressure, assume a linear spring constant models the plate stiffness, and assume the dc bias voltage is close to the pull-in voltage. These assumptions are typically valid for CMUTs similar to those designed for medical imaging.**

I. Introduction

For deflections that are small relative to the plate thickness, an equivalent spring constant, k_1, and mass, m, accurately model the mechanical properties of a clamped circular CMUT plate [1].

$$k_1 = \frac{192\pi D}{a^2} = \frac{16\pi E t^3}{a^2(1-\nu^2)} \tag{1}$$

$$m = 1.84\pi a^2 \rho t \tag{2}$$

We assume the radiation resistance dominates the damping and that it equals the plane-wave radiation impedance given by (3). Typical medical imaging transducers elements that are approximately $\lambda/2$ in size will have a complex radiation impedance whose real part is less than (3); for more accuracy, the calculated radiation impedance for a given design could be used in place of (3).

$$R_b = \pi a^2 R_{rad} \tag{3}$$

To design the cell dimensions it is convenient to think in terms of the mechanical quality factor, Q, and the undamped resonance frequency, f_0, as these are indicative of the transducer's center frequency and bandwidth. We can write these design variables in terms of the CMUT cell dimensions and material properties, where a is the plate radius, ρ is the plate material density, E is Young's modulus, D is the plate's flexural rigidity, and ν is the Poisson ratio.

$$\omega_0 = \frac{10.22}{a^2\sqrt{\rho t/D}} = \frac{2.95t}{a^2}\sqrt{\frac{E}{\rho(1-\nu^2)}} \tag{4}$$

$$= \frac{1.27}{a}\sqrt{\frac{R_{rad}Q}{\rho^{3/2}}\sqrt{\frac{E}{1-\nu^2}}} \tag{5}$$

$$Q = 5.43\frac{t^2}{a^2 R_{rad}}\sqrt{\frac{E\rho}{1-\nu^2}} = 1.84\frac{\rho t}{R_{rad}}\omega_0 \tag{6}$$

Rearranging gives the plate dimensions in terms of the design variables.

$$t = 0.543\frac{R_{rad}Q}{\rho\omega_0} \tag{7}$$

$$a = \frac{1.27}{\omega_0}\sqrt{\frac{QR_{rad}}{\rho^{3/2}}\sqrt{\frac{E}{1-\nu^2}}} \tag{8}$$

To determine the gap height, it is sometimes convenient to design directly in terms of the effective gap height i.e. the gap height minus the insulator thickness divided by the insulator's relative permittivity. For example, the gap height may be dictated by the fabrication process or by a displacement requirement. Other times, it is more convenient to think in terms of the pull-in (collapse) voltage, which maps to the gap height. For example, the pull-in voltage might be dictated by the expected ac and dc operating voltages.

For CMUTs designed to operate in the MHz frequency range in water, atmospheric pressure typically deflects the plate a negligible amount. In this case, at pull-in, the average displacement equals 15% of the gap.

$$g_{pi} = 0.85 g_0 \tag{9}$$

The pull-in voltage is given by (10).

$$V_{PI}|_{P_{atm}=0} = 0.39\sqrt{\frac{g_0^3 k_1}{\pi a^2 \varepsilon_0}} = \frac{1.56}{a^2}\sqrt{\frac{E t^3 g_0^3}{\varepsilon_0(1-\nu^2)}} \tag{10}$$

