

TABLE OF CONTENTS

INTRODUCTION: A SHORT HISTORY OF EARLY DAYS OF MODERN COMPOSITE MATERIALS	1
1 STRESS-STRAIN RELATIONS	13
1.1 Notations	13
1.2 Contracted Notation	14
1.3 Contracted Stiffness	16
1.4 Contracted Compliance	18
1.5 Generalized Hooke's Law	20
1.5.1 Triclinic Symmetry	20
1.5.2 Monoclinic Symmetry	21
1.5.3 Orthotropic Symmetry	23
1.5.4 Transversely Isotropic Symmetry	24
1.5.5 Isotropy	25
1.6 Summary of Material Symmetries	26
1.7 Engineering Constants	28
1.8 Stiffness and Engineering Constants	30
1.9 Plane Stress	31
1.10 Plane Strain	36
1.11 Sample Calculations for Elastic Moduli	38
1.12 Stress and Strain Transformation Equations	40
1.13 Sample Calculations in Stress and Strain	46
1.14 Conclusions	49
1.15 Problems	50

2	PLY STIFFNESS	53
2.1	Transformation of Stiffness	53
2.2	Transformation of Compliance	59
2.3	Transformation in Three Dimensions	60
2.4	Multiple Angle Transformation	62
2.5	Matrix Inversion: Engineering Constants	66
2.6	Elastic Constants of Typical Composite Plies	68
2.7	Quasi-Isotropic Constants	71
2.8	Calculations of Ply Elastic Constants	73
2.9	Transformed Stiffness of Composites	76
2.10	Conclusions	82
2.11	Problems	83
3	IN-PLANE STIFFNESS	87
3.1	Laminate Code	87
3.2	Laminated Plate Theory	88
3.2.1	Matrix Inversion	91
3.2.2	In-Plane Engineering Constants	91
3.3	Stiffness Matrix: Multiple Angle Method	92
3.4	Example of the Cash Register Method	94
3.5	Examples of the Rule-Of-Mixtures Method	96
3.6	Sample Calculations of In-Plane Stiffness	97
3.7	Invariants of In-Plane Moduli	107
3.8	Ply Stress and Ply Strain	108
3.9	Residual Stresses	110

3.10	Residual Strains After Curing	112
3.11	Expansion Coefficients	115
3.12	Sample Calculations in Expansions	117
3.13	Laminate Stiffness by Lamrank	120
3.14	Conclusions	126
3.15	Problems	127
4	FLEXURAL STIFFNESS OF SYMMETRIC LAMINATES	131
4.1	Laminated Plate Theory	131
4.1.1	Stress-Strain Relations	134
4.1.2	Matrix Inversion	135
4.1.3	Engineering Constants	136
4.2	Flexural Stiffness Evaluation	136
4.3	Sandwich Plates	141
4.4	Transformation of Flexural Moduli	143
4.5	Homogenization of Laminated Plates	144
4.6	Stacking Sequence Effects	148
4.7	Ply Stress and Ply Strain	153
4.8	Beams	155
4.9	Tubing	157
4.10	Conclusions	158
4.11	Problems	159
5	STIFFNESS OF GENERAL LAMINATES	161
5.1	Laminated Plate Theory	161
5.1.1	Partial Inversion	165
5.1.2	Laminate Stiffness and Compliance	165

5.2	Repeated Sub-Laminates	166
5.3	Unsymmetric Cross-Ply Laminates	170
5.4	Other Unsymmetric Laminates	176
5.5	Hygrothermal Warpage	180
5.6	Thin Wall Construction	184
5.7	Conclusions	186
5.8	Problems	187
6	MICROMECHANICS	189
6.1	Background	189
6.2	Specific Gravity, Volume and Mass Fractions, and Void	192
6.3	Longitudinal Young's Modulus and Poisson's Ratio	193
6.4	Transverse Modulus	193
6.5	Longitudinal Shear Modulus	198
6.6	Expansion Coefficients	202
6.7	Effective Stiffness of Degraded Plies	202
6.8	Hygrothermal Effects	207
6.9	Micromechanics of Strength	209
6.10	Micromechanics of Woven Composites	213
6.11	Micromechanics of Random Composites	214
6.12	Conclusions	215
6.13	Problems	216
7	FAILURE CRITERIA	217
7.1	Introduction	217

