Simple geometric criterion to predict the existence of surface modes in air-core photonic-bandgap fibers

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Abstract: We propose a simple geometric criterion based on the size of the core relative to the photonic crystal to quickly determine whether an air-core photonic-bandgap fiber with a given geometry supports surface modes. Comparison to computer simulations show that when applied to fibers with a triangular-pattern cladding and a circular air core, this criterion accurately predicts the existence of a finite number of discrete ranges of core radii that support no surface modes. This valuable tool obviates the need for time-consuming and costly simulations, and it can be easily applied to fibers with an arbitrary photonic-crystal structure and core profile.

OCIS codes: (060.2280) Fiber design and fabrication; (060.0060) Fiber optics and optical communications

References and Links

1. D. C. Allan, N. F. Borrelli, M. T. Gallagher, D. Müller, C. M. Smith, N. Venkataraman, J. A. West, Peihong Zhang, and K. W. Koch, "Surface modes and loss in air-core photonic band-gap fibers," *Proc. of SPIE* Vol. 5000, pp. 161-174 (2003).
2. D. Müller, D. C. Allan, N. F. Borrelli, K. T. Gahagan, M. T. Gallagher, C. M. Smith, N. Venkataraman, and K. W. Koch, "Measurement of photonic band-gap fiber transmission from 1.0 to 3.0 µm and impact of surface mode coupling," *Conf. on Lasers and Electro-Optics*, Baltimore, USA, June 2003, paper QtuL2, (2003).
3. H. K. Kim, J. Shin, S. Fan, M. J. F. Digonnet, and G. S. Kino, "Designing air-core photonic-bandgap fibers free of surface modes," to appear in IEEE J. of Quantum Electronics (May 2004).
4. K. Saito, N. A. Mortensen, and M. Koshiba, "Air-core photonic band-gap fibers: the impact of surface modes," Opt. Express Vol. 12, No. 3, 394-400 (Feb. 2004), http://www.opticsexpress.org/abstract.cfm?URI=OPEX-12-3-394.
5. C. M .Smith, N. Venkataraman, M. T. Gallagher, D. Muller, J. A. West, N. F. Borreill, D. C. Allan, K. W. Koch, "Low-loss hollow-core silica/air photonic bandgap fibre," Nature Vol. 424, No. 6949, pp. 657-659 (2003).
6. F. Ramos-Mendieta and P. Halevi, "Surface electromagnetic waves in two-dimensional photonic crystals: effect of position of the surface plane," Phy. Rev. B Vol. 59, p. 15112 (1999).
7. J. D. Joannopoulos, R. D. Meade, and J. N. Winn, *Photonic Crystals: Molding the flow of light,* (Princeton U. Press, Princeton, N.J, 1995), pp. 73-76.
8. A. Yariv, and P. Yeh, *Optical Waves in Crystals: Propagation and Control of Laser Radiation*, (John Wiley & Sons, New York, 1984), p. 210.
9. S. G. Johnson, and J. D. Joannopoulos, "Block-iterative frequency-domain methods for Maxwell's equations in planewave basis," Opt. Expr. Vol. 8, No. 3, pp. 173-190 (2001), http://www.opticsexpress.org/abstract. cfm?URI=OPEX-8-3-173.

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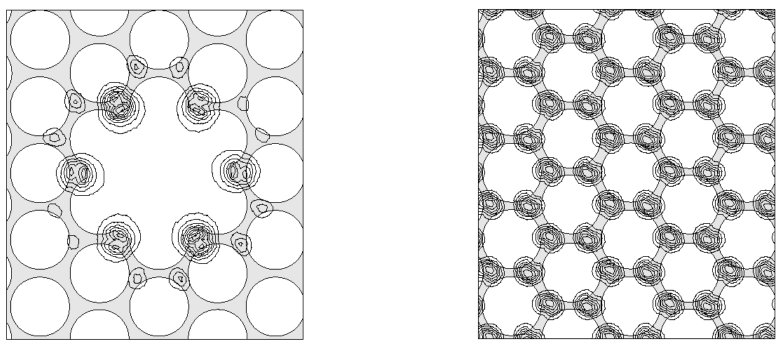
1. Introduction

