

Stimulated Emission from a Single Excited Atom in a Waveguide

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We study stimulated emission from an excited two-level atom coupled to a waveguide containing an incident single-photon pulse. We show that the strong photon correlation, as induced by the atom, plays a very important role in stimulated emission. Additionally, the temporal duration of the incident photon pulse is shown to have a marked effect on stimulated emission and atomic lifetime.

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Introduction.—Stimulated emission, first formulated by Einstein in 1917 [1], is the fundamental physical mechanism underlying the operation of lasers [2] and optical amplifiers [3,4], both of which are of paramount importance in modern technology. In recent years, stimulated emission has been studied in a variety of novel systems, including a surface plasmon nanosystem [5], a single molecule transistor [6], and superconducting transmission lines [7]. Stimulated emission also plays a crucial role in the quantum cloning of photons [3,8–10].

In textbooks [11], stimulated emission is usually described by having a photon interact with an atom in the excited state. As a result of such interaction, the atom emits a second photon that is “identical” to the incident photon. As a concrete experimental example, in Refs. [4,12], photons of a given polarization were sent into a parametric amplifier, and stimulated emission manifested in an enhanced probability of the outgoing photons having the same polarization. In Refs. [4,12], such enhancement was attributed entirely to constructive interference due to photon indistinguishability.

In this Letter, we consider a scenario of stimulated emission that is arguably closest to the textbook description [11]. We consider an atom coupled to a waveguide and study the interaction of the atom with a single incident photon, when the atom is initially in the excited state, as shown in Fig. 1(a). The atom-waveguide system is of current experimental and theoretical interest both for nanophotonics in the optical regime [13–19] and in the studies of superconducting transmission lines in the microwave regime [20,21]. In an atom-waveguide system, the ability to amplify few-photon quantum states is important and has been recently demonstrated experimentally [7]. Thus, the scenario that we consider is of experimental interest.

We observe that the Hamiltonian describing the waveguide-atom system also describes, in 3D, the interaction of a light beam with an atom, when the beam is designed to mode-match the atom’s radiation pattern. This observation was emphasized and exploited in a number of recent papers [6,22–24]. Thus, our result should be relevant to a wide class of recent 3D experiments as well.

In recent years, there has been much advancement in the capacity to deterministically generate single photons [25,26] and to control the shape of the single-photon pulse [27,28]. In both waveguide and free space, a single photon, by necessity, must exist as a pulse [29]. Therefore, our study of stimulated emission at the single-photon level reveals important dynamic characteristics of stimulated emission. For instance, we will show that there exists an optimal spectral bandwidth of the single-photon pulse that results in the largest enhancement of forward scattered light. This is fundamentally different from the conventional study of stimulated emission involving a single-mode continuous radiation field. Finally, our results show that, in the waveguide-atom system, emission enhancement in the stimulated process cannot be entirely attributed to photon indistinguishability but, instead, has a substantial connection to the strong photon correlation induced by the two-level atom.

Model system and the incident single-photon pulse.—We consider the atom-waveguide system shown in Fig. 1. For a waveguide with a linear dispersion relation, the real-space Hamiltonian of the system in the rotating-wave approximation is [13]

$$\begin{aligned} \hat{H}/\hbar = & -iv_g \int dx c_R^\dagger(x) \frac{\partial}{\partial x} c_R(x) + iv_g \int dx c_L^\dagger(x) \frac{\partial}{\partial x} c_L(x) \\ & + \frac{\Omega}{2} \sigma_z + \frac{V}{\sqrt{2}} \int dx \delta(x) [c_R^\dagger(x) \sigma_- + c_L^\dagger(x) \sigma_- \\ & + \sigma_+ c_R(x) + \sigma_+ c_L(x)], \end{aligned} \quad (1)$$

where x is the spatial coordinate along the waveguide’s symmetry axis. $c_R^\dagger(x)$ [$c_L^\dagger(x)$] creates a right- [left-] moving photon, $c_R(x)$ [$c_L(x)$] annihilates a right- [left-] moving photon, and σ_+ (σ_-) is a raising (lowering) operator of the two-level atom. The atom-waveguide coupling strength is V , corresponding to an atom spontaneous emission rate $\Gamma \equiv V^2/v_g$. The transition frequency of the atom is Ω . v_g is the waveguide group velocity, which we set as $v_g = 1$ from here on out.

