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FINITE ELEMENT ANALYSIS OF STRESS STIFFENING EFFECTS IN CMUTS

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***Abstract*—We use finite element analysis (FEA) to model capacitive micromachined ultrasonic transducer (CMUT) cells where stress stiffening affects static deflection, pull-in voltage, resonance frequency, and small-signal sensitivity. Determining the small-signal sensitivity is challenging because it requires a prestressed harmonic response analysis in which the geometric nonlinearities are activated during the static analysis to prestress the structure. The goal is a correct static operation point calculation for CMUT plates that exceed a static deflection-to-thickness ratio of 20%. Assuming only a small AC excitation, we use a linear but updated stiffness matrix to calculate the harmonic response. We achieve this by using a prestressed mode superposition harmonic response analysis, which uses the sum of factored mode shapes obtained from a nonlinear prestressed modal analysis. We test our FEA for two CMUTs, of which one operates in a more membrane-dominated regime. Comparisons to measurements demonstrate that only the FEA that accounts for stress stiffening features good agreement for both designs. Our FEA allows us to model CMUTs that operate in a more membrane-dominated regime.**

I. INTRODUCTION

Interestingly, the term membrane is the established name for the key component of a CMUT – the moving part of the CMUT cell. The term plate, however, is more appropriate because almost every demonstrated CMUT operates in a pure plate regime, in which the bending stiffness (out-of-plane stress) dominates both the static and dynamic behavior. Even CMUTs with vacuum-sealed cavities, biased close to pull-in point, typically have deflection-to-thickness ratios less than 20% (limit of Kirchhoff's linear plate bending theory [1]), which places them in the plate regime.

Stress stiffening is a geometrical nonlinearity that needs to be considered for thin structures that, in addition to bending stiffness, also have axial stiffness [2]. Even for an ideal-clamped plate with uniform thickness that experiences a large deflection due to a uniform load, the calculation of the deflection is complicated. The exact calculation of the deflection of the plate requires the knowledge of the stress-state of the plate, which in turn depends on the deflection. This leads to a nonhomogeneous system of equations (Karman equations, [3], [4]) that can only be attacked by successive approximation techniques.

There are several finite element software packages that are useful for the CMUT design process. ANSYS, CAPA, COMSOL, PZFlex, and COVENTOR are examples. All of these software packages provide the required analysis types for a successful CMUT design: a static structural analysis to calculate the static deflection and stress state of the CMUT plate; a modal analysis to calculate the natural frequencies and mode shapes of the CMUT plate; a harmonic response analysis to calculate the dynamic response for sustained cyclic loading conditions; and a transient dynamic analysis to calculate the dynamic response to general time-dependent loading conditions.

For a CMUT plate, above a vacuum gap and biased with a bias voltage V_{DC}, the harmonic response analysis as well as the modal analysis, only make sense when following a static structural analysis (prestressed analysis [5]). In case of the transient dynamic analysis, the static operation point needs to be taken into account by correct static pressure and DC voltage loading, before the main transient excitation signal of interest can be applied.

During the FEA-based design process of the CMUT, only the harmonic response analysis and the transient dynamic analysis allow direct performance comparisons between different design iterations. Examples are the calculation of displacement, acoustic output pressure, electrical input impedance, and receive and transmit sensitivity. The transient dynamic analysis, however, requires significant more computation time compared to a harmonic response analysis, in particular for systems with large quality factors. The measured quality factors for the displacement of the devices considered in this work are in the range of hundred to several hundreds. Although for circular shaped cells the axial symmetry can be exploited by using 2D models, these quality factors limit the transient dynamic analysis to impulse response calculations. For example, if one wants to calculate the response to burst or CW signals, finding the exact excitation frequency for determining the maximum displacement is a challenge.

Therefore, after discussing nonlinear static and nonlinear modal analysis in the next section, we focus on a prestressed harmonic response calculation with nonlinear static solution steps before we compare our simulation results to measurements.

II. Static and Modal Analysis

We use the software package ANSYS to model stress stiffening effects in CMUTs, mainly because of the availability of the lumped transducer element trans126 (Fig. 1) [6].

In this work, we consider two CMUT designs (A and B) for airborne ultrasound applications in the 50 kHz range [7]. Design A has a 40-μm-thick CMUT plate and a 36-μm gap; design B has a 60-μm-thick CMUT plate and a 16-μm gap. For both designs the plate material is single crystal silicon, the plate diameter is $4000\ \mu$m, and the insulation layer (thermal grown silicon dioxide) thickness at the bottom of cavity is $3.3\ \mu$m. After the fabrication we compared the measured static plate deflections to the calculated ones, obtained from the FEM shown in Fig. 1(a). If stress stiffening effects are not considered in the FEM (stress stiffening effects turned off, nlgeom command in ANSYS), only for the 60-μm-thick plate, good agreement can be observed (Fig. 2).

