# MATHEMATICS

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Courses offered by the Department of Mathematics have the subject code MATH, and are listed in the "Mathematics (MATH) Courses" section of this bulletin.

The Department of Mathematics offers programs leading to the degrees of Bachelor of Science, Master of Science, and Doctor of Philosophy in Mathematics, and participates in the program leading to the B.S. in Mathematical and Computational Science. The department also participates in the M.S. and Ph.D. degree programs in Scientific Computing and Computational Mathematics and the M.S. degree program in Financial Mathematics.

## UNDERGRADUATE PROGRAMS IN MATHEMATICS

### ADVANCED PLACEMENT IN MATHEMATICS FOR FRESHMEN

Students of unusual ability in mathematics often take one or more semesters of college-equivalent courses in mathematics while they are still in high school. Under certain circumstances, it is possible for such students to secure both advanced placement and credit toward the bachelor's degree. A decision as to placement and credit is made by the department after consideration of the student's performance on the Advanced Placement Examination in Mathematics (forms AB or BC) of the College Entrance Examination Board, and also after consideration of transfer credit in mathematics from other colleges and universities.

The department does not give its own advanced placement examination. Students can receive either 5 or 10 units of advanced placement credit, depending on their scores on the CEEB Advanced Placement Examination. Entering students who have credit for two quarters of single variable calculus (10 units) are encouraged to enroll in MATH 51-53 in multivariable mathematics, or the honors version 51H-53H. These three-course sequences, which can be completed during the freshman year, supply the necessary mathematics background for most majors in science and engineering. They also serve as excellent background for the major or minor in Mathematics, or in Mathematical and Computational Science. Students who have credit for one quarter of single variable calculus (5 units) should take MATH 42 in Autumn Quarter and 51 in Winter Quarter. Options available in Spring Quarter include MATH 52, or 53. For proper placement, contact the Department of Mathematics.

### BACHELOR OF SCIENCE IN MATHEMATICAL AND COMPUTATIONAL SCIENCE

The Department of Mathematics participates with the departments of Computer Science, Management Science and Engineering, and Statistics in a program leading to a B.S. in Mathematical and Computational Science. See the "Mathematical and Computational Science" section of this bulletin.

#### INTRODUCORY AND UNDERGRADUATE COURSES

The department offers two sequences of introductory courses in single variable calculus.

MATH 41, 42 present single variable calculus. Differential calculus is covered in the first quarter, integral calculus in the second.

1. MATH 19, 20, 21 cover the material in 41, 42 in three quarters instead of two.

There are options for studying multivariable mathematics:

MATH 51, 52, 53 cover differential and integral calculus in several variables, linear algebra, and ordinary differential equations. These topics are taught in an integrated fashion and emphasize application. MATH 51 covers differential calculus in several variables and introduces matrix theory and linear algebra; 52 covers integral calculus in several variables and vector analysis; 53 studies further topics in linear algebra and applies them to the study of ordinary differential equations. This sequence is strongly recommended for incoming freshmen with 10 units of advanced placement credit.

2. MATH 51H, 52H, 53H cover the same material as 51, 52, 53, but with more emphasis on theory and rigor.

The department offers four classes on linear algebra: 51 (or 51H), 104, 113, and 114.

### BACHELOR OF SCIENCE IN MATHEMATICS

The following department requirements are in addition to the University's basic requirements for the bachelor's degree:

Students wishing to major in Mathematics must satisfy the following requirements:

Department of Mathematics courses (other than MATH 100) totaling at least 49 units credit; such courses must be taken for a letter grade. For the purposes of this requirement, STATS 116, PHIL 151, and PHIL 152 count as Department of Mathematics courses.

3. Additional courses taken from Department of Mathematics courses numbered 101 and above or from approved courses in other disciplines with significant mathematical content, totaling at least 15 units credit. At least 9 of these units must be taken for a letter grade.
4. A Department of Mathematics adviser must be selected, and the courses selected under items '1' and '2' above must be approved by the department's director of undergraduate study, acting under guidelines laid down by the department's Committee for Undergraduate Affairs. The Department of Mathematics adviser can be any member of the department's faculty.
5. To receive the department's recommendation for graduation, a student must have been enrolled as a major in the Department of Mathematics for a minimum of two full quarters, including the quarter immediately before graduation. In any case, students are strongly encouraged to declare as early as possible, preferably by the end of the sophomore year.