In recent months there has been mounting evidence that surface modes impose serious limitations in air-core photonic-bandgap fibers (PBFs) [1-4]. Surface modes, which do not exist in conventional fibers, are defect modes that form at the boundary between the air core and the photonic-crystal cladding. A typical surface mode is displayed in Fig. 1(a) for the purpose of illustration. This mode was calculated for a triangular-pattern air-core PBF made of silica, with a core radius $R = 1.15\Lambda$ and air holes of radius $\rho = 0.47\Lambda$, where Λ is the period of the photonic crystal. The supercell size used in these calculations was 10x10 and a grid resolution of $\Lambda/16$. Unless suitably designed, a fiber will support many such surface modes [3]. The propagation constants of surface modes often fall close to or can even be equal to the propagation constant of the fundamental core mode [2,3]. The fundamental mode can thus easily be coupled to surface modes, for example by random perturbations in the fiber cross-section. Because surface modes are inherently lossy due to their high energy density in the dielectric, this coupling represents a loss mechanism. Recent findings indeed demonstrate conclusively that surface modes are at the origin of the reduced transmission bandwidth in Corning's 13-dB/km air-core PBF [5]. They are also likely responsible for the remaining loss of air-core PBFs.



(a) (b)

Fig. 1. (a) Example of a surface mode calculated for a triangular-pattern PBF with an air hole radius $\rho = 0.47\Lambda$ and a core radius of 1.15Λ, and (b) the highest frequency bulk mode of the same fiber in the absence of core. Both were calculated at $k_z\Lambda/2\pi = 1.7$.

Direct detection of surface modes is experimentally difficult. At present their existence can only be predicted by time-consuming and costly numerical simulations on supercomputers. For example, generating the surface mode of Fig. 1(a) (together with the other modes of that fiber) took about 6 hours using 16 parallel processors [3]. Given the significance of surface modes, it was both timely and important to devise expedient modeling alternatives. In earlier work we showed that the presence of surface modes can be predicted on the basis of whether the air-core surface intersects the high intensity lobes of the highest frequency bulk mode of the lower band of the photonic crystal [3]. In this paper, we present a much simpler, more intuitive existence criterion that requires no advanced simulations at all. We show that surface modes are created when the surface of the core intersects one or more of the dielectric corners of the photonic crystal. Based on this observation, we propose a fast and simple geometric criterion to evaluate whether a particular fiber design supports surface modes. We apply this criterion to triangular-pattern PBFs with a circular air core and find that in spite of its simplicity, this approximate model yields quantitative predictions in remarkable agreement with computer simulations.

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2. Physical origin of surface modes

Surface modes can occur when an infinite photonic crystal is abruptly terminated, as happens for example at the edges of a crystal of finite dimensions. Terminations introduce a new set of boundary conditions, which result in the creation of surface modes that satisfy these conditions and are localized at the termination [6]. In a photonic crystal, the existence of surface modes depends strongly on the location of the termination [6-8]. For example, in photonic crystals made of dielectric rods in air, surface modes are induced only when the termination cuts through rods. A termination that cuts only through air is too weak to induce surface modes [7].