$$= 0.39\sqrt{\frac{g_0^3 R_{rad} Q \omega_0}{\varepsilon_0}} \tag{11}$$

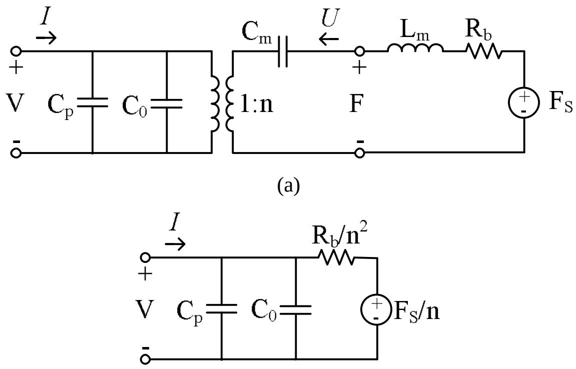

Fig. 1. Equivalent circuits used to calculate transmit and receive performance. (a) Standard CMUT equivalent circuit. (b)Simplified equivalent circuit assuming frequencies close to the undamped resonance frequency.

Rearranging (10) gives the gap in terms of V_{pi}.

$$g_0 = 1.87(\frac{\varepsilon_0 V_{pi}^2}{R_{rad} Q \omega_0})^{1/3} \tag{12}$$

Using (4)-(12), a designer can easily compute the CMUT cell dimensions that will give a desired resonance frequency, mechanical quality factor, and pull-in voltage. Since the application often dictates the transducer's desired center frequency, it is important to understand how quality factor and pull-in voltage affect the transducer's transmit and receive performance.

II. Computing a Simplified Equivalent Circuit

The CMUT equivalent circuit (Fig. 1) is a useful tool for calculating small-signal transmit and receive behavior. For this analysis, we are interested in the CMUT's performance at the center frequency, which is often similar to its performance at the natural resonance frequency. At the resonance frequency, the series L and C that model the plate's mechanical impedance cancel, and the circuit simplifies to a parallel resistor and capacitor. The ratio of parasitic capacitance, C_p, to the CMUT capacitance, C_0, is given by γ.

By assuming the applied dc voltage, V_{dc} equals the pull-in voltage, V_{pi}, we can calculate the values of the equivalent circuit in terms of the design variables. In practice, the applied dc voltage is typically between 50% and 90% of V_{pi} so this assumption overestimates the transducer's performance.

For V_{dc} equal to V_{pi}, the transformer ratio equals (13).

$$n_{pi} = 0.81 \pi a^2 \sqrt{\frac{\varepsilon_0 R_{rad} Q \omega_0}{g_0}} \tag{13}$$

$$= 0.81 A \sqrt{\frac{\varepsilon_0 R_{rad} Q \omega_0}{g_0}} \tag{14}$$

$$= 0.59 A (\frac{\varepsilon_0 (R_{rad} Q \omega_0)^2}{V_{pi}})^{1/3} \tag{15}$$

Since the transformer ratio scales with the number of cells, A equals the total element area and not necessarily the area of a single cell.

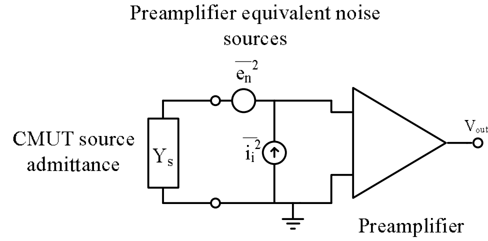

Fig. 2. Circuit used to compute the noise figure. Computation of the noise figure uses the CMUT's electrical admittance and the equivalent noise sources of the amplifier. Correlated noise sources in the amplifier are neglected.

The parallel resistance and capacitance are given by (16) and (17).

$$R_{pi} = 1.54 \frac{g_0}{\varepsilon_0 A Q \omega_0} = \frac{2.88}{A} (\frac{V_{pi}}{\varepsilon_0 \sqrt{R_{rad}} Q^2 \omega_0^2})^{2/3} \tag{16}$$

$$C_{pi} = 1.22 \frac{\varepsilon_0 A}{g_0} = 0.65 A (\frac{\varepsilon_0^2 Q R_{rad} \omega_0}{V_{pi}^2})^{1/3} \tag{17}$$

Note that the resistance and reactance are related by a simple term.

$$R_{pi} = 1.54 \frac{g_0}{\varepsilon_0 A Q \omega_0} = 1.26 (\frac{V_{pi}}{\varepsilon_0 A \sqrt{R_{rad}} Q^2 \omega_0^2})^{2/3} \tag{18}$$

III. Computing Receive Noise Figure

The noise figure of the amplifier relative to the resistive component of the CMUT's electrical impedance is a good indication of receiver performance. For a given amplifier equivalent voltage and current noise, we can optimize the transducer design for a low noise figure. Using classic two-port noise figure analysis (Fig. 2) [2] and the simplified equivalent circuit we can compute the noise figure.