Students are normally expected to complete either the sequence 19, 20, 21 or the sequence 41, 42 (but not both). Students with an Advanced Placement score of at least 4 in BC math or 5 in AB math may receive 10 units credit and fulfill requirement '1' by taking at least 39 units of Department of Mathematics courses numbered 51 and above. Students with an Advanced Placement score of at least 3 in BC math or at least 4 in AB math may receive 5 units credit and fulfill requirement '1' by taking at least 44 units of Department of Mathematics courses numbered 42 and above.

Sophomore seminar courses may be counted among the choice of courses under item '1'. Other variations of the course requirements laid down above (under items '1' and '2') may, in some circumstances, be allowed. For example, students transferring from other universities may be allowed credit for some courses completed before their arrival at Stanford. However, at least 24 units of the 49 units under item '1' above and 9 of the units under item '2' above must be taken at Stanford. In all cases, approval for variations in the degree requirements must be obtained from the department's Committee for Undergraduate Affairs. Application for such approval should be made through the department's director of undergraduate study. The policy of the Mathematics Department is that no courses other than the MATH 50 series and below may be double-counted toward any other University major or minor.

It is to be emphasized that the above regulations are minimum requirements for the major; students contemplating graduate work in mathematics are strongly encouraged to include the courses 116, 120, 121, 147 or 148, and 171 in their selection of courses, and in addition, take at least three Department of Mathematics courses over and above the minimum requirements laid out under items '1' and '2' above, including at least one 200-level course. Such students are also encouraged to consider the possibility of taking the honors program, discussed below.

To help develop a sense of the type of course selection (under items '1' and '2' above) that would be recommended for math majors with various backgrounds and interests, see the following examples. These represent only a few of a very large number of possible combinations of courses that could be taken in fulfillment of the Mathematics major requirements:

*Example 1*—A general program (a balanced program of both pure and applied components, without any particular emphasis on any one field of mathematics or applications) as follows:

A. either MATH 19, 20, and 21, or 41 and 42 (or satisfactory Advanced Placement credit); 51, 52, 53; 104 or 113; 106; 109; 110; 111; 115; 118
B. plus any selection of at least eight of the following courses, including three Department of Mathematics courses: MATH 108, 131, 132, 143, 146, 147, 148, 152, 161; CS 137; ECON 50; PHYSICS 41, 43, 45; STATS 116. These courses from other departments are only meant as examples; there are many suitable courses in several departments that can be taken to fulfill part or all of requirement '2.'

*Example 2*—A theoretical program recommended for those contemplating possible later graduate work providing an introduction to the main areas of mathematics both broader and deeper than the general program outlined above; see, also, the discussion of the honors program below:

C. either MATH 19, 20 and 21, or 41 and 42 (or satisfactory Advanced Placement credit)
D. either the sequence 51, 52, 53, or the sequence 51H, 52H, 53H; 106 or 116; 113; 120; 171
E. plus nine or more of the following courses, including at least one from each group: algebra sequence 114, 121, 152, 156; analysis sequence 131, 132, 135, 151, 174A,B, 175; geometry/topology sequence 143, 145, 146, 147, 148; logic and set theory sequence PHIL 151, 152; MATH 161.

In addition, those contemplating eventual graduate work in Mathematics should consider including at least one graduate-level math course such as MATH 205A, 210A, or 215A or B. Such students should also consider the possibility of entering the honors program.

*Example 3**—An applied mathematics program:

F. either MATH 19, 20, and 21; or 41 and 42 (or satisfactory Advanced Placement credit); 51, 52, 53; 104; 106; 108; 109; 110; 111; 115; 118; 131; STATS 116
G. plus at least 15 units of additional courses in Applied Mathematics, including, for example, suitable courses from the departments of Physics, Computer Science, Economics, Engineering, and Statistics.

* Students with interests in applied mathematics, but desiring a broader-based program than the type of program suggested in Example 3, including significant computational and/or financial and/or statistical components, are encouraged to also consider the Mathematics and Computational Science program.

### HONORS PROGRAM

The honors program is intended for students who have strong theoretical interests and abilities in mathematics. The goal of the program is to give students a thorough introduction to the main branches of mathematics, especially analysis, algebra, and geometry. Through the honors thesis, students may be introduced to a current or recent research topic, although occasionally more classical projects are encouraged. The program provides an excellent background with which to enter a master's or Ph.D. program in Mathematics. Students completing the program are awarded a B.S. in Mathematics with Honors.

It is recommended that the sequence 51H, 52H, 53H be taken in the freshman year. Students who have instead taken the sequence 51, 52, 53 in their freshman year may be permitted to enter the honors program, but such entry must be approved by the Department of Mathematics Committee for Undergraduate Affairs.