Similarly, in a PBF the core acts as a defect that perturbs the crystal and may introduce surface modes at the core's edge. Whether surface modes appear, and how many appear, depends on how the photonic crystal is terminated, which determines the magnitude of the perturbation introduced by the defect. In the absence of core, a PBF carries only bulk modes. An example of bulk mode is illustrated in Fig. 1(b). This mode was calculated for the same triangular-pattern air-core PBF as in Fig. 1(a). Contour lines represent equal intensity lines. This particular bulk mode consists in a series of narrow intensity lobes centered on each and every one of the thicker dielectric corners of the photonic crystal. Here we define corners as the thicker portions of the dielectric photonic crystal separated by three adjacent holes (see Fig. 2(a)). The thinner portions of dielectric that connect corners, i.e., the portions separated by any two adjacent holes, are referred to as membranes (see Fig. 2(a)). Other bulk modes have different lobe distributions, e. g. with all lobes centered on membranes. When an air core is introduced, the core locally replaces dielectric material with air. The portions of the core surface that cut through the cladding air holes (see Fig. 2(a)) replace air by air and thus, just like in the case of a planar photonic crystal [7], they do not induce much of a perturbation. On the other hand, the portions of core that cut through dielectric regions of the crystal (see Fig. 2(a)), which replace dielectric by air, perturb the bulk modes strongly enough to potentially induce surface modes. Since a core of any size and shape always cuts through dielectric material, this perturbation is always present. The sign of the perturbation is such that in the ω-k diagram, the bulk modes all shift *up* in frequency from their respective unperturbed position. For a silica/air PBF, the perturbation is comparatively weak, this shift is thus small and almost all perturbed bulk modes remain in a bulk-mode band, i.e., they do not induce surface modes. The exception is modes from the highest frequency bulk-mode band of the lower band (HFBM in short). Because they are located just below the bandgap in the ω-k diagram, the perturbation moves them into the bandgap as surface modes [7]. Surface modes can always be written as an expansion of bulk modes. For the weak perturbation considered here, it can be shown that the main term in this expansion is the HFBM, as expected given the origin of these surface modes. The HFBM is the mode shown in Fig. 1(b). The fact that its lobes are all centered on corners of the crystal has two important consequences. First, because surface modes are induced by a perturbation of this bulk mode, their lobes are also centered on corners (see fig. 1(a), for example). Second, for the HFBM to be perturbed strongly enough to yield surface modes, the perturbation must occur in regions of the photonic crystal that carry a sizable HFBM intensity, i.e., corners of the photonic crystal. In other words, for a surface mode to be created the air core must intersect corners of the photonic crystal. This is true, for example, for the core of radius R_1 shown in Fig. 2(a), but the core of Fig. 2(b) (radius R_2) does not support surface modes.

In earlier work, we established that surface modes are indeed strongly correlated with the magnitude of the perturbation introduced by the air core on the HFBM [3]. If the core surface intersects lobes of the HFBM in the dielectric, the perturbation is large and surface modes are induced. The number of surface modes is then proportional to the highest intensity intersected by the core in the dielectric. Conversely, if the core surface does not intersect any of the lobes of this bulk mode, no surface modes are created. By comparison to exact simulations, we

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showed that this bulk-mode-based criterion predicts the presence or absence of surface modes fairly accurately [3]. Note that although some of the solid edges at the core boundary have abrupt corners (see Fig. 2), we do believe that the effects of such sharp corners on the existence of surface modes are minimal. The best proof is that when the core cuts through the membranes of the photonic crystal, sharp corners are present yet no surface mode is induced.

