

Bankruptcy: is it enough to forgive or must we
also forget?¹

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Preliminary

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Abstract

In many countries, lenders are not permitted to use information about past defaults after a specified period of time has elapsed. We model this provision and determine conditions under which it is optimal.

We develop a model in which entrepreneurs must repeatedly seek external funds to finance a sequence of risky projects under conditions of both adverse selection and moral hazard. We show that forgetting a default makes incentives worse, *ex-ante*, because it reduces the punishment from failure. However, following a default it is generally good to forget, because pooling riskier agents with safer ones makes exerting high effort to preserve their (undeservedly good) reputation more attractive.

Our key result is that if agents are sufficiently patient, and low effort is not too inefficient, then the optimal law would prescribe some amount of forgetting — that is, it would not permit lenders to fully utilize past information. We also show that such a law must be enforced by the government — no lender would willingly agree to forget.

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I Introduction

In studying the “fresh start” provisions of personal bankruptcy law, economists typically focus on the *forgiveness* of debts. However, another important feature is the *forgetting* of past defaults. In many countries, lenders are not permitted to use information about past defaults after a specified period of time has elapsed; for example, in the United States information about a bankruptcy cannot be used after ten years.

In this paper we model this provision and determine conditions under which it is optimal. We develop a model — related to Diamond (1989) — in which entrepreneurs must repeatedly seek external funds to finance a sequence of risky projects under conditions of both adverse selection and moral hazard. In this model, reputation effects encourage agents to exert high effort; however, it is typically the case that reputation is not efficacious until agents have accumulated a long enough credit history to make default unattractive.

We show that forgetting a default makes incentives worse, *ex-ante*, because it reduces the punishment from failure; as a result, it delays the salutary impact of reputation. However, following a default it is generally good to forget, because pooling riskier agents with safer ones makes exerting high effort to preserve their (undeservedly good) reputation more attractive.

Our key result is that if agents are sufficiently patient, and low effort is not too inefficient, then the optimal law would prescribe some amount of forgetting — that is, it would not permit lenders to fully utilize past information. We also show that this law must be enforced by the government — no lender would willingly agree to forget.

The focus of this paper is the effect of laws governing bankruptcy on investment. We thus have in mind a world of small entrepreneurs who finance their business ventures with loans for which they are personally liable. Data

from the 1993 National Survey of Small Business Finance (NSSBF) suggests that the majority of small businesses do indeed finance themselves with some sort of personal loan or guarantee; see also Berger and Udell (1995). These entrepreneurs are also three times as likely to file for personal bankruptcy as their counterparts in the general population — see Sullivan, Warren and Westbrook (1989). In such a setting we are then naturally led to explore the incentive effects of these laws; this seems to be where the greatest economic impact should be found. An alternative approach, however, might be to focus on the risk-sharing and redistributive impact of these laws on consumers.

The U.S. Fair Credit Reporting Act (FCRA) prescribes that a personal bankruptcy filing may remain on credit files for up to ten years, after which it must be removed (other derogatory information can stay a maximum of seven years). Similar provisions exist in many other countries — in the European Union the median time which a bankruptcy can stay on records is even shorter — only six years (see Jentzsch 2004, for a comprehensive summary of the international evidence). Another feature of some European laws is that they sometimes restrict disclosure to only *negative* information (such as defaults), rather than “positive” (such as balances); this is modeled by Padilla and Pagano (2000). It should also be noted that information release is also restricted after a certain time in other contexts, such as motor vehicle records and criminal convictions.

Musto (2004) finds that these restrictions are binding. He shows that for those bankrupts who are more creditworthy, access to credit increases significantly when the bankruptcy ‘flag’ is dropped from their credit file, when the 10-year period expires; for the less creditworthy bankrupts dropping the flag has little impact, because they have many other derogatory indicators in their file. He also finds that these individuals are subsequently likelier than average to default on this new credit; he interprets this as evidence that these laws are suboptimal. Our model will be consistent with this behavior,

although we will show that it need not be evidence of inefficiency.

II Previous Literature

There is of course a large literature on the potential inefficiency of public revealing information, dating back at least to Hirshleifer (1971). Hirshleifer argues that revealing information may be socially inefficient, because it can preclude insurance opportunities.¹ Our paper is closer to another strand of the literature, which examines the effect that revealing information can have on the punishments a principal can impose on an agent. Crémer (1995) shows that using an inefficient monitoring technology can sometimes improve incentives because by monitoring less efficiently a principal ensures that he will have less information about a default and can thereby commit to punish more severely (because he will not have enough information to renegotiate punishments).² In a similar vein, Vercammen (1995) presents an example in which a socially optimal policy involves restricting the memory of a credit bureau; his model resembles that of Holmstrom (1982) (and also Crémer, 1995) in that incentives become *worse* over time as a good agents' type is revealed (so that what is needed is really to forget *successes* rather than failures). Our paper is different, in that for us forgetting actually *weakens* the punishment incurred by a defaulter; as such it tends to make incentives worse, ex-ante. However, ex-post having a weaker punishment can be better; the optimal policy trades off these two effects. This tradeoff between current and future incentives is also unique to our paper. We also believe that our model is more appropriate as a description of a credit market in that (like Diamond, 1989) incentives are worst at the beginning of an agent's life.

¹See also Marin and Rahi (2000).

²The idea that restricting renegotiation can be beneficial has also been used to motivate corporate bankruptcy laws — see Berkovitch, Israel and Zender (1998).