To graduate with a B.S. in Mathematics with Honors, the following conditions apply in addition to the usual requirements for math majors:

The selection of courses under items '1' and '2' above must include all the math courses 106 or 116, 120, 171 and also must include seven or more additional courses, with at least one from each of the groups: algebra sequence 114, 121, 152, 154, 155, 156; analysis sequence 131, 132, 135, 136, 151, 172, 174A, 174B, 175, 176; geometry/topology sequence 143, 145, 146, 147, 148; logic and set theory sequence PHIL 151, PHIL 152, and MATH 161.

6. Students in the honors program must write a senior thesis. In order to facilitate this, the student must, by the end of the junior year, choose an undergraduate thesis adviser from the Department of Mathematics faculty, and map out a concentrated reading program under the direction and guidance of the adviser. During the senior year, the student must enroll in MATH 197 for a total of 6 units (typically spread over two quarters), and work toward completion of the thesis under the direction and guidance of the thesis adviser. The thesis may contain original material, or be a synthesis of work in current or recent research literature. The 6 units of credit for MATH 197 are required in addition to the course requirements laid out under items '1' and '2' above and in addition to all other requirements for math majors.

In addition to the minimum requirements laid out above, it is strongly recommended that students take at least one graduate-level course (that is, at least one course in the 200 plus range). MATH 205A, 210A, and 215A or B are especially recommended in this context.

Students with questions about the honors program should see the director of undergraduate advising.

### MINOR IN MATHEMATICS

To qualify for the minor in Mathematics, a student should complete, for a letter grade, at least six Department of Mathematics courses (other than MATH 100) numbered 51 or higher, totaling a minimum of 24 units. It is recommended that these courses include either the sequence 51, 52, 53 or the sequence 51H, 52H, 53H. At least 12 of the units applied toward the minor in Mathematics must be taken at Stanford. The policy of the Mathematics Department is that no courses other than the MATH 50 series and below may be double-counted toward any other University major or minor.

## GRADUATE PROGRAMS IN MATHEMATICS

### TEACHING CREDENTIALS

For information concerning the requirements for teaching credentials, see the "School of Education" section of this bulletin or address inquiries to Credential Secretary, School of Education.

### MASTER OF SCIENCE IN MATHEMATICS

The University's basic requirements for the master's degree are discussed in the "Graduate Degrees" section of this bulletin. Students should pay particular attention to the University's course requirements for graduate degrees. The following are specific departmental requirements:

Candidates must complete an approved course program of 45 units of courses beyond the department requirements for the B.S. degree, of which at least 36 units must be Mathematics Department courses, taken for a letter grade. The Mathematics courses must include at least 18 units numbered 200 or above. The candidate must have a grade point average (GPA) of 3.0 (B) over all course work taken in Mathematics, and a GPA of 3.0 (B) in the 200-level courses considered separately. Course work for the M.S. degree must be approved during the first quarter of enrollment in the program by the department's Director of Graduate Studies.

For the M.S. degree in Financial Mathematics, see the "Financial Mathematics" section of this bulletin.

### DOCTOR OF PHILOSOPHY IN MATHEMATICS

The University's basic requirements for the doctorate (residence, dissertation, examinations, etc.) are discussed in the "Graduate Degrees" section of this bulletin. The following are specific departmental requirements.

To be admitted to candidacy, the student must have successfully completed 27 units of graduate courses (that is, courses numbered 200 and above). In addition, the student must pass qualifying examinations given by the department.

Beyond the requirements for candidacy, the student must complete a course of study approved by the Graduate Affairs Committee of the Department of Mathematics and submit an acceptable dissertation. In accordance with University requirements, Ph.D. students must complete a total of 135 course units beyond the bachelor's degree. These courses should be Department of Mathematics courses or approved courses from other departments. The course program should display substantial breadth in mathematics outside the student's field of application. The student must receive a grade point average (GPA) of 3.0 (B) or better in courses used to satisfy the Ph.D. requirement. In addition, the student must pass the Department area examination and the University oral examination.

Experience in teaching is emphasized in the Ph.D. program. Each student is required to complete nine quarters of such experience. The nature of the teaching assignment for each of those quarters is determined by the department in consultation with the student. Typical assignments include teaching or assisting in teaching an undergraduate course or lecturing in an advanced seminar.

For further information concerning degree programs, fellowships, and assistantships, inquire of the academic associate of the department.

### PH.D. MINOR IN MATHEMATICS

The student should complete both of the following:*

MATH 106 *or* 116, 131, 132

7. MATH 113, 114, 120 *or* 152

These courses may have been completed during undergraduate study, and their equivalents from other universities are acceptable.