At $t = 0$, we assume that the atom is in the excited state. At the same time, a right-going single-photon pulse starts

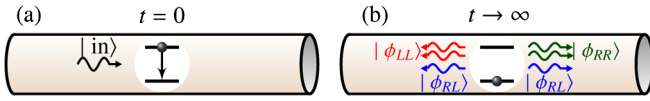


FIG. 1 (color online). Schematic of the atom-waveguide system studied here. (a) The initial state at $t = 0$, consisting of an incident single-photon pulse and an atom in the excited state. (b) At $t \rightarrow \infty$ the atom is in the ground state, and the photon field is in a superposition of two-photon states $|\phi_{RR}\rangle$, $|\phi_{LL}\rangle$, and $|\phi_{RL}\rangle$ (green, red, and blue, respectively).

to interact with the atom. The in state of the system is therefore

$$|\text{in}\rangle = \int dx \psi(x) c_R^\dagger(x) \sigma_+ |0\rangle. \quad (2)$$

At $t \rightarrow \infty$ the atom has decayed to the ground state, and the out state contains only two-photon states, as given by [Fig. 1(b)]

$$|\text{out}\rangle = |\phi_{RR}\rangle + |\phi_{LL}\rangle + |\phi_{RL}\rangle,$$

where e.g., $|\phi_{RR}\rangle = \int dx_1 dx_2 \phi_{RR}(x_1, x_2) (1/\sqrt{2}) c_R^\dagger(x_1) \times c_R^\dagger(x_2) |0\rangle$ denotes an outgoing two-photon state with two right-moving photons and $P_{RR} \equiv \langle \phi_{RR} | \phi_{RR} \rangle$ is its respective probability. Here, we use ϕ to denote all outgoing states and use subscripts to distinguish these various states. Since the incident photon is right-moving, to study stimulated emission we will be particularly interested in comparing behaviors of P_{RR} , where both outgoing photons are right-moving, to other outcomes as described by P_{RL} and P_{LL} .

For computation in this Letter, we choose an input wave function

$$\psi(x) = i\sqrt{\alpha\Gamma} e^{i(\Omega - i\alpha\Gamma/2)x} \theta(-x). \quad (3)$$

For $\alpha = 1$, $\psi(x) = \phi_{sp}(x)$, where $\phi_{sp}(x)$ is the wave function of a photon spontaneously emitted by the excited atom. Variation of α allows us to probe various time and frequency scales, as well as the connection between spontaneous and stimulated emission in radiation dynamics.

The out state.—To calculate the out state for an arbitrary initial state as described by Eq. (2), it is useful to define $c_e^\dagger(x) \equiv (1/\sqrt{2})[c_R^\dagger(x) + c_L^\dagger(-x)]$ and $c_o^\dagger(x) \equiv (1/\sqrt{2}) \times [c_R^\dagger(x) - c_L^\dagger(-x)]$. With these definitions, the Hilbert space is separated into even and odd subspaces, each described by a Hamiltonian:

$$\begin{aligned} \hat{H}_e/\hbar &= -i \int dx c_e^\dagger(x) \frac{\partial}{\partial x} c_e(x) \\ &\quad + V \int dx \delta(x) [c_e^\dagger(x) \sigma_- + \sigma_+ c_e(x)] + \frac{\Omega}{2} \sigma_z, \\ \hat{H}_o/\hbar &= -i \int dx c_o^\dagger(x) \frac{\partial}{\partial x} c_o(x). \end{aligned}$$

We note that $H = H_e + H_o$ and $[H_e, H_o] = 0$. Similarly, we decompose the in state:

$$\begin{aligned} |\text{in}\rangle &= |\text{in}\rangle_{eo} + |\text{in}\rangle_{ee} \\ &= \frac{1}{\sqrt{2}} \iint dx_1 dx_2 \psi(x) c_o^\dagger(x) \sigma_+ |0\rangle \\ &\quad + \frac{1}{\sqrt{2}} \iint dx_1 dx_2 \psi(x) c_e^\dagger(x) \sigma_+ |0\rangle. \end{aligned}$$

We note that, since $[\sigma_+, \hat{H}_o] = 0$, the state $\sigma_+ |0\rangle$ is contained in the even subspace.