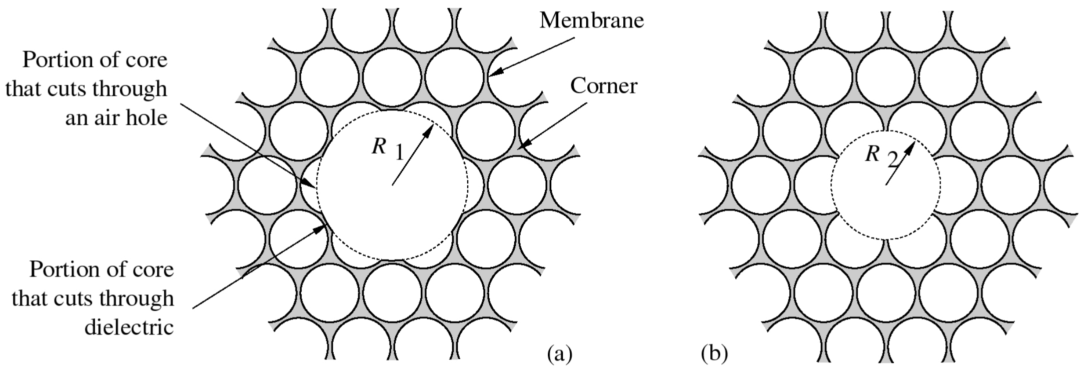

Fig. 2. Example of a circular core (a) that intersects corners of the photonic-crystal cladding (surface modes expected), and (b) that intersects only membranes of the photonic crystal (no surface modes expected).

3. Geometric criterion

In the present work we simplify this criterion by recognizing that the intensity lobes of the HFBM being nearly azimuthally symmetric (see Fig. 1(b)), the portion of each lobe confined in a dielectric corner can be approximated by the circle inscribed in this corner. This inscribed circle is illustrated as a black circle in Fig. 3. Its radius is related to the crystal's parameters by $a = \Lambda/\sqrt{3} - \rho$. Thus we approximate the portions of the HFBM confined to the dielectric by a two-dimensional array of circles centered on all the photonic-crystal corners. This array is illustrated by the small black circles in Fig. 4, plotted for a triangular pattern and $\rho = 0.47\Lambda$. This approximation makes it possible to formulate a new, simpler existence criterion for surface modes, namely that surface modes are predicted to exist when and only when the core surface intersects one or more of these circles. Of course it should be kept in mind that many other kinds of perturbations can induce surface modes in a photonic crystal, so that the above condition for the absence of surface modes is necessary but not sufficient.

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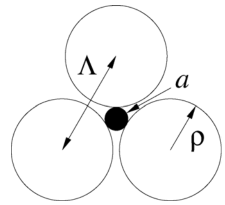

Fig. 3. Schematic of the rod (small black circle) inscribed within a corner of a photonic-crystal cladding (larger open circles), drawn for $\rho = 0.47\Lambda$.

Interestingly, the same geometric criterion can be derived with coupled-mode theory. In light of the symmetry of the lower-band bulk modes, each corner can be approximated by a dielectric rod inscribed in this corner, and which extends the length of the PBF. Each isolated rod is surrounded by air and constitutes a dielectric waveguide. This waveguide carries a fundamental mode with strong fields in the rod that decay evanescently into the air, so it looks much like the individual lobes of the HFBM (see Fig. 1(b)). In the periodic array of rods of Fig. 4, the waveguide modes of individual rods are weakly coupled to each other due to the proximity of neighboring rods and form the bulk modes. The HFBM is just one particular superposition of individual waveguide modes. If an air core that cuts into one or more rods is introduced, the removal of dielectric perturbs the waveguide modes in the opposite direction to that forming bulk modes. The waveguide modes of the ring of perturbed rods located at the core surface are then coupled to each other and form a surface mode. The latter is supported by the ring of rods and has fields that fall off outside each rod, as evidenced in the exemplary surface mode of Fig. 1(a). If the core cuts only through membranes instead of corners, the rods are unperturbed, they couple to each other much as they did without the core, and no surface mode is formed. In this description, surface modes exist if and only if the core surface intersects rods. This is of course the same criterion as derived by approximating the HFBM lobes by inscribed circles.

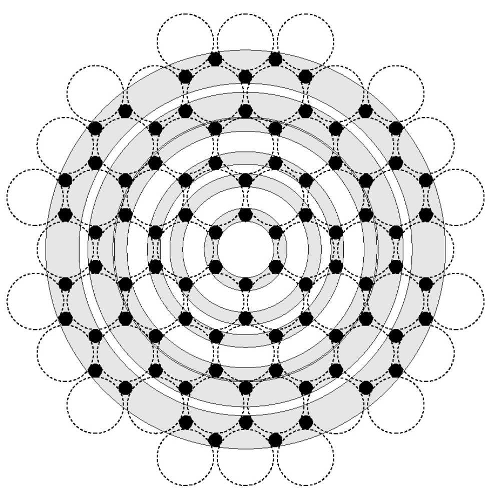

Fig. 4. The gray regions represent the ranges of core radii that intersect rods, and thus support surface modes, and the white regions between them the surface-mode-free bands. See text for details.