There is also a literature which studies the outcomes which arise when lenders can punish defaulting agents with varying degrees of exclusion from financial markets.³ An early example of this is Kehoe and Levine (1993), who assume that when an agent fails he is excluded from all future credit markets. More recent work relaxes this assumption; for example, a recent paper by Bond and Krishnamurthy (2004) develops a model in which an optimal law would only exclude defaulters until they repay their original debt. Kletzer and Wright (2000) study the outcome when any punishment is required to be renegotiation-proof. Finally, in the paper closest to ours, Krueger and Uhlig (2003) also require any punishment to be renegotiation-proof, but add two further assumptions. First of all, they assume one-sided commitment: while principals can commit to long-term contracts, agents are always free to walk away from their existing contract. However, if they do so, then they must seek a new contract in a competitive market. The requirements that contracts be renegotiation-proof and that the market for contracts is competitive are shared by our model. Despite these similarities, an important difference between our model and this literature is that in our model lenders face both adverse selection and an incentive problem on the part of the agents. Another difference is that rather than directly controlling the degree of exclusion, we make assumptions on the sharing of information which generate a particular pattern of exclusion in equilibrium.

Finally, our basic model shares many aspects with Diamond (1989). Like in his, entrepreneurs must repeatedly seek external funds to finance a project under conditions of both adverse selection and moral hazard. The renegotiation-proof equilibrium in our model — in the special case of perfect memory —

³Although exclusion arises naturally in our setting because of the moral hazard problem (and indeed it is an equilibrium outcome), in other models different penalties might be more appropriate. For example, in a recent paper by DeMarzo and Fishman (2004) the “threat of losing control of the project is the key to inducing the agent to share the cash flow with investors.”

shares many similarities with his; in particular, in both cases the outcome is characterized by a period of low effort followed by one in which risky agents exert high effort. One difference is that our model has an infinite horizon; this leads to a cleaner and slightly more tractable setup. More fundamentally, our goals are of course quite different; we are interested in analyzing the effect of forgetting past defaults, an issue which is not addressed in Diamond (1989).

III Model

A Introduction

Consider an economy made up of risk-neutral *entrepreneurs*, who live forever and discount the future at the rate $\beta \leq 1$. In each period an entrepreneur receives a new project, which requires one unit of financing in order to be undertaken. This project returns either R (success) or 0 (failure). Output is non-storable, so entrepreneurs must seek external financing in each period.

There is also a perfectly competitive lending sector, which is prepared to lend at a rate which guarantees an expected rate of return of 0; we will let r denote the gross interest rate charged to a particular borrower. As in Diamond (1989), entrepreneurs meet a new lender in every period, so long-term contracts are not feasible.

There are two types of entrepreneurs. There is a set of measure 1 of *riskless* agents, whose projects always succeed (i.e. return R .) In addition, there is a set of *risky* agents, with measure $p > 0$, for whom the project may fail with some probability. If the project fails then we assume that since the return is 0 the current debt is forgiven (discharged), while if the project succeeds lenders may seize the return of R if the entrepreneur does not repay the loan. The risky agent's projects are independent and identically

distributed.

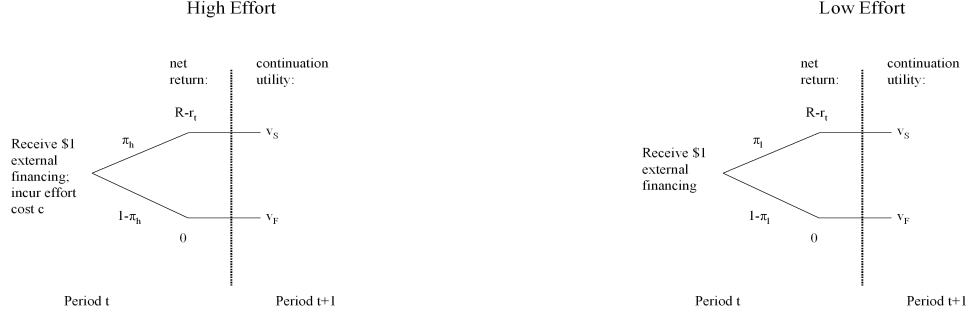
The risky agents also have an effort choice. They may choose to exert high effort, in which case the success probability will be $\pi_h > 1/R$ (i.e. the project NPV is positive); we also assume that $\pi_h < 1$ to distinguish these agents from the riskless ones. The cost of exerting high is $c > 0$, and we assume $\pi_h R - 1 > c$, so that the entrepreneur would derive positive benefit from the project. Alternatively, the risky entrepreneur may choose low effort. Low effort is costless, but the success probability under low effort is only π_l ; we assume that $\pi_l < 1/R$ so that the project has a negative NPV under low effort.

There is an incentive problem in that it is only worth lending to the risky agent if he exerts high effort, but effort is non-contractible. Since the entrepreneur does not pay anything when he fails, he may well prefer to exert low effort even though it is inefficient. One feature which will sometimes help improve incentives is *reputation* — i.e. that failing will brand an entrepreneur as risky (since the riskless agents never fail).

The timeline of a single period is as follows. The entrepreneur receives his project. He requests a loan of 1 from the lender he meets in this period. He undertakes the project (funds lent cannot be diverted to consumption). If he succeeds, then he repays (we will see that when lenders are willing to advance funds then $r < R$) and if it fails then he defaults. Note that — purely for convenience — we assume that lenders repay in the same period in which they borrow.

To make the model interesting, we will assume that incentives are sufficiently poor that we need reputational effects to elicit high effort. That is, we will assume that even if we charge the lowest possible interest rate ($r = 1$), then high effort is not incentive-compatible for an risky agent who is playing a one-shot game.