We can now calculate the out state by evaluating $\lim_{t \rightarrow \infty} e^{-iHt} |\text{in}\rangle$. From this point on, with the variable t we always assume that we are in the $t \rightarrow \infty$ limit. We consider $|\text{in}\rangle_{eo}$ and $|\text{in}\rangle_{ee}$ separately:

$$\begin{aligned} e^{-iHt} |\text{in}\rangle_{eo} &= \frac{1}{\sqrt{2}} \iint dx_1 dx_2 \phi_{sp}(x_1 - t) \psi(x_2 - t) |x_1, x_2\rangle_{eo} \\ &\equiv \frac{1}{\sqrt{2}} \iint dx_1 dx_2 \phi_{eo}(x_c - t, x_d) |x_1, x_2\rangle_{eo}, \quad (4) \end{aligned}$$

where $x_c \equiv \frac{x_1 + x_2}{2}$, $x_d \equiv x_1 - x_2$, and $|x_1, x_2\rangle_{eo} = c_e^\dagger(x_1) \times c_o^\dagger(x_2) |0\rangle$. Similarly,

$$\begin{aligned} e^{-iHt} |\text{in}\rangle_{ee} &= \frac{1}{\sqrt{2}} \iint dk_1 dk_2 \int dx e^{-iH_e t} |k_1 k_2^+\rangle \\ &\quad \times \langle k_2^+ k_1 | \psi(x) c_e^\dagger(x) a^\dagger |0\rangle \\ &= \iint dx_1 dx_2 \phi_{ee}(x_c - t, x_d) |x_1, x_2\rangle_{ee}, \quad (5) \end{aligned}$$

where $|k_1 k_2^+\rangle$ is the two-excitation interacting eigenstate of \hat{H}_e in the two-excitation manifold, as determined in Ref. [15], and $|x_1, x_2\rangle_{ee} = (1/\sqrt{2}) c_e^\dagger(x_1) c_e^\dagger(x_2) |0\rangle$.

For the specific initial state as defined in Eq. (2), $\phi_{ee}(x_c - t, x_d)$ and $\phi_{eo}(x_c - t, x_d)$ are calculated in Ref. [30], and as a result we have

$$\begin{aligned} P_{RR,LL}(\alpha) &= \frac{1}{4} \left[1 + \frac{2\alpha}{(1+\alpha)^2} \pm \frac{(1+\alpha)^3 - 8\alpha}{(\alpha-1)(1+\alpha)^2} \right], \\ P_{RL}(\alpha) &= \frac{1}{2} \left[1 - \frac{2\alpha}{(1+\alpha)^2} \right], \end{aligned}$$

which we plot in Fig. 2(a).

Analysis of stimulated emission.—When $\alpha \ll 1$, $P_{RR}(\alpha) \rightarrow 0$, and $P_{RL}(\alpha)$, $P_{LL}(\alpha) \rightarrow 0.5$. In this limit, the temporal duration of the incident photon pulse is much longer than the spontaneous emission lifetime. Thus, the initial excitation of the atom decays spontaneously, and the pulse is completely reflected by an atom largely in the ground state [13]. When $\alpha \gg 1$, $P_{RR}(\alpha)$, $P_{RL}(\alpha) \rightarrow 0.5$, and $P_{LL}(\alpha) \rightarrow 0$. In this case, the photon pulse is extremely short; there is therefore no atom-photon interaction. As a result, the incident photon pulse is fully transmitted, and the atom decays spontaneously after the pulse passes through. This limit of $\alpha \gg 1$ corresponds to a scenario where the transmission of the incident photon and the decay of the atom occur independently. In this “independent” scenario, $P_{RR} = 0.5$. Therefore, $P_{RR} > 0.5$