To verify the validity of this new criterion, we applied it to the most widely studied class of air-core PBFs, namely fibers with circular air holes in a triangular pattern, as illustrated in

Fig. 4. The core is a larger circular air hole of radius R at the center of the fiber. Again, we postulate that when R is such that the core surface intersects one or more rods, (1) surface modes exist, and (2) the number of surface modes is proportional to the number of rods intersected. This scaling law is expected because as the number of intersected rods increases the perturbation magnitude increases and so does the number of surface modes. Conversely, when the core does not intersect any rods no surface modes occur. A simple diagram of the fiber cross section (e.g., Fig. 4) makes the application of this criterion to any fiber geometry very easy.

Table 1. Location of the 14 bands of core radii that support no surface modes in triangular PBFs with $\rho = 0.47\Lambda$ (see text for details).

Band number	**Range (from criterion)**	**Range (from simulations)**	**Width**
#1	0.685—1.047	0.65^{*}—1.05^{*}	0.363
#2	1.262—1.420	$1.27^{\wedge}$—1.45^{*}	0.158
#3	1.635—1.974	1.65^{*}—2.05^{*}	0.339
#4	2.189—2.202		0.013
#5	2.624—2.779		0.155
#6	3.322—3.405		0.083
#7	3.619—3.679		0.059
#8	3.893—3.934		0.071
#9	4.271—4.402		0.131
#10	5.239—5.400		0.161
#11	6.218—6.244		0.026
#12	6.914—6.916		0.0022
#13	7.875—7.914		0.039
#14	8.844—8.856		0.0113

All entries are in units of Λ. Uncertainties: $^{*} = \pm 0.1$; $^{\wedge} = \pm 0.04$.

The result of this geometric analysis is graphed in Fig. 4 for a triangular pattern. The shaded rings represent the ranges of core radii that intersect rods and thus support surface modes. The white rings between them represent ranges of radii that intersect no rods (no surface modes). The dependence of the number of surface modes on core radius, calculated straightforwardly by applying elementary trigonometry to Fig. 4 to track down how many rods a core of given radius crosses, is plotted in Fig. 5 (solid curve). Our simple postulate predicts the important result that there are several bands of radii for which this type of PBF supports no surface modes at all, across the entire bandgap, as we established previously with numerical simulations [3]. Six bands occur in the range covered in Fig. 5 (R up to 3.5Λ), not counting the band below $R = 0.47\Lambda$, for which the radii are too small to support a core mode. Another eight bands occur for radii larger than 3.5Λ, the last one being at $R \approx 8.86\Lambda$. Table 1 lists the boundaries and width of these 14 bands. The first band is the widest; it is also the most important, because it is the only one that falls in the single-mode range of this PBF ($R < 1.2\Lambda$ for $\rho = 0.47\Lambda$). All other bands, except for the third one, are substantially narrower, generally getting narrower as the core size increases. Note that by nature of the rod approximation, these values are independent of the dielectric refractive index.