Assumption 1: $c > (\pi_h - \pi_l) \times (R - 1)$



To see that this is the appropriate assumption, notice that the utility accruing from high effort is $[\pi_h \times (R - r) + (1 - \pi_h) \times 0] - c$. Similarly, the utility from low effort is $[\pi_l \times (R - r) + (1 - \pi_l) \times 0]$. Assumption 1 obtains once we recognize that for high effort to be incentive-compatible the utility from high effort must be no less than that from low effort, and substitute $r = 1$.

More generally, suppose that the current interest rate is r , and that the risky agent's continuation utility (next period) if the project succeeds is v_S and that it is v_F if the project fails. Then it is easy to see that the incentive-compatibility constraint which must be satisfied to ensure high effort in this period becomes:

$$c \leq (\pi_h - \pi_l) \times [R - r + \beta(v_S - v_F)] \quad (1)$$

Given that the probabilities and project return are fixed, the two requirements for this to be satisfied are that r is sufficiently low, and that $v_S - v_F$ be sufficiently high; i.e. that there be a “wedge” between the continuation utility upon success and failure.

Before we proceed any further, we introduce an equilibrium refinement.

Assumption 2: We restrict attention to equilibria which are *renegotiation-proof* (RP).

What this means is that the (continuation) utility which an agent receives must be maximal, conditional on the information that we have about that agent in the node reached. As Krueger and Uhlig (2003) point out, this assumption is quite natural when the lending sector is perfectly competitive, since (by contrast with Kletzer and Wright, 2000) there is no way for lenders to collude in punishing the agent by driving his utility down.

As we will argue in Lemma 1 below, the requirement that equilibrium be renegotiation-proof implies that if an agent is known to be risky, then he will not receive a loan. Empirically it is known that although post-bankruptcy credit is available (Staten, 1993), access to credit is still very restricted for those with bankruptcy flags in their files (Musto, 2004). So our next result can be viewed as a stylized representation of this fact.

Lemma 1 In any period, if an agent is known to be risky, then he receives no financing and cannot undertake his project.

Proof :

Once we are certain that an agent is risky, renegotiation-proofness implies that we must have $v_S = v_F$; that is, the continuation utilities must be the same no matter what the outcome of the agent's project (and, more importantly, no matter what the agent does in this period). As a result, the incentive-compatibility condition in (1) becomes

$$c \leq (\pi_h - \pi_l) \times [R - r]$$

Now, when an agent is risky, then the lowest interest-rate for which lenders break-even, in expected terms, is $r = 1/\pi_h > 1$. By Assumption 1, IC cannot

be satisfied with this interest rate. Since we have assumed that the project has a negative NPV under low effort, no lender would be willing to advance financing under these terms since there would be no way to recoup this from other agents since the lendingmarket is competitive. \square

Since only risky agents can fail at their project, the following corollary is immediate.

Corollary 1 If we remember failures forever, then once an agent fails at his project, he can no longer obtain financing.

Observation: This refinement gives us the same equilibrium outcome in this case as in Lemma 5 of Diamond (1989). In that paper, however, the horizon was assumed to be finite and the result obtained via backward induction. We favor our approach because it simplifies the analysis and gives us a cleaner characterization of the equilibrium (below).

We now slightly rewrite the incentive-compatibility conditions. For any period n let v_n denote the utility in period n if the agent acts optimally. Also, let r_n denote the equilibrium interest rate in this case. Now note that if high effort is incentive compatible, then we have $v_n = \pi_h(R - r_n) - c + \pi_h\beta v_{n+1}$, where we have used the fact that the agent is no longer financed once he fails. Similarly, if low effort is incentive-compatible, then $v_n = \pi_l(R - r_n) + \pi_l\beta v_{n+1}$. Then substituting these expressions into the incentive compatibility condition (1) above, we get the following result.

Corollary 2 High effort is incentive compatible if and only if $v_n \geq c \frac{\pi_l}{\pi_h - \pi_l}$

Observation: Notice that this implies that high effort yields more utility than low effort.

In order to focus attention on the more interesting cases, we add one more assumption — namely that the measure of risky agents is sufficiently small that the project return R exceeds the highest possible interest rate.

Assumption 3: $p < \frac{R-1}{\pi_l R-1}$

Comment: To see that this is the right assumption, first notice that since the riskless agents never fail, the worst possible case occurs when all of the risky agents are in the pool — they have measure p . Furthermore, the worst case occurs when the risky agents exert low effort. In this case, the interest rate r satisfies $1 + p = 1 \times r + p[\pi_l \times r + (1 - \pi_l) \times 0]$. Our assumption is obtained by restricting this interest rate to be smaller than R .

Observation: Under this assumption, the lenders are always willing to advance funds as long as they are not certain that the agent is risky. This assumption rules out the less interesting case of a collapse of the credit market (which could be treated separately in a straightforward manner).

Now we can ask what will ensure that the risky agent exerts high effort? Examining the IC constraint (1), one can see that one way to achieve this is *pooling* with the riskless agent. The reason this helps is that (i) it lowers the interest rate and (ii) it creates a “punishment” for failure, because it will imply that $v_S > v_F$. Looking ahead, this will be why forgetting a past default may be optimal.

Before we treat the question of the optimal level of “forgetting”, we first characterize the equilibrium when past defaults are remembered forever.

B Perfect Memory

When the lending sector is permitted to remember a (past) default forever, the renegotiation-proof equilibrium can be characterized by the following lemma.

Lemma 2 When lenders are permitted to remember all defaults, the renegotiation-

proof equilibrium has the following properties:

- At period $n = 0$, all agents (risky, riskless) are pooled.
- Any time an agent fails, he is excluded forever after (and is no longer in the “pool”).
- There exists a cutoff period N ($0 \leq N \leq \infty$) such that if $n < N$ then any risky agents still in the pool exert low effort, and if $n \geq N$ then all risky agents remaining in the pool exert high effort.