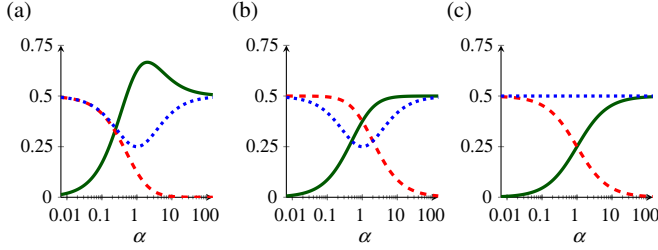


FIG. 2 (color online). (a) $\{P_{RR}, P_{LL}, P_{RL}\}$ in solid green, dashed red, and dotted blue lines for a two-level atom. (b) $\{P_{RR}^{\text{cavity}}, P_{LL}^{\text{cavity}}, P_{RL}^{\text{cavity}}\}$ in solid green, dashed red, and dotted blue lines. (c) $\{P_{RR}^{\text{classical}}, P_{LL}^{\text{classical}}, P_{RL}^{\text{classical}}\}$ in solid green, dashed red, and dotted blue lines.

indicates an enhanced probability that both photons go to the right and is therefore a direct indication of stimulated emission.

In Fig. 2(a), we indeed observe $P_{RR} > 0.5$ in the intermediate region of α . In fact, P_{RR} maximizes to $2/3$ at $\alpha = 2$ —when the temporal decay rate of the incident photon pulse is twice the atom’s spontaneous emission rate. Moreover, the enhancement of P_{RR} can be observed with other incident pulse shapes. For example, numerically, we have found that an incident photon with half-Gaussian wave function $\psi(x) = \frac{(2\alpha^2\Gamma^2)^{1/4}}{\pi} e^{-(\alpha\Gamma)^2 x^2/4} \theta(-x)$ attains a similar maximum of $P_{RR} \approx 0.65$ for $\alpha \approx 1.6$.

To better understand the stimulated emission behavior in Fig. 2(a), we consider the general expression of the three probabilities P_{RR} , P_{LL} , and P_{RL} , for an arbitrary incident wave function $\psi(x)$. From Eqs. (4) and (5), one can show that

$$\begin{aligned} P_{RL} &= 0.5 - 0.25 \int dx_1 dx_2 \phi_{eo}^*(x_c - t, -x_d) \phi_{eo}(x_c - t, x_d) \\ &= 0.5 - 0.25 \left[\int dx \psi^*(x) \phi_{sp}(x) \right]^2, \end{aligned}$$

where to obtain the first line, we note that the cross terms between $\phi_{eo}(x_c - t, x_d)$ and $\phi_{ee}(x_c - t, x_d)$ happen to cancel, and in obtaining the second line we have used Eq. (4). We define a *photon indistinguishability factor*

$$F \equiv \left[\int dx \psi^*(x) \phi_{sp}(x) \right]^2, \quad (6)$$

which measures the indistinguishability between the incident single-photon pulse and the single-photon pulse from spontaneous emission of an excited atom. When $\alpha = 1$, F maximizes at 1. Consequently, $P_{RL}(\alpha)$ is minimal. Moreover, since $0 \leq F \leq 1$, we have $0.25 \leq P_{RL} \leq 0.5$, establishing an upper bound of $3/4$ for P_{RR} .

Similarly, from Eqs. (4) and (5), we can derive

$$\begin{aligned} P_{RR,LL} &= \frac{1}{4} + \frac{1}{8}F \pm \frac{1}{8} \iint dx_1 dx_2 \{ [\phi_{eo}^*(x_c - t, x_d) \\ &\quad + \phi_{eo}^*(x_c - t, -x_d)] \phi_{ee}(x_c - t, x_d) \\ &\quad + 2\phi_{ee}^*(x_c - t, x_d) \phi_{eo}(x_c - t, x_d) \}. \end{aligned} \quad (7)$$

In Eq. (7), we see that P_{RR} and P_{LL} (with + and – signs, respectively) are influenced in the same way by the photon indistinguishability factor F . Thus, the most prominent feature of stimulated emission in terms of $P_{RR} > 0.5$ cannot be attributed to photon indistinguishability.