In addition, the student should complete 21 units of 200-level courses in Mathematics. These must be taken at Stanford and approved by the Department of Mathematics Ph.D. minor adviser.

* A third coherent sequence designed by the student, subject to the approval of the graduate committee, may be considered as a substitute for items '1' or '2'.

# MATHEMATICS (MATH) COURSES

For information on undergraduate and graduate programs in the Department of Mathematics see the "Mathematics" section of this bulletin.

## UNDERGRADUATE COURSES IN MATHEMATICS

The department offers two sequences of introductory courses in single variable calculus. MATH 41,42 present single variable calculus; differential calculus is covered in the first quarter, integral calculus in the second. MATH 19,20,21 cover the material in 41,42 in three quarters instead of two. There are two options for studying multivariable mathematics. MATH 51,52,53 cover differential and integral calculus in several variables, linear algebra, and ordinary differential equations. These topics are taught in an integrated fashion and emphasize application. MATH 51 covers differential calculus in several variables and introduces matrix theory and linear algebra, 52 covers integral calculus in several variables and vector analysis, 53 studies further topics in linear algebra and applies them to the study of ordinary differential equations. This sequence is recommended for incoming freshmen with 10 units of advanced placement credit. MATH 51H,52H,53H cover the same material as 51,52,53, but with more emphasis on theory and rigor. The introductory course in linear algebra is 103 or 113. There are no formal prerequisites for these courses, but appropriate mathematical maturity is expected. Much of the material in 103 is covered in the sequence 51,52,53.

**MATH 15. Overview of Mathematics**
Broad survey of mathematics; its nature and role in society. GER:DB-Math
*3 units, not given this year*

**MATH 19. Calculus**
The content of MATH 19, 20, 21 is the same as the sequence MATH 41, 42 described below, but covered in three quarters, rather than two. GER:DB-Math
*3 units, Aut (Butscher, A), Win (Staff), Sum (Staff)*

**MATH 20. Calculus**
Continuation of 19. Prerequisite: 19. GER:DB-Math
*3 units, Win (Butscher, A), Spr (Staff)*

**MATH 21. Calculus**
Continuation of 20. Prerequisite: 20. GER:DB-Math
*4 units, Spr (Butscher, A)*

**MATH 41. Calculus**
Introduction to differential and integral calculus of functions of one variable. Topics: review of elementary functions including exponentials and logarithms, rates of change, and the derivative. Introduction to the definite integral and integration. Prerequisites: algebra, trigonometry. GER:DB-Math
*5 units, Aut (Lucianovic, M)*

**MATH 41A. Calculus ACE**
Students attend MATH 41 lectures with different recitation sessions, four hours instead of two, emphasizing engineering applications. Prerequisite: application; see http://soe.stanford.edu/edp/programs/ace.html. GER:DB-Math
*6 units, Aut (Lucianovic, M)*

**MATH 42. Calculus**
Continuation of 41. Methods of symbolic and numerical integration, applications of the definite integral, introduction to differential equations. Infinite series. Prerequisite: 41 or equivalent. GER:DB-Math
*5 units, Aut (Staff), Win (Lucianovic, M)*

**MATH 42A. Calculus ACE**
Students attend MATH 41 lectures with different recitation sessions, four hours instead of two, emphasizing engineering applications. Prerequisite: application; see http://soe.stanford.edu/edp/programs/ace.html. GER:DB-Math
*6 units, Aut (Staff), Win (Lucianovic, M)*

**MATH 51. Linear Algebra and Differential Calculus of Several Variables**
Geometry and algebra of vectors, systems of linear equations, matrices, vector valued functions and functions of several variables, partial derivatives, gradients, chain rule in several variables, vector fields, optimization. Prerequisite: 21, 42, or a score of 4 on the BC Advanced Placement exam or 5 on the AB Advanced Placement exam, or consent of instructor. GER:DB-Math
*5 units, Aut (Staff), Win (Staff), Spr (Staff), Sum (Staff)*

**MATH 51A. Linear Algebra and Differential Calculus of Several Variables, ACE**
Students attend MATH 51 lectures with different recitation sessions: four hours per week instead of two, emphasizing engineering applications. Prerequisite: application; see http://soe.stanford.edu/edp/programs/ace.html. GER:DB-Math
*6 units, Aut (Staff), Win (Staff), Spr (Staff)*

**MATH 51H. Honors Multivariable Mathematics**
For prospective Mathematics majors in the honors program and students from other areas of science or engineering who have a strong mathematics background. Three quarter sequence covers the material of 51, 52, 53, and additional advanced calculus and ordinary

and partial differential equations. Unified treatment of multivariable calculus, linear algebra, and differential equations with a different order of topics and emphasis from standard courses. Students should know one-variable calculus and have an interest in a theoretical approach to the subject. Prerequisite: score of 5 on BC Advanced Placement exam, or consent of instructor. GER:DB-Math

*5 units, Aut (Simon, L)*

**MATH 51M. Introduction to MATLAB for Multivariable Mathematics**

Corequisite: MATH 51.