Proof: Suppose that this is not the case. I.e., that there exists a period n such that in period n agents exert high effort, but that in period $n + 1$ they exert low effort.

Now, from the IC constraint (1), we must then have

$$c/(\pi_h - \pi_l) \leq R - r_n + \beta v_{n+1}$$

but

$$c/(\pi_h - \pi_l) > R - r_{n+1} + \beta v_{n+2}$$

where r_n is the equilibrium interest rate in period n and v_{n+1} is the utility beginning in period $n + 1$, should the entrepreneur succeed in period n . The definitions for period $n + 1$ are analogous.

Now, we will argue that it is profitable for an individual agent to deviate from this equilibrium and exert high effort in period $n + 1$. Let p_{n+1} denote the measure of risky agents remaining in the pool in period $n + 1$ (note that $p_{n+1} = \pi_h p_n$) if high effort is exerted in period n . Now, notice that the interest rate r_{n+1}^G this agent would offer would satisfy $1 + p_{n+1} = 1 \times r_{n+1}^G + p_{n+1}[\pi_h r_{n+1}^G + (1 - \pi_h)0]$, so that $r_{n+1}^G = \frac{1+p_{n+1}}{1+\pi_h p_{n+1}}$. Implicitly, we are assuming that when an agent deviates it is assumed that he is of average quality (this is the refinement Diamond (1989) also considers.)

Since $\pi_h < 1$, it is immediate that $p_{n+1} < p_n$, and so $r_{n+1}^G < r_n$; that is, the interest rate is lower. In addition, also note that under the maintained equilibrium low effort is exerted by the pool in period $n + 1$; since the population of risky agents still in the pool shrinks over time, it must therefore be that $r_{n+1} > r_{n'}$ for all $n' > n + 1$.

In addition, we also claim that this implies that $v_{n+2} > v_{n+1}$. To see this, first notice that under the maintained equilibrium,

$$v_{n+1} = \pi_l(R - r_{n+1}) + \pi_l\beta v_{n+2}.$$

Now, suppose this were not true, i.e. that actually $v_{n+2} \leq v_{n+1}$. Then we would have

$$\pi_l(R - r_{n+1}) + \beta v_{n+2} \geq v_{n+2}$$

or

$$v_{n+2} \leq \frac{\pi_l(R - r_{n+1})}{1 - \beta\pi_l}.$$

The righthand side of this inequality is the utility from exerting low effort in every period beginning in period 2, and always being charged an interest rate of r_{n+1} . This expression is necessarily below v_{n+2} , however, for the following reasons. First of all, from above we know that since the pool of risky agents declines over time, $r_{n+1} > r_{n'}$ for all $n' > n + 1$ — i.e. interest rates in the future will always be lower. In addition, we know from corollary (2) that if the agents ever choose high effort in some period(s) $n' > n + 1$, v_{n+2} will be even higher. \square

The following corollary follows by noting that $r_n \rightarrow 1$ as $n \rightarrow \infty$.

Corollary 3 If $c < \frac{(\pi_h - \pi_l)(R-1)}{1 - \pi_l\beta}$, then high effort begins at $N < \infty$

C Imperfect Memory

We will now allow for “imperfect memory”. Why might “forgetting a default” be a good idea? Observe that in the previous section, once we pass period N incentives continue to improve as the interest rate r_n falls and the continuation utility v_{n+1} rises (also since the future interest rates fall). So excluding agents who fail after this period seems inefficient; while allowing some risky agents back into the pool would raise the interest rate, if this is done in a moderate fashion it would still be possible to elicit high effort from these agents and so we would have a larger pool of agents undertaking efficient projects. On the other hand, the cost of such a policy would be that, in anticipating this forgetting, agents would have worse incentives (ex-ante). This will have the effect of raising N — i.e. of extending the period in which low effort is exerted.

We model forgetting as follows. When a risky agent’s project fails, there is a probability $q \in [0, 1]$ that this failure is forgotten, and the agent is reinserted the following period into the “pool”. We assume that this amnesia is immediate and permanent — that is, once forgotten, this failure is never recalled (of course an agent may well fail in the future, and that failure would not necessarily be forgotten). We also assume that q is constant over time — we will have more to say on this below. A similar approach is taken by Padilla and Pagano (2000).

This model is meant to capture — in a stylized fashion — the idea that a default is forgotten after a certain number of years — the use of a probability of exclusion rather than a period of time greatly simplifies the analysis because it obviates the need to keep track of an agent’s entire history. (A higher value for q would then be interpreted as forgetting more quickly.) For the moment we assume that q is imposed exogenously — we will discuss this further below.

Now, when a default is forgotten, then the agent is returned to the “pool”

of agents who have not defaulted, and receives continuation utility v_S . Recalling that any agent who is known to be risky (i.e. for whom the default is *not* forgotten) does not receive financing and so has a continuation utility of 0, the incentive compatibility constraint becomes:

$$c \leq (\pi_h - \pi_l) \times [R - r + \beta(1 - q)v_S,] \quad (2)$$

where now v_S is the continuation utility both for those still remaining in the pool - i.e., both those who succeeded in the this period as well as those who failed but for whom all was forgotten. Notice that incentives are now worse (holding v_S fixed), because the chance that a default will be forgotten makes low effort less costly. The net effect is uncertain, because v_S may increase as we raise q . [However, we will see that if p is small then $N(q)$ is increasing in q .]

When $q = 0$, then a default is never forgotten and we are in the case discussed in the previous section. When $q > 0$ then the characterization of equilibrium is the same as above. That is, there exists $N(q)$ for which agents exert low effort when $n < N(q)$ and high effort when $n \geq N(q)$.