We first compute the admittance of the CMUT equivalent circuit, Y_s.

$$Y_s = 0.65 \frac{\varepsilon_0 A \omega_0}{g_0} (Q + j1.872(\gamma + 1)) \tag{19}$$

$$= 0.347 A (\frac{\varepsilon_0 \omega_0^2 \sqrt{Q R_{rad}}}{V_{pi}})^{2/3} (Q + j1.872(\gamma + 1)) \tag{20}$$

Next we compute the current noise, i_s, and voltage noise, v_s, of the parallel resistor.

$$i_s^2 = 2.6 \frac{\varepsilon_0 A k T Q \omega_0}{g_0} = 1.39 A k T (\frac{\varepsilon_0 \sqrt{R_{rad}} Q^2 \omega_0^2}{V_{pi}})^{2/3} \tag{21}$$

$$e_s^2 = 6.16 k T \frac{g_0}{\varepsilon_0 A Q \omega_0} \frac{1}{1 + \frac{3.5(\gamma+1)^2}{Q^2}} \tag{22}$$

$$= 11.52 \frac{kT}{A} (\frac{V_{pi}}{\varepsilon_0 \sqrt{R_{rad}} Q^2 \omega_0^2})^{2/3} \frac{1}{1 + \frac{3.5(\gamma+1)^2}{Q^2}} \tag{23}$$

The noise figure is computed as follows.

$$F = 1 + \frac{\overline{|i_n + Y_s e_n|^2}}{i_s^2} = 1 + \frac{\overline{i_n^2} + |Y_c + Y_s|^2 \, \overline{e_n^2}}{i_s^2} \tag{24}$$

$$F = 1+\frac{\overline{i_n^2}}{i_s^2}+\frac{|Y_s|^2\,\overline{e_n^2}}{i_s^2} = 1+\frac{R_{pi}}{4kT}\overline{i_n^2}+\frac{1}{4kTR_{pi}}(1+\frac{3.5(\gamma+1)^2}{Q^2})\overline{e_n^2} \quad (25)$$

$$F = 1 + 0.385\frac{g_0}{kT\varepsilon_0\omega_0 QA}\overline{i_n^2}+ \quad (26)$$

$$0.162\frac{\varepsilon_0\omega_0 A}{g_0 kT}(Q+\frac{3.5(\gamma+1)^2}{Q})\overline{e_n^2} \quad (27)$$

$$= 1 + 0.72\frac{1}{AkT}(\frac{V_{pi}}{\varepsilon_0\sqrt{R_{rad}}Q^2\omega_0^2})^{2/3}\overline{i_n^2}+ \quad (28)$$

$$0.0868\frac{A}{kT}(\frac{\varepsilon_0 Q^2\omega_0^2\sqrt{R_{rad}}}{V_{pi}})^{2/3}\times \quad (29)$$

$$(1+\frac{3.5(\gamma+1)^2}{Q^2})\overline{e_n^2} \quad (30)$$

IV. Total Source Pressure

To compute total transmit pressure in Pascals, T_{tot}, we assume an ac excitation voltage equal to a fixed percentage (20%) of the pull-in voltage. The total source pressure thus equals V_{pi}/5 times the transmit sensitivity.

$$T_{tot} \approx 0.59(\frac{\varepsilon_0(R_{rad}Q\omega_0)^2}{V_{pi}})^{1/3}\frac{V_{pi}}{5} \quad (31)$$