*1 unit, Aut (Staff)*

**MATH 52. Integral Calculus of Several Variables**

Iterated integrals, line and surface integrals, vector analysis with applications to vector potentials and conservative vector fields, physical interpretations. Divergence theorem and the theorems of Green, Gauss, and Stokes. Prerequisite: 51. GER:DB-Math

*5 units, Aut (Tzou, L), Win (Kerckhoff, S), Spr (Brumfiel, G)*

**MATH 52H. Honors Multivariable Mathematics**

Continuation of 51H. Prerequisite: 51H. GER:DB-Math

*5 units, Win (Simon, L)*

**MATH 53. Ordinary Differential Equations with Linear Algebra**

Linear ordinary differential equations, applications to oscillations, matrix methods including determinants, eigenvalues and eigenvectors, matrix exponentials, systems of linear differential equations with constant coefficients, stability of non-linear systems and phase plane analysis, numerical methods, Laplace transforms. Integrated with topics from linear algebra (103). Prerequisite: 51. GER:DB-Math

*5 units, Aut (Lisi, S), Win (Iyer, G), Spr (Staff), Sum (Staff)*

**MATH 53H. Honors Multivariable Mathematics**

Continuation of 52H. Prerequisite: 52H. GER:DB-Math

*5 units, Spr (Brendle, S)*

**MATH 80Q. Capillary Surfaces: Explored and Unexplored Territory**

Stanford Introductory Seminar. Preference to sophomores. Capillary surfaces: the interfaces between fluids that are adjacent to each other and do not mix. Recently discovered phenomena, predicted mathematically and subsequently confirmed by experiments, some done in space shuttles. Interested students may participate in ongoing investigations with affinity between mathematics and physics.

*3 units, Win (Finn, R)*

**MATH 87Q. Mathematics of Knots, Braids, Links, and Tangles**

Stanford Introductory Seminar. Preference to sophomores. Types of knots and how knots can be distinguished from one another by means of numerical or polynomial invariants. The geometry and algebra of braids, including their relationships to knots. Topology of surfaces. Brief summary of applications to biology, chemistry, and physics.

*3 units, Spr (Wieczorek, W)*

**MATH 88Q. The Mathematics of the Rubik's Cube**

Stanford Introductory Seminar. Preference to sophomores. Group theory through topics that can be illustrated with the Rubik's cube: subgroups, homomorphisms and quotient groups, the symmetric and alternating groups, conjugation, commutators, and Sylow subgroups.

*3 units, Win (Bump, D)*

**MATH 100. Mathematics for Elementary School Teachers**

Mathematics and pedagogical strategies. Core mathematical content in grades K-6, classroom presentation, how to handle student errors, and mathematical issues that come up during instruction.

*4 units, Spr (Milgram, R)*

**MATH 104. Applied Matrix Theory**

Linear algebra for applications in science and engineering: orthogonality, projections, the four fundamental subspaces of a matrix, spectral theory for symmetric matrices, the singular value decomposition, the QR decomposition, least-squares, the condition number of a matrix, algorithms for solving linear systems. Prerequisites: MATH 51 and MATH 52 or 53. GER:DB-Math

*3 units, Aut (Demanet, L), Win (Liu, T)*

**MATH 106. Functions of a Complex Variable**

Complex numbers, analytic functions, Cauchy-Riemann equations, complex integration, Cauchy integral formula, residues, elementary conformal mappings. Prerequisite: 52. GER:DB-Math

*3 units, Win (Nedelec, L), Sum (Brumfiel, G)*

**MATH 108. Introduction to Combinatorics and Its Applications**

Topics: graphs, trees (Cayley's Theorem, application to phylogony), eigenvalues, basic enumeration (permutations, Stirling and Bell numbers), recurrences, generating functions, basic asymptotics. Prerequisites: 51 or 103 or equivalent. GER:DB-Math