If $q = 1$ (and $\beta < 1$) then defaults are always forgotten, and the incentive compatibility constraint becomes $c \leq (\pi_h - \pi_l)[R - r]$. From Assumption 1 this constraint can never be satisfied and the risky agents would always exert low effort. This is clearly not optimal. The goal of this section, however, is to determine conditions under which $q > 0$ is optimal. The objective function we will consider is *aggregate welfare*. That is, if $N(q)$ is the period in which agents first exert high effort (given q), we would like to choose q so as to maximize

$$\sum_{n=0}^{N(q)-1} \beta^n [\pi_l + (1 - \pi_l)q]^n (\pi_l R - 1) \quad (3)$$

$$+\beta^{N(q)} [\pi_l + (1 - \pi_l)q]^{N(q)} \sum_{n=0}^{\infty} \beta^n [\pi_h + (1 - \pi_h)q]^n (\pi_h R - 1 - c)$$

Observe that we are subtracting the investment of 1 each time the agent is financed, so we can ignore the riskless agents (as all they are doing is subsidizing the risky ones to a greater or lesser degree).

We will prove the following proposition:

Proposition 1 If agents are sufficiently close patient (β close to 1), and if low effort is not too inefficient ($\pi_l R - 1$ not too small), then the socially optimal value of q is above 0 (but below 1).

The intuition behind these conditions is clear. Since raising q will extend the period of low effort, we want low effort not to be too inefficient. In addition, we want agents to be patient, so that the future rewards of forgetting (more agents undertaking efficient projects) are not swamped by the extra periods of low effort.

Proof : We will take the following approach to the proof of this Proposition. We will assume that $\pi_l R - 1 = 0, \beta = 1$ and $q < 1$ and show that it is optimal to take $q \rightarrow 1$; of course this implies that $q > 0$ is optimal. We will then argue that, by continuity, when we are close to these values (i.e., β close to 1 and $\pi_l R - 1$ negative but close to 0) then $q > 0$ must also be optimal (recall however that we already know that in this case that $q = 1$ is necessarily suboptimal).

Define $W(N, q)$ to be the aggregate (risky-agent) welfare when risky agents first start exerting high effort in period N and the memory is q . This is well-defined when $q < 1$ (even if $\beta = 1$). From (4) above, under our assumptions we have

$$W(N, q) = [\pi_l + (1 - \pi_l)q]^N \sum_{n=0}^{\infty} [\pi_h + (1 - \pi_h)q]^n (\pi_h R - 1 - c)$$

$$= \frac{[\pi_l + (1 - \pi_l)q]^N}{(1 - q)(1 - \pi_h)} \times (\pi_h R - 1 - c)$$

Given our assumptions, it is easy to see that $W(N, q)$ is decreasing in N and increasing in q .

Now, we would like to show that there is an equilibrium in which $q > 0$ dominates $q = 0$. I.e., we would like to show that there exists $q > 0$ such that $W(N(q), q) > W(N(0), 0)$. Since it is difficult to solve explicitly for $N(q)$, however, our strategy will be to show that for q sufficiently close to 1, we can find a value of \tilde{N} which we can be sure is both larger than $N(q)$ and for which welfare dominates that with $q = 0$, i.e. for which $W(\tilde{N}, q) > W(N(0), 0)$. Since $W(N, q)$ is decreasing in N , this will mean that that we necessarily have $W(N(q), q) > W(N(0), 0)$, as desired.

Let $\tilde{N}(q)$ be the maximal value of N such that $W(N, q) \geq W(N(0), 0)$, where recall that $N(0)$ is the time until the risky agents start exerting high effort when $q = 0$, i.e. when a default is never forgotten. That is, $\tilde{N}(q)$ is the longest delay for which q would still (weakly) dominate perfect memory. Given our assumptions — that $\beta = 1$ and $\pi_l R - 1 = 0$ — we can compute $\tilde{N}(q)$ to be the largest integer less than or equal to $\frac{\log[(1-q)\pi_l^{N(0)}]}{\log[\pi_l + q(1 - \pi_l)]}$.

Observe that in fact $\tilde{N}(q)$ is the greatest integer less than or equal to

$$\frac{\log[(1 - q)\pi_l^{N(0)}]}{\log[\pi_l + q(1 - \pi_l)]} = N(0) \frac{\log(\pi_l)}{\log[\pi_l + q(1 - \pi_l)]} + \frac{\log(1 - q)}{\log[\pi_l + q(1 - \pi_l)]}. \quad (4)$$

From this expression it can be seen that: (i) $\tilde{N}(q) \geq N(0)$; (ii) $\tilde{N}(q)$ is finite when $q < 1$; and (iii) $\tilde{N}(q) \rightarrow \infty$ as $q \rightarrow 1$.⁴

Also, given q , let $r(n, q)$ be the equilibrium interest rate in period n ; q is relevant both because it affects the measure of risky agents who remain in the pool as well as their incentives. Also let $v(n, q)$ denote the risky-agent

⁴Recall that π_l , $1 - q$, and $\pi_l + q(1 - \pi_l)$ are all less than 1.

equilibrium utility beginning in period n .

We have defined $\tilde{N}(q)$ to be the maximal delay for which aggregate welfare with a probability of forgetting of q weakly dominates welfare when we have perfect memory. We can also define $IC(q)$ to be the righthand side of the incentive compatibility constraint in this period. That is, let

$$IC(q) = (\pi_h - \pi_l) \left[R - r(\tilde{N}(q), q) + \beta(1 - q)v(\tilde{N}(q) + 1, q) \right].$$

Incentive compatibility is satisfied in this period if and only if $c \leq IC(q)$.