The last term in Eq. (7) contains overlap integrals between the out state $\phi_{eo}(x_c - t, x_d)$ in the eo subspace—which is a product of the incident wave function $\psi(x - t)$ and a spontaneously emitted photon $\phi_{sp}(x - t)$ —and the out state $\phi_{ee}(x_c - t, x_d)$ in the ee subspace. Since a single two-level atom cannot simultaneously absorb two photons, the two-photon wave function $\phi_{ee}(x_c - t, x_d)$ in the ee subspace exhibits complex entangled structures in spatial, spectral, and temporal domains [15]. It is known that the two-photon wave function in the ee subspace has a direct connection to the Bethe-ansatz wave functions that have been commonly used to describe strongly correlated quantum systems [15]. Hence, here we refer to the effect arising from the complex structure in $\phi_{ee}(x_c - t, x_d)$ as a strong photon correlation effect. The form of Eq. (7) therefore directly points to the role that the strong correlation induced by the atom plays in stimulated emission in addition to the effect of photon indistinguishability.

The role of photon indistinguishability in stimulated emission is a topic of current interest. In Ref. [4], Sun *et al.* studied this question by comparing two scenarios. In the first scenario, an N -photon state is sent into a parametric down-converter, and the amplification results in the creation of an $N + 1$ photon state in the output. In the second scenario, the amplification process is instead emulated by combining the N -photon state with an extra photon at a beam splitter. The authors then showed experimentally that these two scenarios produce the same photon statistics. Thus, the crucial aspect of stimulated emission in their system is the indistinguishability between the photons.

Motivated by their work, here we also consider an alternative scenario where we replace the atom with a side-coupled cavity. We emulate the stimulated emission process by injecting a single-photon pulse into the waveguide while there is a photon in the cavity. Mathematically, this system as well as the initial state can be described by replacing $\sigma_- (\sigma_+)$ with $a (a^\dagger)$ and σ_z with $2a^\dagger a - 1$ in Eqs. (1) and (2). Here $a (a^\dagger)$ is the annihilation (creation) operator of a cavity photon, and $[a, a^\dagger] = 1$. Also, we note that the waveguide-atom and waveguide-cavity systems have identical behaviors in the single-excitation manifold.

In the absence of the injected single photon in the waveguide, the photon in the cavity decays spontaneously. Thus, photon indistinguishability in this case refers to how close the photon pulse matches the spontaneously decayed photon pulse from the cavity, as characterized by the same F factor in Eq. (6). For the waveguide-cavity system, the output probabilities for various outcomes are

$$P_{RR,LL}^{\text{cavity}} = \frac{1}{4} \left[1 + \frac{2\alpha}{(1+\alpha)^2} \pm \frac{\alpha-1}{\alpha+1} \right],$$

$$P_{RL}^{\text{cavity}}(\alpha) = P_{RL}(\alpha),$$

which we plot in Fig. 2(b).

We compare the waveguide-cavity system to a classical system consisting of a waveguide and an ancilla. We consider an incident single-photon pulse with a probability spectrum $|\psi(k)|^2 = \frac{1}{2\pi} \frac{\alpha\Gamma}{(k-\Omega)^2 + (\alpha\Gamma/2)^2}$, where $\psi(k)$ is the momentum-space representation of $\psi(x)$ in Eq. (3). The ancilla emits a classical photon with probability $1/2$ in each direction and, moreover, reflects an incident photon with a probability spectrum $|r_k|^2 = \frac{(\Gamma/2)^2}{(k-\Omega)^2 + (\Gamma/2)^2}$. The ancilla therefore has the same intensity response function to a single photon as either an atom or a cavity. For this classical case we have $P_{RR}^{\text{classical}}(\alpha) = \frac{1}{2} \frac{\alpha}{1+\alpha}$, $P_{LL}^{\text{classical}}(\alpha) = \frac{1}{2} \frac{1}{1+\alpha}$, and $P_{RL}^{\text{classical}}(\alpha) = 1/2$, as plotted in Fig. 2(c). We see that the waveguide-cavity system has an enhanced probability P_{RR}^{cavity} compared to the classical system $P_{RR}^{\text{classical}}$. Photon indistinguishability, which is unique to the quantum system, certainly plays a role here.