*3 units, Spr (Kahle, M)*

**MATH 109. Applied Group Theory**

Applications of the theory of groups. Topics: elements of group theory, groups of symmetries, matrix groups, group actions, and applications to combinatorics and computing. Applications: rotational symmetry groups, the study of the Platonic solids, crystallographic groups and their applications in chemistry and physics. GER:DB-Math, WIM

*3 units, Aut (Ionel, E)*

**MATH 110. Applied Number Theory and Field Theory**

Number theory and its applications to modern cryptography. Topics: congruences, finite fields, primality testing and factorization, public key cryptography, error correcting codes, and elliptic curves, emphasizing algorithms. GER:DB-Math

*3 units, Spr (Lucianovic, M)*

**MATH 111. Computational Commutative Algebra**

Introduction to the theory of commutative rings, ideals, and modules. Systems of polynomial equations in several variables from the algorithmic viewpoint. Groebner bases, Buchberger's algorithm, elimination theory. Applications to algebraic geometry and to geometric problems. GER:DB-Math

*3 units, not given this year*

**MATH 113. Linear Algebra and Matrix Theory**

Algebraic properties of matrices and their interpretation in geometric terms. The relationship between the algebraic and geometric points of view and matters fundamental to the study and solution of linear equations. Topics: linear equations, vector spaces, linear dependence, bases and coordinate systems; linear transformations and matrices; similarity; eigenvectors and eigenvalues; diagonalization. GER:DB-Math

*3 units, Aut (Staff), Win (Cohen, R), Spr (Tzou, L)*

**MATH 114. Linear Algebra and Matrix Theory II**

Advanced topics in linear algebra such as: invariant subspaces; canonical forms of matrices; minimal polynomials and elementary divisors; vector spaces over arbitrary fields; inner products; Jordan normal forms; Hermitian and unitary matrices; multilinear algebra; and applications. Prerequisite: 51H or 113. GER:DB-Math

*3 units, Spr (Katznelson, Y)*

**MATH 115. Functions of a Real Variable**

The development of real analysis in Euclidean space: sequences and series, limits, continuous functions, derivatives, integrals. Basic point set topology. Honors math majors and students who intend to do graduate work in mathematics should take 171. Prerequisite: 51. GER:DB-Math

*3 units, Aut (Toussaint, A), Win (Katznelson, Y), Sum (Brumfiel, G)*

**MATH 116. Complex Analysis**

Analytic functions, Cauchy integral formula, power series and Laurent series, calculus of residues and applications, conformal mapping, analytic continuation, introduction to Riemann surfaces, Fourier series and integrals. Prerequisites: 52, and 115 or 171. GER:DB-Math

*3 units, Win (Li, J)*

**MATH 118. Mathematics of Computation**

Notions of analysis and algorithms central to modern scientific computing: continuous and discrete Fourier expansions, the fast Fourier transform, orthogonal polynomials, interpolation, quadrature, numerical differentiation, analysis and discretization of initial-value and boundary-value ODE, finite and spectral elements. Prerequisites: MATH 51 and 53. GER:DB-Math

*3 units, Win (Demanet, L)*

**MATH 120. Modern Algebra**
Basic structures in algebra: groups, rings, and fields. Elements of group theory: permutation groups, finite Abelian groups, p-groups, Sylow theorems. Polynomial rings, principal ideal domains, unique factorization domains. GER:DB-Math, WIM
*3 units, Aut (Vakil, R), Spr (Soundararajan, K)*

**MATH 121. Modern Algebra II**
Continuation of 120. Fields of fractions. Solvable and simple groups. Elements of field theory and Galois theory. Prerequisite: 120. GER:DB-Math
*3 units, Win (Staff)*

**MATH 131M. Partial Differential Equations I**
More theoretical version of MATH 131P; suitable for Mathematics majors and mathematically inclined Physics majors . Topics include first order equations, physical and mathematical sources of PDE's, method of characteristics, D'Alembert's formula, maximum principles, heat kernel, Duhamel's principle, separation of variables, and Fourier series. Introduction to PDE in multiple space dimensions. Emphasis is on a mathematically rigorous treatment of the subject. Students may not take both 131M and 131P. Prerequisite: 53. GER:DB-Math
*3 units, Win (Schoen, R)*

**MATH 131P. Partial Differential Equations I**
A more applied version of MATH 131M; suitable for non-Math majors. Topics include physical examples of PDE's, method of characteristics, D'Alembert's formula, maximum principles, heat kernel, Duhamel's principle, separation of variables, Fourier series, Harmonic functions, Bessel functions, spherical harmonics. Students may not take both 131M and 131P. Prerequisite: 53. GER:DB-Math
*3 units, Aut (Iyer, G)*