The design with the 40-μm-thick plate has a deflection-to-thickness ratio of more than 65%, i.e. this CMUT plate, in addition to bending stiffness, also is affected by membrane stiffness. Tensile membrane forces (plate is also stretched due to the deflection) lead to a reduced bending deformation. Fig. 2 confirms that our nonlinear FEA shows good agreement for both designs.

Stress stiffening not only affects the static behavior of the CMUT plate, it also increases the resonance frequency of the device. For example, a linear and nonlinear prestressed (pressure only) modal analysis for the (0,1) mode for plate thicknesses ranging from $10\ \mu$m to $90\ \mu$m (Fig. 3) illustrates why the deflection-to-thickness ratio is worth considering during the design process.

Another prestressed (pressure and bias voltage) modal analysis (Fig. 4), in which the bias voltage is varied from 0 to 1100 V, demonstrates that besides increasing the resonance frequency, stress stiffening can dramatically increase the pull-in voltage.

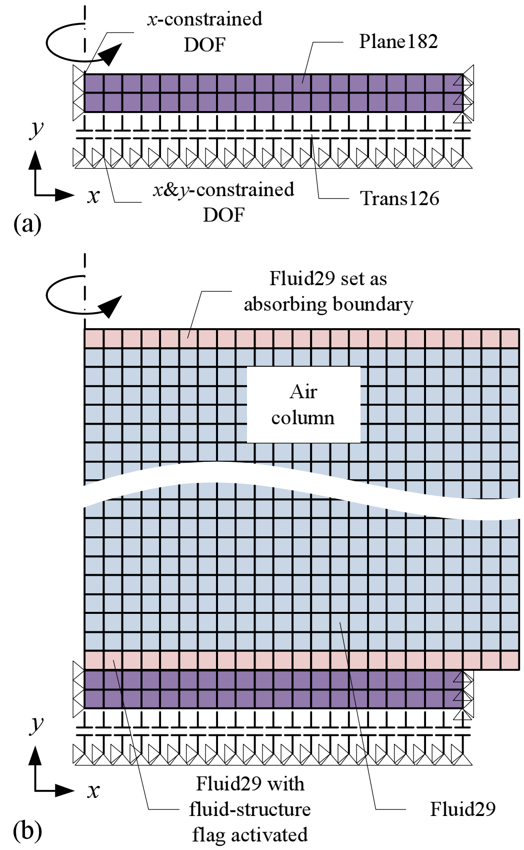

Fig. 1. Schematics of the two finite element models (FEMs) for the calculation of the static, modal, and harmonic response of circular CMUT plates, including stress stiffening effects.

III. Harmonic Response Calculation with Nonlinear Static Solution Step

A harmonic response analysis is a small signal analysis, which is linear per definition. It is a main requirement that the structure has constant or frequency-dependent stiffness, damping, and mass effects. The prestressed harmonic response analysis requires two parts. First, a static solution needs to be obtained (pstres command in ANSYS) and saved for the second subsequent part, the harmonic response analysis.

As soon as geometrical nonlinearities are present in the structure, the prestressed harmonic response analysis is not possible per se. In case one turns on the geometrical nonlinearities during the static analysis, ANSYS does not support the required subsequent harmonic response analysis and stops with the error message: “A prestressed analysis may not be performed when the previous analysis used NLGEOM,ON.”

For CMUT plates, which, in addition to bending stiffness, also are dominated by membrane stiffness, this is a major limitation. In such cases, the correct DC bias point

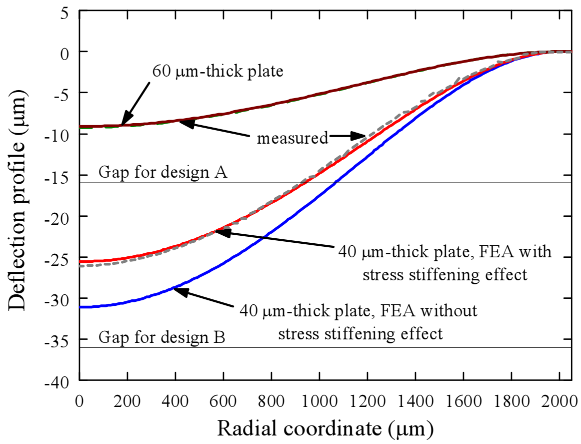

Fig. 2. Measured versus calculated plate deflection profiles for two designs A and B [7]. Design A has a deflection-to-thickness ratio of 65.2%, for B it is 15.2%.

