

THEORY OF COMPOSITES DESIGN

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Our favorite Focus Group: CDW-9, Shanghai, April 2004 with Hongneng Cai, Mr. and Mrs. Iwashita of KIT, Ran and Jane Kim, Iris Tsai, Sangwook, Eunmi and Amy Sihh, Dick and Kristy Christensen, Sung Ha, Noriko Miyano (head bowed), Lydia Tay, Yves-Henri and Francois Grunvald, Ms. Xiaobing Tong, and Francis Collombet