**MATH 132. Partial Differential Equations II**
Laplace's equation and properties of harmonic functions. Green's functions. Distributions and Fourier transforms. Eigenvalue problems and generalized Fourier series. Numerical solutions. Prerequisite: 131M or 131P (formerly 131). GER:DB-Math
*3 units, Spr (Nedelec, L)*

**MATH 135. Nonlinear Dynamics and Chaos**
Topics: one- and two-dimensional flows, bifurcations, phase plane analysis, limit cycles and their bifurcations. Lorenz equations, fractals and strange attractors. Prerequisite: 51 and 53 or equivalent. GER:DB-Math
*3 units, not given this year*

**MATH 136. Stochastic Processes**
(Same as STATS 219.) Introduction to measure theory, Lp spaces and Hilbert spaces. Random variables, expectation, conditional expectation, conditional distribution. Uniform integrability, almost sure and Lp convergence. Stochastic processes: definition, stationarity, sample path continuity. Examples: random walk, Markov chains, Gaussian processes, Poisson processes, Martingales. Construction and basic properties of Brownian motion. Prerequisite: STATS 116 or MATH 151 or equivalent. Recommended: MATH 115 or equivalent. GER:DB-Math
*3 units, Aut (Ross, K)*

**MATH 137. Mathematical Methods of Classical Mechanics**
Newtonian mechanics. Lagrangian formalism. E. Noether's theorem. Oscillations. Rigid bodies. Introduction to symplectic geometry. Hamiltonian formalism. Legendre transform. Variational principles. Geometric optics. Introduction to the theory of integrable systems. Prerequisites: 51, 52, 53, or 51H, 52H, 53H. GER:DB-Math
*3 units, Win (Eliashberg, Y)*

**MATH 138. Celestial Mechanics**
Mathematically rigorous introduction to the classical N-body problem: the motion of N particles evolving according to Newton's law. Topics include: the Kepler problem and its symmetries; other central force problems; conservation theorems; variational methods; Hamilton-Jacobi theory; the role of equilibrium points and stability; and symplectic methods. Prerequisites: 53, and 115 or 171. GER:DB-Math
*3 units, not given this year*

**MATH 143. Differential Geometry**
Geometry of curves and surfaces in three-space and higher dimensional manifolds. Parallel transport, curvature, and geodesics. Surfaces with constant curvature. Minimal surfaces. GER:DB-Math
*3 units, Win (Nedelec, L)*

**MATH 145. Algebraic Geometry**
Real algebraic curves, Hilbert's nullstellensatz, complex affine and projective curves, Bezout's theorem, the degree/genus formula, Riemann surfaces, Riemann-Roch theorem. Prerequisites: 106 or 116, and 109 or 120. Recommended: familiarity with surfaces equivalent to 143, 146, 147, or 148. GER:DB-Math
*3 units, Spr (Li, J)*

**MATH 146. Analysis on Manifolds**
Differentiable manifolds, tangent space, submanifolds, implicit function theorem, differential forms, vector and tensor fields. Frobenius' theorem, DeRham theory. Prerequisite: 52 or 52H. GER:DB-Math
*3 units, Win (Ionel, E)*

**MATH 147. Differential Topology**
Smooth manifolds, transversality, Sards' theorem, embeddings, degree of a map, Borsuk-Ulam theorem, Hopf degree theorem, Jordan curve theorem. Prerequisite: 115 or 171. GER:DB-Math
*3 units, Spr (Staff), alternate years, not given this year*

**MATH 148. Algebraic Topology**
Fundamental group, covering spaces, Euler characteristic, homology, classification of surfaces, knots. Prerequisite: 109 or 120. GER:DB-Math
*3 units, Spr (Diaconis, P), alternate years, not given next year*

**MATH 151. Introduction to Probability Theory**
Counting; axioms of probability; conditioning and independence; expectation and variance; discrete and continuous random variables and distributions; joint distributions and dependence; central limit theorem and laws of large numbers. Prerequisite: 52 or consent of instructor. GER:DB-Math
*3 units, Win (Carlsson, G)*

**MATH 152. Elementary Theory of Numbers**
Euclid's algorithm, fundamental theorems on divisibility; prime numbers, congruence of numbers; theorems of Fermat, Euler, Wilson; congruences of first and higher degrees; Lagrange's theorem and its applications; quadratic residues; introduction to the theory of binary quadratic forms. GER:DB-Math
*3 units, Aut (Soundararajan, K)*