7.2	Basic Strength Data	219
7.3	Quadratic Criterion in Stress Space	221
7.4	Failure Mode Interactions	228
7.5	Admissible Values for the Interaction Term	231
7.6	Quadratic Criterion in Strain Space	237
7.7	Transformation of Failure Envelopes	241
7.8	Strength/Stress Ratio and Failure Index	245
7.9	Experimental Data	250
7.10	Conclusions	251
7.11	Problems	251
8	STRENGTH OF LAMINATES	253
8.1	First Ply Failure Envelopes	253
8.1.1	Stress-Space Failure Envelopes	255
8.2	Ply-By-Ply Stress and FPF Strength Analysis	257
8.3	Uniaxial Strengths of Angle-Ply Laminates	259
8.4	Hygrothermal Stresses	261
8.5	Strength After First Ply Failure	264
8.5.1	Progressive Failures	266
8.5.2	Intact and Degraded Plies	268
8.5.3	Selective Degradation by Micro Cracking	269
8.5.4	Progressive Failures of a [0/90] Laminate	270
8.5.5	Ultimate Failure Envelopes	273
8.6	Progressive Versus Simultaneous Degradation	274
8.7	Conclusions	277
8.8	Problems	278
9	THE INVARIANT-BASED APPROACH TO STIFFNESS	279

9.1	Stiffness Invariants - Trace	279
9.2	Trace Normalized Stiffness – Master Ply	281
9.3	Transformation of Trace-Normalized Stiffness and Their Invariants	282
9.4	Trace-Normalized Stiffness of Laminates	285
9.5	Hard and Soft Laminates	287
9.6	Sensitivity of Trace Normalized Properties to Constituents	293
9.7	Sensitivity of Trace and Trace-Normalized Properties to Temperature and Moisture	296
9.8	Trace for General Laminates	298
9.9	Experimental Determination of Trace	300
9.10	Conclusions	306
9.11	Supplemental Information	307
9.11.1	Traces of Tensors Used in Composites Mechanics	307
9.11.2	Trace of CFRP Compared to Metals	308
9.11.3	Cross-Ply Laminates and Fabrics	309
9.11.4	2D vs. 3D TRACE	312
9.11.5	Tensile and Compressive Traces	316
9.12	Problems	319
10	THE INVARIANT-BASED APPROACH TO STRENGTH	321
10.1	Omni Strain Failure Envelopes	323
10.2	Aplication Examples	330
10.3	Omni Envelopes Based on Non-Interactive Failure Theories	334
10.4	Testing Laminates for The Omni Envelope	337
10.5	Effect of Matrix Degradation on Omni LPF Envelopes	340
10.6	Stiffness of Degraded Laminates	344
10.7	Envelopes in Stress Space	348

10.8	Progressive Versus Simultaneous Failure	351
10.9	Comparison With Experimental Data	359
10.10	Non-Omni Laminates	363
10.11	Open Hole Coupons	370
10.12	Master Stress-Strain Template	373
10.13	Conclusions	384
11	NEW OPPORTUNITIES IN DESIGN AND TESTING	385
11.1	The Building Block Approach	385
11.2	Discrete and Continuous Laminates	393
11.3	C-Ply by Chomarat	407
11.4	Opportunities In Tape Laying	411
11.5	Conclusions	420
12	MECHANICAL TESTING OF COMPOSITES	421
12.1	Common Testing of Composite Materials	421
12.1.1	Tensile Testing	422
12.1.2	Compression Testing	428
12.1.3	Shear Testing	431
12.2	Digital Image Correlation	433
12.2.1	Introduction	433
12.2.2	Correlation principles in 2D with global approach	433
12.2.3	2D DIC Examples	436
12.2.4	Stereo (3D) Correlation	437
12.2.5	Stereo Correlation of Composites with Correli ^{stc}	440
12.3	Simplified Testing of Composites with DIC	444
12.3.1	Comparison of DIC and Strain Gaged Composites	445
12.3.2	Determination of Material Properties from Open Hole Tension with DIC	447
12.4	Conclusion	448