V. Some Design Guidelines

From the expressions for noise figure, (26), and total source pressure, (31), we can derive some guidelines for choosing Q and the pull-in voltage for a given resonance frequency. Increasing Q results in a stiffer equivalent spring constant and thus a smaller gap for the same pull-in voltage. The resulting increased electric field in the gap increases sensitivity and thus total output pressure. Furthermore, the smaller gap results in a lower electrical input impedance that is less sensitive to current noise. However, increasing Q typically comes at the expense of bandwidth.

For minimizing the voltage noise contribution to noise figure, (26) shows that an an optimum value of Q exists; the optimal value depends on the parasitic capacitance given by γ as seen in Fig. 3.

Since we assume the dc and ac voltages are proportional to the pull-in voltage, increasing the pull-in voltage requires larger ac and dc voltages. Increasing the pull-in voltage effectively increases the gap resulting in less transmit sensitivity. However, (31) shows that the increased ac voltage compensates for the larger gap resulting in a net increase in total output pressure. The larger gap results in a higher electrical impedance which increases the current-noise contribution to the noise figure and reduces the voltage-noise contribution.

Finally, (26) shows that a larger transducer, which has lower electrical impedance, is more sensitive to voltage noise and less sensitive to current noise.

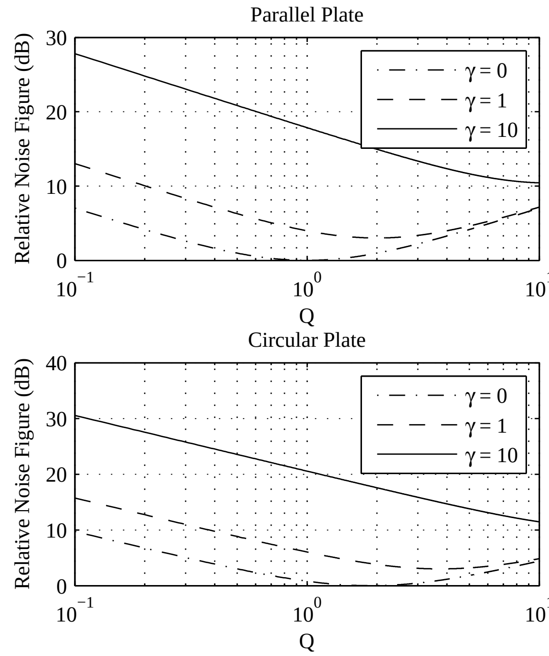

Fig. 3. Contribution of voltage noise to the noise figure as a function of mechanical Q and parasitic capacitance. An optimal value of Q minimizes the voltage noise contribution to noise figure. The optimum value depends on the amount of parasitic capacitance.

VI. Analyzing an Ideal Parallel-Plate CMUT Cell

Repeating the noise-figure and source pressure analysis for an ideal parallel-plate capacitor shows the extent that we can improve the CMUTs performance with a more piston-like plate (e.g. [3]).

For a parallel-plate capacitor, pull-in occurs when the deflection equals 1/3 the gap. The gap at pull-in thus equals (32).

$$g_{pi} = \frac{2}{3}g_0 \quad (32)$$

Pull-in voltage maps to a larger gap for the parallel-plate transducer.