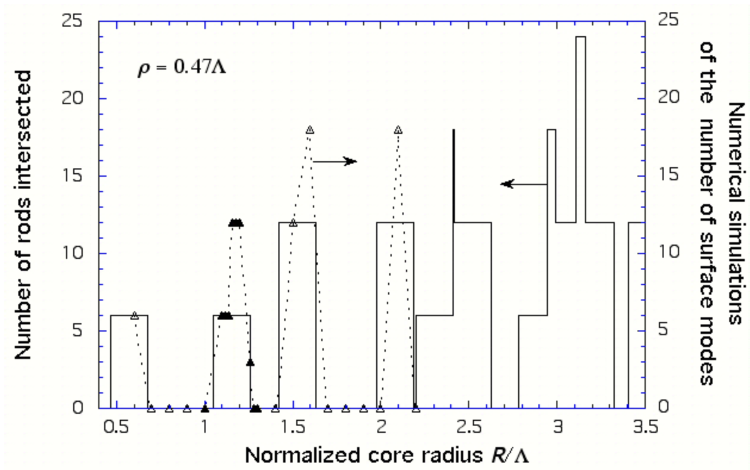

Fig. 5. Dependence of the number of surface modes on core radius predicted by numerical simulations (dashed curves with triangles) and by the proposed geometric criterion (solid curve).

4. Comparison to simulations

To evaluate the accuracy of these quantitative predictions, we conducted numerical simulations of the surface modes of this same class of PBFs on a supercomputer using a full-vectorial plane wave expansion method [9]. The dielectric was taken to be silica and ρ = 0.47Λ. Simulations results are plotted in Fig. 5 (open triangles joined by dashes). The grid size used to generate the filled triangles was Λ/16. To generate additional points in the more interesting range of core radii between 1.1Λ and 1.3Λ, we had to reduce the grid size to Λ/32 (open triangles). As a result, the absolute number of surface modes predicted in this range does not scale the same way as in the rest of the graph, which is inconsequential since we are only interested in the boundaries of the surface mode regions. Figure 5 shows that the agreement with the predictions of the geometric criterion is excellent. This is further apparent by comparing in Table 1 the boundary values of the first three surface-mode-free bands generated by the geometric criterion (second column) and by simulations (third column): the accuracy of the criterion is better than 5%. It is worth pointing out that the exact boundary radii were computed in limited numbers and with a limited number of digits because simulations are very time consuming (again, about six hours per radius). The accuracy of these simulated radii is also limited (see uncertainties listed in Table 1) by the finite sampling and the grid size. In contrast, the geometric criterion provided far more information with a greater accuracy (within the approximation of the model) in a considerably shorter time. Also note that although the geometric criterion does not accurately predict the number of surface modes (see Fig. 5), it does exhibit the right trend, namely that surface modes generally become more numerous with increasing R, which supports our earlier hypothesis.

5. Effect of filling factor

The effect of the fiber air-filling ratio on the presence of surface modes can also be quickly evaluated with our criterion by simply recalculating the boundary radii for different values of the hole radius ρ. The result is shown in Fig. 6, where the ranges of core radii that support (gray) and do not support (white) surface modes are plotted against ρ. Possible values for ρ

are constrained between ~0.43Λ, below which the photonic crystal has no bandgap, and 0.50Λ, above which the thickness of the membranes drops to zero. Figure 6 shows that larger holes (higher air-filling ratios) yield wider surface-mode-free bands. The reason is that as the hole size is increased, the rods get smaller, thus the ranges of core radii that intersect them narrow and the bands of surface-mode-free radii widen.

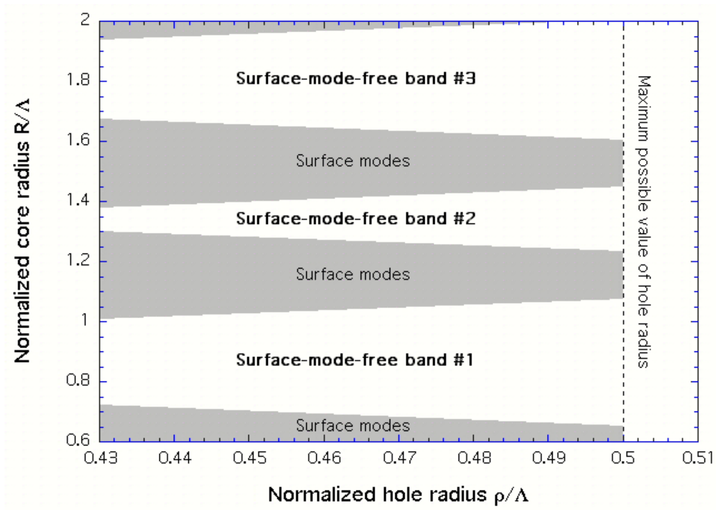

Fig. 6. Evolution of the surface-mode-free bands with increasing hole radius predicted by the geometric criterion.