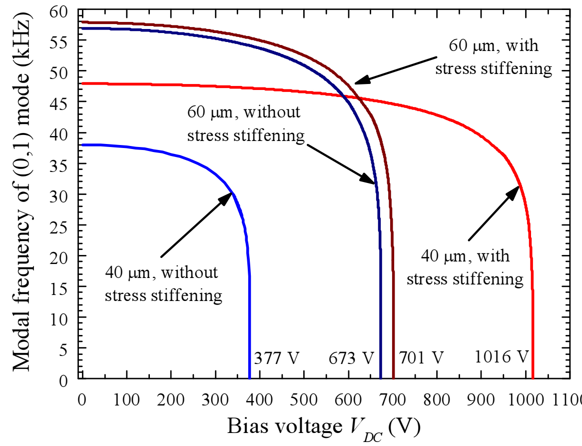

Fig. 4. Calculated modal frequency for mode (0,1) for design A and B as a function of the applied bias voltage V_{DC}.

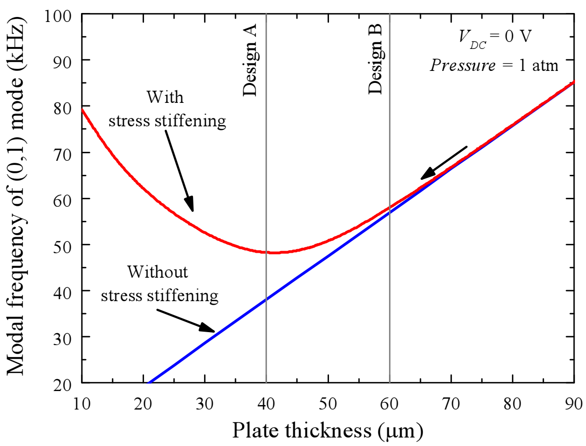

Fig. 3. Calculated modal frequency for mode (0,1) of a circular single crystal CMUT plate with a diameter of 4000 μm as a function of its thickness.

calculation (static deflection due to atmospheric pressure and applied DC bias voltage) in the prestressing step requires the nonlinearities to be turned on. If we assume the following AC excitation to be small enough to result in small AC displacement of the CMUT plate only, we can argue that the stress state of the CMUT plate can be assumed to be constant around that DC operation point. This allows us to use a linear but updated stiffness matrix for the calculation of the harmonic response. The stiffness matrix is updated in terms of the static stress state of the large deflected CMUT plate. Such a FEA that can perform a prestressed harmonic response calculation with a nonlinear static solution step is required for CMUT designs when stress-stiffening effects are present. It will help predict the frequency response and allow performance comparisons of design variations in terms of transmit and receive sensitivity without running a computationally intensive transient analysis.

In ANSYS we realized the prestressed harmonic response calculation with the nonlinear static solution step by using a prestressed mode superposition harmonic response analysis. In this type of analysis the sum of factored mode shapes (eigenvectors) obtained from a nonlinear prestressed modal analysis is used to calculate the harmonic response. The procedure is as follows. First, a nonlinear static deflection is calculated to prestress the structure. Then, a prestressed modal analysis is performed to obtain all mode shapes in the frequency range of interest. In the last step, the mode superposition harmonic solution is obtained by adding all mode shapes.

Such a prestressed mode superposition analysis cannot directly be applied to our problem statement of a CMUT plate moving in a medium, such as air or water, because of a limitation for the required prestressed modal analysis step. The solid fluid interaction will introduce damping to the system, i.e. non-symmetric element matrices in the FEM, which are not compatible with the only mode extraction method (QR damp mode extraction method [5]) that supports damping.

We circumvent this problem by using both FEMs shown in Fig. 1. The second FEM has the solid fluid interaction included [Fig. 1(b)] and we use the waveguide approach as described in [8].

Our procedure to calculate the prestressed harmonic response calculation with the nonlinear static solution step is as follows: First, we use the FEM from Fig. 1(b) to calculate the prestressed harmonic response of the CMUT cell with a linear static solution step. The goal of this calculation is to determine the constant damping