$$g_0 = \frac{3}{2}(\frac{\varepsilon_0 V_{pi}^2}{R_{rad}Q\omega_0})^{1/3} \quad (33)$$

$$V_{pi} = \sqrt{\frac{8}{27}\frac{g_0^3 k_1}{A\varepsilon_0}} = 0.54\sqrt{\frac{g_0^3 k_1}{A\varepsilon_0}} \quad (34)$$

$$= 0.54\sqrt{\frac{g_0^3 R_{rad}Q\omega_0}{\varepsilon_0}} \quad (35)$$

$$n_{pi} = 1.225A\sqrt{\frac{\varepsilon_0 Q R_{rad}\omega_0}{g_0}} = A(\frac{\varepsilon_0(R_{rad}Q\omega_0)^2}{V_{pi}})^{1/3} \quad (36)$$

$$R_{pi} = \frac{0.67}{A}\frac{g_0}{\varepsilon_0 Q\omega_0} = \frac{1}{A}(\frac{V_{pi}}{\varepsilon_0\sqrt{R_{rad}}Q^2\omega_0^2})^{2/3} \quad (37)$$

TABLE I
EXAMPLE DESIGN PARAMETERS

f_0	10 MHz
Q	0.3
V_{pi}	100 V
e_n	5.8 nV/vHz
i_n	10 fA/vHz
γ	25%
A	250 m × 250 m
R_{rad}	1.5 MRayls

TABLE II
RESULTING CELL GEOMETRY AND PERFORMANCE

	Circular Cell	Parallel Plate
t	1.8 μm	-
a	26 μm	26 μm
g_0	270 nm	220 nm
R_{pi}	40 kΩ	14 kΩ
C_{pi}	2.5 pF	3.77 pF
F	6.1 dB	5.6 dB
Tx	488 kPa	827 kPa

$$C_{pi} = \frac{3}{2}\frac{\varepsilon_0 A}{g_0} = A(\frac{\varepsilon_0^2 Q R_{rad}\omega_0}{V_{pi}^2})^{1/3} \tag{38}$$

$$Y_s = 1.49\frac{\varepsilon_0\omega_0 A}{g_0}(Q + j(1+\gamma)) \tag{39}$$

$$i_s^2 = 6\frac{kT\varepsilon_0 Q\omega_0 A}{g_0} = 4AkT(\frac{\varepsilon_0\sqrt{R_{rad}}Q^2\omega_0^2}{V_{pi}})^{2/3} \tag{40}$$

$$F = 1 + 0.167\frac{g_0}{kT\varepsilon_0 Q\omega_0 A}\overline{i_n^2}+ \tag{41}$$

$$0.375\frac{\varepsilon_0\omega_0 A}{g_0 kT}(Q + \frac{(1+\gamma)^2}{Q})\overline{e_n^2} \tag{42}$$

$$= 0.25\frac{1}{AkT}(\frac{V_{pi}}{\varepsilon_0\sqrt{R_{rad}}Q^2\omega_0^2})^{2/3}\overline{i_n^2} \tag{43}$$

$$+0.25\frac{A}{kT}(\frac{\varepsilon_0\sqrt{R_{rad}}Q^2\omega_0^2}{V_{pi}})^{2/3}(1 + \frac{(1+\gamma)^2}{Q^2})\overline{e_n^2} \tag{44}$$

$$T_{tot} \approx (\frac{\varepsilon_0(R_{rad}Q\omega_0)^2}{V_{pi}})^{1/3}\frac{V_{pi}}{5} \tag{45}$$

VII. EVALUATING A TYPICAL DESIGN

Table I gives some design variables for a typical 5-MHz 2D transducer array element. Choosing a higher resonance frequency than the desired center frequency compensates for the frequency shift caused by damping. A Q value of 0.3 is a compromise between wide bandwidth and good sensitivity. A pull-in voltage of 100 V is appropriate for pulse-voltages on the order of 20 V, like those used in [4]. Finally, the amplifier noise parameters are typical for a low-power amplifier.

Table II gives the resulting designs and their estimated performance. Compared to the circular cell, the parallel-plate cell produces about 70% more pressure and has a 0.5-dB lower noise figure.

VIII. CONCLUSION

Designing in terms of Q, f_0, and V_{pi} enables quick estimation of cell dimensions and electronics requirements. Furthermore, it allows a direct comparison between different CMUT structures such as a conventional circular cell and an ideal parallel-plate cell.

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