Since, when $q = 0$, incentive compatibility necessarily holds in period $N(0)$ (since $N(0) = \tilde{N}(0)$), we will want to show that $IC(q) > IC(0)$ for q sufficiently close to 1; this will imply that incentive compatibility holds for these values of q in period $\tilde{N}(q)$. This will mean that for these values of q equilibrium aggregate welfare is no lower than with $q = 0$. To show that we can actually strictly improve upon $q = 0$, we will show that incentive compatibility is also satisfied in period $\tilde{N}(q) - 1$ (hence the need to show that IC is slack in period $\tilde{N}(q)$; since $W(N, q)$ is strictly decreasing in N this will give us the desired result.

Now, we know (above) that when incentive-compatibility is satisfied in some period n , then it holds for all subsequent periods $n' > n$. So if IC holds in period $\tilde{N}(q)$, then

$$\begin{aligned} \beta(1 - q)v(\tilde{N}(q) + 1, q) &= \beta(1 - q) \times \sum_{n=0}^{\infty} \beta^n [\pi_h + (1 - \pi_h)q]^n \left[\pi_h(R - r(\tilde{N}(q) + 1 + n, q) - c) \right] \\ &= \frac{1}{1 - \pi_h} (\pi_h R - c) - (1 - q) \times \sum_{n=0}^{\infty} [\pi_h + (1 - \pi_h)q]^n r(\tilde{N}(q) + 1 + n, q) \end{aligned}$$

since $\beta = 1$.

Observe that the first part of this expression does not depend on q , so in order to show that IC holds it suffices to focus on interest rates and show

that

$$-r(\tilde{N}(q), q) - (1 - q) \times \sum_{n=0}^{\infty} [\pi_h + (1 - \pi_h)q]^n r(\tilde{N}(q) + 1 + n, q) > \\ -r(N(0), 0) - \sum_{n=0}^{\infty} \pi_h^n r(N(0) + 1 + n, 0).$$

Our strategy will be to show that the interest rates making up the lefthand side of this inequality (only) shrink as $q \rightarrow 1$. Recall that it is the case that (holding n fixed), an interest rate $r(n, q)$ increases as q approaches 1 (since more risky agents remain in the pool). On the other hand, for fixed q it is the case that $r(n, q) \rightarrow 1$ as $n \rightarrow \infty$ and from the definition of $\tilde{N}(q)$ above, one can see that as $q \rightarrow 1$, then $\tilde{N}(q) \rightarrow \infty$. So we have two opposing effects as we increase q .

Now, assuming that IC holds in period $\tilde{N}(q)$, the highest value of $r(\tilde{N}(q), q)$ is achieved when $N(q) = 0$ — i.e., when agents exert high effort in the very first period (because then the measure of risky agents in the pool would be greatest). In this case we would have

$$r(\tilde{N}(q), q) = \frac{1 + p[\pi_h + (1 - \pi_h)q]^{\tilde{N}(q)}}{1 + \pi_h p[\pi_h + (1 - \pi_h)q]^{\tilde{N}(q)}}.$$

From the expression for $\tilde{N}(q)$ in (4), we can see that the $\tilde{N}(q) \rightarrow \infty$ effect dominates; as $q \rightarrow 1$, we have $r(\tilde{N}(q), q) \rightarrow 1$.⁵

Regarding the rest of the interest rates, observe that $r(n', q) < r(\tilde{N}(q), q)$ for all $n' > \tilde{N}(q)$, since from above interest rates are decreasing once we reach

⁵The intuition is as follows. From the expression for $W(N, q)$ in (4), we can see that under our assumptions $\tilde{N}(q)$ must grow sufficiently quickly that $[\pi_l + (1 - \pi_l)q]^{\tilde{N}(q)}$ approaches zero at the rate $1 - q$ as $q \rightarrow 1$. This will also be rapid enough to ensure that $[\pi_h + (1 - \pi_h)q]^{\tilde{N}(q)} \rightarrow 0$ (this can be seen by taking the logarithm of this expression). Finally, recall that the interest rate in question is $\frac{1 + p[\pi_h + (1 - \pi_h)q]^{\tilde{N}(q)}}{1 + \pi_h p[\pi_h + (1 - \pi_h)q]^{\tilde{N}(q)}}$.

a period in which high effort is exerted. So as $q \rightarrow 1$, we can conclude that

$$-r(\tilde{N}(q), q) - (1 - q) \times \sum_{n=0}^{\infty} [\pi_h + (1 - \pi_h)q]^n r(\tilde{N}(q) + 1 + n, q) \rightarrow -1 - \frac{1}{1 - \pi_h}.$$

As far as the other side of the inequality, we know that $r(N(0) + 1 + n, 0) > 1$ for all n ; this is not a function of q , moreover. So the righthand side of the inequality can also be bounded:

$$-r(N(0), 0) - \sum_{n=0}^{\infty} \pi_h^n r(N(0) + 1 + n, 0) < -r(N(0), 0) - \sum_{n=0}^{\infty} \pi_h^n \times 1 = -r(N(0), 0) - \frac{1}{1 - \pi_h}.$$

Since $r(N(0), 0) > 1$, this means that the inequality must be satisfied when q is sufficiently large.

In addition, since this inequality is slack, and since $r(\tilde{N}(q), q) \rightarrow 1$, it is easy to see that for q sufficiently close to 1 it must also be satisfied in the previous period — i.e., period $\tilde{N}(q) - 1$.

Thus for q sufficiently large, high effort is exerted in period $\tilde{N}(q) - 1$, and hence the welfare for q dominates that with perfect memory. \square

Although it was not needed for the proof of the main proposition, we have asserted that the cost of increasing q is an additional period of delay until agents begin to exert high effort. The following Lemma demonstrates this.