6. Discussions

Other interesting observations can be made from this study. First, in experimental PBFs the core is typically created by removing the central seven or 19 tubes from the preform. These configurations correspond to core radii R of ~1.2Λ and ~2.1Λ, respectively. The geometric criterion confirms the predictions of exact simulations [3] that both of these configurations unfortunately exhibit surface modes (see Fig. 5), which explains at least in part the high propagation loss of these fibers. To reduce this loss, it is therefore important to design alternative configurations and assess their surface mode behavior, a task that can be greatly simplified and expedited by making use of our criterion.

Second, the simulated curve in Fig. 5 shows that a small change in core radius is all it takes to move from a surface-mode-free PBF to a PBF that carries surface modes. The abruptness of these transitions is in keeping with the perturbation process that creates surface modes, and it lends credence to the rod approximation.

Third, the trends in Table 1 discussed earlier can be explained with simple physical arguments. As the core radius increases, adjacent concentric layers of rods get closer to each other (see Fig. 4), and it becomes increasingly more difficult to find a circular radius that avoids all rods. Also, a given radius tends to intersect more rods, and thus the number of surface modes generally increases. A manifestation of this effect can readily be seen in the 5th and 6th layers of rods in Fig. 4: they overlap radially and thus merge into a single, wider zone of surface modes. The same is true of the 7th, 8th, and 9th layers. Conversely, as R increases the surface-mode-free bands become narrower and narrower, as predicted by Table 1. In fact, we expect intuitively that cores with a radius larger than some critical value R_c will all support surface modes, and thus that there should be a finite number of surface-mode-free bands. This is all consistent with the results of Table 1: for the structure under study and $\rho = 0.47\Lambda$, there is a limited number of bands (14), and there is a critical radius ($R_c \approx 8.86\Lambda$) above which surface modes form a continuum. The last four bands are in fact so narrow (ΔR of a few percent of Λ) that they are probably unusable in practice. A corollary of this observation is that multimode PBFs with this particular geometry and R > 5.4 will likely be plagued with surface modes.

It is important to remember that the criterion derived in this paper is specifically developed to describe the effects of photonic-crystal truncation on the creation of surface modes. In the geometry shown in Fig. 2, the only perturbations to the periodic lattice are the presence of the air core and the resulting photonic-crystal truncation. In this situation the geometrical criterion completely predicts the existence or non-existence of the fiber modes. In all manufactured air-core PBFs reported to date, in addition to this truncation, severe distortions of the crystal periodicity and irregular hole dimensions occur in the vicinity of the core. These perturbations can lead to additional surface modes that are not described by the geometric criterion presented here. As stressed earlier, many kinds of perturbations will induce surface modes in a PBF; improper termination of the core surface is only one of them, and that is what the geometrical criterion developed here captures.

A final observation is that surface modes can always be avoided in principle, for any core size, by selecting a non-circular core shape that does not intersect any rods.

7. Conclusions

We have provided physical arguments for a simple geometric criterion to quickly evaluate whether an air-core PBF exhibits surface modes. Comparison to numerical simulations demonstrate that when applied to fibers with a triangular-pattern cladding and a circular core, this criterion accurately predicts the presence of a finite number of core radius bands that support no surface modes. For large enough circular cores, i.e., for radii above the largest of these bands, the fiber supports surface modes for any core radius. This versatile criterion provides an expedient new tool to analyze the existence of surface modes in photonic-crystal fibers with an arbitrary crystal structure and core profile.

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