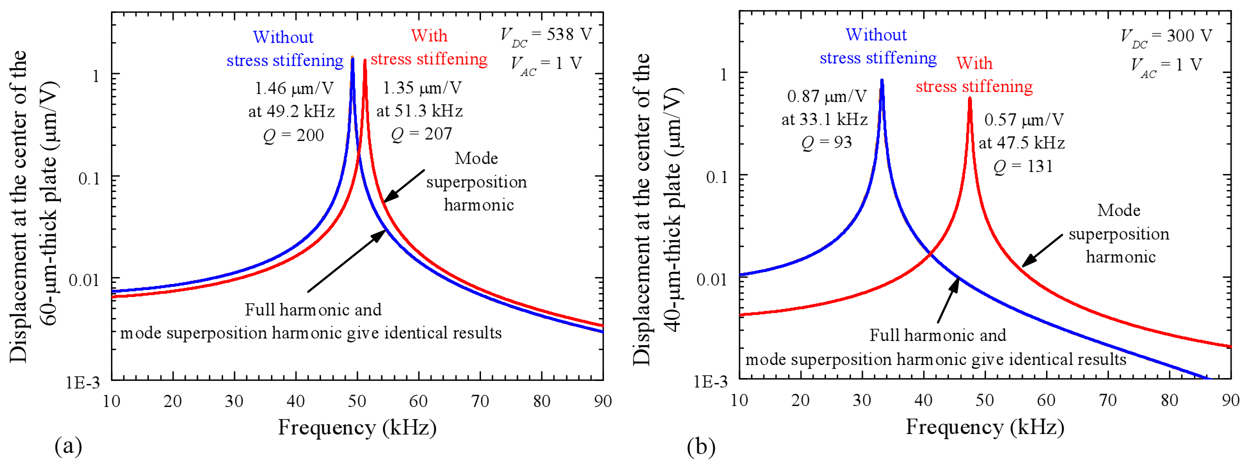

Fig. 5. Direct comparison for design B (left) and design A (right) of prestressed harmonic response analysis with linear static operation point calculation (blue) and prestressed mode superposition harmonic response analysis with nonlinear static operation point calculation (red).

ratio $\xi_i = \frac{1}{2Q_i}$, where Q_i is the quality factor at the resonance frequency ω_i. The next step is to run a prestressed modal analysis with a nonlinear static solution step to determine the resonance frequency ω_j of the CMUT plate for the case when stress stiffening effects are present. By neglecting mass damping effects, we can simply use the ratio $\frac{\omega_j}{\omega_i}$ as linear scaling factor to determine the corresponding constant damping ratio ξ_j [5], because for this case the damping ratio is proportional to the natural frequency of the system. Then we can use ξ_j in combination with the FEM from Fig. 1(a) to calculate the prestressed mode superposition harmonic response analysis with nonlinear static solution step.

IV. Results

As an example, we show the calculated center displacement for 1-V AC excitation for both designs (Fig. 5), biased at 80% of the pull-in voltage. Note that for the direct comparison (FEA with/without stress stiffening effects) we had to use the pull-in voltage for the linear FEA (Fig. 4), otherwise the bias voltage would have been beyond the pull-in voltage for the linear case for design A. In both cases, we verified our FEA by comparing the prestressed harmonic analysis to the prestressed mode superposition harmonic analysis for the linear cases. The curves overlap each other, as expected. For the nonlinear cases, the direct comparison to the prestressed modal analysis (Fig. 3) shows excellent agreement in terms of the resonance frequency. Furthermore, our calculated center frequencies (51.3 kHz for design B, and 47.5 kHz for design A) show good agreement to the measured center frequencies (55 kHz for design B, and 46 kHz for design A [7]).

V. Conclusion

Only a model that considers stress stiffening effects is useful for the design of CMUTs that operate in a more membrane dominated regime. The key is a prestressed harmonic response calculation with a nonlinear static solution step. Ignoring stress stiffening leads to large errors in predicted static deflection, pull-in voltage, resonance frequency, and sensitivity.

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References

[1] E. Ventsel and T. Krauthammer, "Thin plates and shells," 1st ed., New York: Marcel Dekker, Inc., 2001.

[2] E. S. Huang and S. D. Senturia, "Extending the travel range of analog-tuned electrostatic actuators," *Journal of Microelectromechanical Systems*, vol. 8, no. 4, pp. 497-505, 1999.

[3] T. von Karman, "The engineer grapples with non-linear problems," in Bull. *Amer. Math. Soc.*, 1940, pp. 615–683.

[4] S. Way, "Bending of circular plate with large deection," in Trans. *ASME*, 1934, pp. 627–636.

[5] ANSYS 11.0 Manual, Ansys Inc., Canonsburg, PA.

[6] M. Gyimesi and D. Ostergaard, "Electro-mechanical capacitor element for MEMs analysis in ANSYS," in Proc. *Modeling and Simulation of Microsystems*, 1999, pp. 270–273.

[7] I. O. Wygant, M. Kupnik, J. C. Windsor, W. M. Wright, G. Yaralioglu, M. S. Wochner, M. F. Hamilton, and B. T. Khuri-Yakub, "50-kHz capacitive micromachined ultrasonic transducers for generating highly directional sound with parametric arrays," in Proc. *IEEE Ultrason. Symp.*, 2007, pp. 519–522.

[8] A. Lohfink, P.-C. Eccardt, W. Benecke, and H. Meixner, "Derivation of a 1D CMUT model from FEM results for linear and nonlinear equivalent circuit simulation," in Proc. *IEEE Ultrason. Symp.*, 2003, pp. 465–468.