Lemma 3 $N(q)$ is (weakly) increasing in q for $\beta < 1$.

Proof: (Sketch) Suppose that this is not the case. That is, suppose that $N' \equiv N(q') < N(q) \equiv N$ for $q < q'$. We will show that there is nevertheless an equilibrium with q in which the agent begins to put in high effort in period N' , which leads to a contradiction (because of the renegotiation-proofness assumption).

Now, our assumption that $N > N'$ means that incentive-compatibility must be satisfied in period N' for q' but not for q . Recall the incentive-compatibility condition:

$$\frac{c}{\pi_h - \pi_l} \leq R - r(N', q') + \beta(1 - q')v(N', q')$$

We will now show that there is nevertheless an equilibrium in which the agent exerts high effort in period N' under q .

To see this, first note that since for both q and q' the agent exerts low effort for periods prior to n' , it is the case that since $q < q'$, there are strictly fewer risky agents in period N' with q than with q' ; this implies that $r(N', q) < r(N', q')$.

To show that high effort is incentive-compatible with q , we must also show that $(1 - q)v(N', q) \leq (1 - q')v(N', q')$. Now, recall that the assumption that $N(q') = N'$ (i.e. that high effort is first exerted in period N') implies that the continuation utility for the agent is given by:

$$v(N', q') = \sum_{n=1}^{\infty} (\pi_h + (1 - \pi_h)q')^n \beta^n (R - r(N' + n, q')).$$

Now, $(1 - q') \sum_{n=1}^{\infty} (\pi_h + (1 - \pi_h)q')^n \beta^n R < (1 - q) \sum_{n=1}^{\infty} (\pi_h + (1 - \pi_h)q)^n \beta^n R$ when $\beta < 1$. Furthermore, under the assumption that the agent also exerts high effort in period N' with q , $r(n, q') > r(n, q)$ for all n by the same argument made above.

So it is easy to see that this indeed implies that $(1 - q')v(N', q') < (1 - q)v(N', q)$, in which case it follows that there must be an equilibrium in which the agent first exerts high effort in period N' under q .

This implies that we must indeed have $N(q) \leq N(q')$. □

D Discussion

Forgetting a bankruptcy can be beneficial for two reasons. First of all, it relaxes the renegotiation-proofness condition which prevents agents from obtaining any financing once they are discovered to be of low quality. The other, related, benefit of forgetting is that by pooling the risky agents with the riskless ones, there is a cross-subsidy (in the form of lower interest rates), which improves the risky agents' incentives to exert high effort.

It is now also possible to discuss whether our results are consistent with the empirical evidence in Musto (2004), and whether we concur with his assessment that removing this flag is suboptimal. Musto (2004) finds that those who receive credit after their default is forgotten are likelier to default in their future and their credit quality (as measured by their FICO score) declines over time.

This is also a conclusion of our model. Both before and after period $N(q)$, the only defaulters are risky agents; when they are reinserted into the pool they are always more likely than the average to default in the future (since the pool always includes riskless entrepreneurs as well). Indeed, they will eventually default with probability 1, since $\pi_h < 1$.

Nevertheless, our Proposition suggest that this is not necessarily suboptimal. The reason is that while these agents are riskier than average, their projects can nevertheless have positive NPV when they are pooled anew, which would not be the case if they are separated.

E Is a Law Necessary?

We have shown that under certain circumstances it is optimal to forget a past default. It is not difficult to see that lenders would not do this on their own, but rather that a law is needed to enforce this. The reason is that a lender can always profit by refusing to lend to entrepreneurs whom he knows to

be bad; moreover, since we impose a period-by-period break-even constraint on lenders, there is no way for lenders to make up any losses in the current period with future rents.

Moreover, any agent who requested *ex-ante* that lenders forget a default would be signaling that he is of the risky type, and so would not obtain financing (hence no entrepreneur would suggest this); this is reminiscent of Aghion and Hermalin (1990), who argue that restricting the ability of agents to contract privately can sometimes be optimal, because otherwise agents would try to use the contractual form to inefficiently signal that they are of a good type.

IV Examples

In this section we present a few examples to illustrate the results of the previous section.

Let $R = 3$, $\pi_h = 0.5$, and $\pi_l = 0.3$. We will restrict attention to effort costs $c \in (0.4, 0.5]$. The reason is that in order for high effort to be efficient, we must have $c \leq 0.5$. For reputational effects to be needed to enforce incentive-compatibility (and hence for the examples to be interesting), we further restrict attention to $c > (\pi_h - \pi_l)(R - 1) = 0.4$.

- Suppose $c=0.456$ and $\beta = 0.96$.

Then IC is satisfied with $q = 0$ and $N = 0$ (i.e. high effort is exerted immediately with perfect memory). In fact it is easy to verify that IC binds here so one cannot raise q without hurting incentives.

Suppose we raise it to $q = 0.6$; then $N(q) = N(0.6) = 1$; i.e. there would be a delay of one period until interest rates drop sufficiently so that agents exert high effort. Similarly, $N(0.705) = 2$; i.e. this value of q would require a two-period delay.

As far as per-capita welfare $W(N, q)$ is concerned, we have

$$W(0, 0) > W(1, 0.6) > W(2, 0.705),$$

so $q = 0$ is socially optimal.

In Figure 1 we plot the incentive-compatibility constraints (in period N) as a function of q assuming high effort is delayed until period N (for $N = 1, 2, 3$). To be precise, the function graphed is $R - r + \beta(1 - q)v^S - \frac{c}{\pi_h - \pi_l}$, so for IC to hold this function must be nonnegative. We also plot welfare as a function of q ; notice that $q = 0$ is optimal.