However, $P_{RR}^{\text{cavity}}(\alpha)$ is always smaller than 0.5. Thus, the excess probability of having both outgoing photons propagating to the right, i.e., the fact that $P_{RR}(\alpha) > 0.5$, cannot be achieved in the waveguide-cavity system and, therefore, cannot be attributed to photon indistinguishability. It follows that the key difference between the atom and the parametric down-converter in Ref. [4] is the photon correlation induced by the atom due to its inability to absorb more than one photon at a time.

In the waveguide-atom system, the effect of stimulated emission is also manifested in the lifetime of the atomic excitation. We define such a lifetime as

$$\tau_a \equiv \frac{1}{2} \int_0^\infty dt \langle \sigma_z + 1 \rangle.$$

With this definition, a spontaneously decaying atom yields $\tau_a = 1/\Gamma$, as expected. In Fig. 3, we plot τ_a as a function of α , which one may recall from Eq. (3) characterizes the temporal decay rate of the incident photon pulse. In the limit $\alpha \gg 1$, the atom lifetime is identical to the spontaneous emission lifetime $1/\Gamma$. In this regime, the incident photon's duration is extremely short, which has no effect on the atom's lifetime. In the limit $\alpha \ll 1$, $\tau_a = 3/\Gamma$. In this regime, the atom decays spontaneously, and the photon interacts with a ground-state atom, exciting the atom, which then decays again. When $\alpha = 3$, the lifetime of

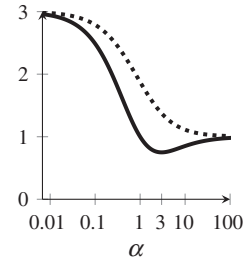


FIG. 3. $\Gamma\tau_a$ ($\Gamma\tau_c$) vs photon decay parameter α for an atom (cavity) in the solid (dashed) curve.

the atom is minimal and equals $\tau_a = 0.75/\Gamma$. This corresponds to the condition for a maximum in the photon correlation term in Eq. (7), demonstrating that stimulated emission in this system is associated with a shortening of the atom lifetime. In contrast, the lifetime of a cavity $\tau_c \equiv \int_0^\infty dt \langle a^\dagger a \rangle$ is always longer than $1/\Gamma$, which it approaches from above as $\alpha \rightarrow \infty$.

It is interesting to note that the maximum probability $P_{RR} = 2/3$, achieved at $\alpha = 2$, is equal to the maximum probability of cloning a quantum state in a universal quantum cloning machine [3,31]. However, the system here is not a universal quantum cloning machine. If we inject a single photon in the even subspace, both of the outgoing photons will be in the even subspace. Moreover, the initial single-photon wave function is not cloned in the outgoing two-photon wave function. Despite this fact, we can consider the stimulated emission process studied here by using the language of photon cloning, if one is interested only in the outgoing direction of the photons and not the detailed photon wave function. Then, the photon cloning fidelity [3] maximizes at $\alpha = 3$, yielding a fidelity of $\eta = P_{RR} + 1/2P_{RL} = 0.8125$. The photon-number amplification factor is maximal at $\alpha = 2$, with a value of $2P_{RR} = 4/3$.

In addition to the coupling between the atom and waveguide, the atom can also couple into nonguided modes, which are considered a loss mechanism. In this case we define the β factor as $\beta \equiv \frac{\Gamma}{\Gamma + \gamma_{\text{ng}}}$, where γ_{ng} is the coupling rate into nonguided modes. Large β factors of 0.9 and higher have been experimentally measured with the same geometry studied here [32–34]. In our analysis, a less-than-unity β factor can be accounted for by making the replacement $\Omega \rightarrow \Omega - i\gamma_{\text{ng}}/2$ in the loss-free results. For $\beta = 0.9$, the results are modified only slightly, with the maximum of P_{RR} now occurring at $\alpha \approx 2.04$, resulting in $P_{RR} \approx 0.63$.

In summary, we have provided a fully quantum-mechanical theoretical study of stimulated emission at its most basic level, involving an incident single-photon pulse and an atom in its excited state. The study provides insights into this fundamental process of nature, including the role of photon pulse dynamics, and the photon correlation induced by the atom due to its inability to absorb more

than one photon at a time. Such a study may prove useful as one seeks to exploit such systems for manipulation of quantum states.

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