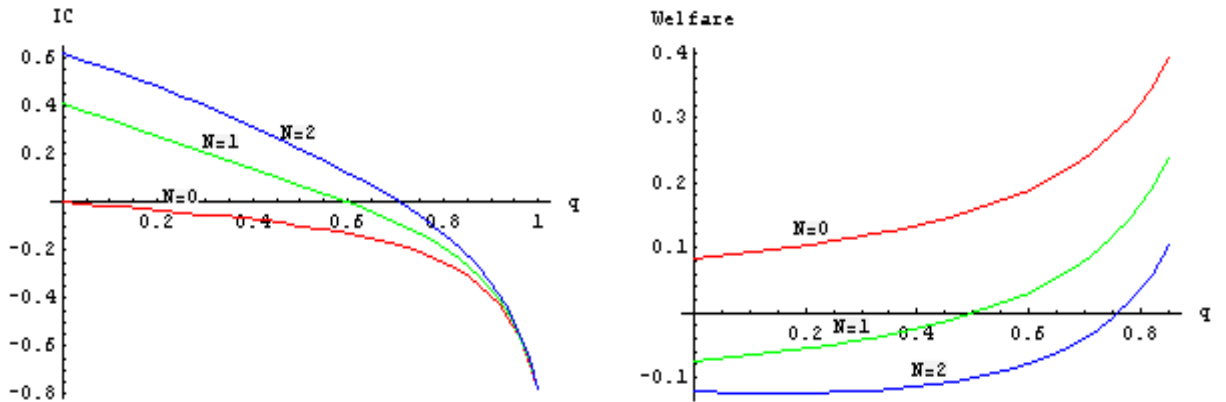


Figure 1: $q = 0$ optimal

- Now suppose that $\beta = 0.99$ and also $c = 0.4637$ (we raised c so that incentives would still bind with $q = 0$)

Agents are now more patient, so the delay due to low effort will be less costly. We will see that, as expected, this makes forgetting a default more attractive.

IC is once again satisfied with $q = 0$ and $N = 0$ (i.e. high effort immediately with perfect memory). We can also see from Figure 2 that $N = 1$ is consistent with $q = 0.7105$ and $N = 2$ with $q = 0.801$.

Welfare (per capita) is as follows:

- $N = 0, q = 0$: 0.0719
- $N = 1, q = 0.7105$: 0.0869
- $N = 2, q = 0.801$: 0.0569

Now the social optimum is $q = 0.7105$ (which leads to a 1-period delay for high effort in equilibrium). Figure 2 represents this example graphically.

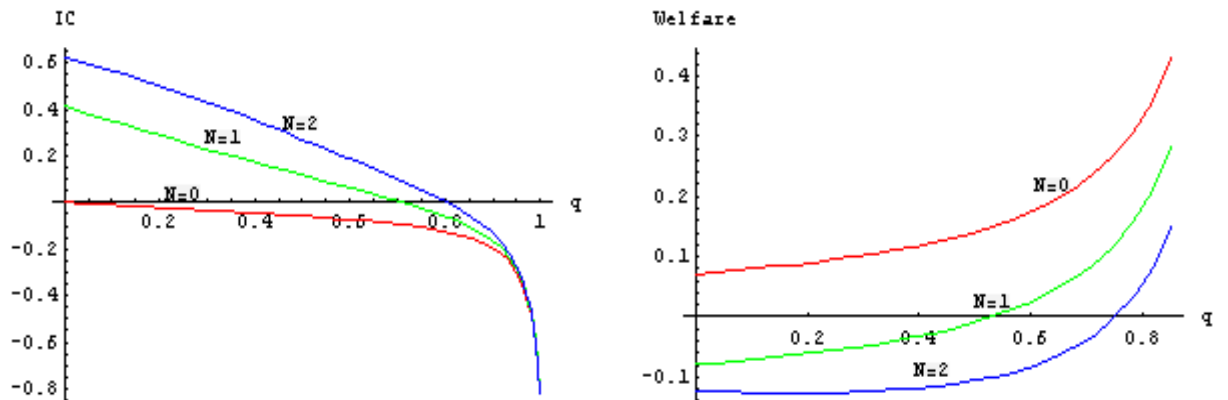


Figure 2: $q = 0.7105$ optimal

V Conclusion

We have developed a model in which entrepreneurs must repeatedly seek external funds to finance a sequence of risky projects under conditions of both adverse selection and moral hazard. We are interested in determining the optimal amount of “forgetting” in this economy; and in particular whether lenders should not be permitted to make use of past defaults. Forgetting a default makes incentives worse, *ex-ante*, because it reduces the punishment from failure. However, following a default it is generally good to forget, because pooling riskier agents with safer ones makes exerting high effort to preserve their (undeservedly good) reputation more attractive. The optimal policy trades off these effects.

Our key result is that if agents are sufficiently patient, and low effort is not too inefficient, then the optimal law would necessarily prescribe some amount of forgetting — that is, it would not permit lenders to fully utilize past information. We also show that this law must be enforced by the government — no lender would willingly agree to forget.

One direction in which this model might be extended is to explore the robustness to our assumptions. In particular, the assumption that only risky agents can fail means that when a default is observed (and remembered) then the defaulting agent is excluded. This clearly makes the model much more tractable. If the “riskless” agents could also default, then exclusion would no longer follow after the first failure, although experiencing sufficiently many failures would surely preclude further financing. Nevertheless, we conjecture that the qualitative nature of our results would not be that different — and that for reasonable parameter values forgetting would still be an optimal policy.

Another extension would involve understanding cross-country differences in laws governing the memory of the credit reporting system. As mentioned

in the introduction, European countries tend to forget defaults more quickly. This may be due to differences in the economic environment, or perhaps to differences in other laws governing bankruptcy.

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