**MATH 154. Algebraic Number Theory**
Properties of number fields and Dedekind domains, quadratic and cyclotomic fields, applications to some classical Diophantine equations; introduction to elliptic curves. Prerequisites: 120, 121. GER:DB-Math
*3 units, Win (Conrad, B), alternate years, not given next year*

**MATH 155. Analytic Number Theory**
Topics such as the distribution of prime numbers, the prime number theorem, twin primes and Goldbach's conjecture, the theory of quadratic forms, Dirichlet's class number formula, Dirichlet's theorem on primes in arithmetic progressions, and the fifteen theorem. Prerequisite: 152, or familiarity with the Euclidean algorithm, congruences, residue classes and reduced residue classes, primitive roots, and quadratic reciprocity. GER:DB-Math
*3 units, alternate years, not given this year*

**MATH 156. Group Representations**
Group representations and their characters, classification of permutation group representations using partitions and Young tableaux, group actions on sets and the Burnside ring, and spherical space forms. Applications to geometric group actions and to combinatorics. Prerequisites: linear algebra (51 and 53, or 103 or 113) and group theory (109 or 120). GER:DB-Math
*3 units, Spr (Staff)*

**MATH 161. Set Theory**
Informal and axiomatic set theory: sets, relations, functions, and set-theoretical operations. The Zermelo-Fraenkel axiom system and the special role of the axiom of choice and its various equivalents. Well-orderings and ordinal numbers; transfinite induction and transfinite recursion. Equinumerosity and cardinal numbers; Cantor's Alephs and cardinal arithmetic. Open problems in set theory. GER:DB-Math
*3 units, Spr (Kahle, M)*

**MATH 162. Philosophy of Mathematics**
(Same as PHIL 162, PHIL 262. Graduate students register for PHIL 262.) 20th-century approaches to the foundations and philosophy of mathematics. The background in mathematics, set theory, and logic. Schools and programs of logicism, predicativism, platonism, formalism, and constructivism. Readings from leading thinkers. Prerequisite: PHIL151 or consent of instructor.
*4 units, not given this year*

**MATH 171. Fundamental Concepts of Analysis**
Recommended for Mathematics majors and required of honors Mathematics majors. Similar to 115 but altered content and more theoretical orientation. Properties of Riemann integrals, continuous functions and convergence in metric spaces; compact metric spaces, basic point set topology. Prerequisites: 51 and 52, or 51H and 52H. GER:DB-Math, WIM
*3 units, Aut (Ryzhik, L), Spr (Vasy, A)*

**MATH 172. Lebesgue Integration and Fourier Analysis**
Similar to 205A, but for undergraduate Math majors and graduate students in other disciplines. Topics include Lebesgue measure on Euclidean space, Lebesgue integration, L^p spaces, the Fourier transform, the Hardy-Littlewood maximal function and Lebesgue differentiation. Prerequisite: 171 or consent of instructor. GER:DB-Math
*3 units, Spr (Iyer, G)*

**MATH 174A. Topics in Analysis and Differential Equations with Applications**
For students planning graduate work in mathematics or physics, and for honors math majors and other students at ease with rigorous proofs and qualitative discussion. Topics may include: geometric theory of ODE's with applications to dynamics; mathematical foundations of classical mechanics including variational principles, Lagrangian and Hamiltonian formalisms, theory of integrable systems; theorems of existence and uniqueness; Sturm-Liouville theory. Prerequisite: 53H or 171, or consent of instructor. GER:DB-Math
*3 units, not given this year*

**MATH 174B. Honors Analysis**
Continuation of 174A. Topics may include: introduction to PDEs including transport equations, Laplace, wave, and heat equations; techniques of solution including separation of variables and Green's functions; Fourier series and integrals; introduction to the theory of distributions; mathematical foundations of quantum mechanics. Prerequisite: 174A. GER:DB-Math
*3 units, not given this year*

**MATH 175. Elementary Functional Analysis**
Linear operators on Hilbert space. Spectral theory of compact operators; applications to integral equations. Elements of Banach space theory. Prerequisite: 115 or 171. GER:DB-Math
*3 units, Spr (Simon, L)*

**MATH 180. Introduction to Financial Mathematics**
Financial derivatives: contracts and options. Hedging and risk management. Arbitrage, interest rate, and discounted value. Geometric random walk and Brownian motion as models of risky assets. Initial boundary value problems for the heat and related partial differential equations. Self-financing replicating portfolio. Black-Scholes pricing of European options. Dividends. Implied volatility. Optimal stopping and American options. Prerequisite: 53. Corequisites: 131, 151 or STATS 116. GER:DB-Math
*3 units, Aut (Toussaint, A)*

**MATH 197. Senior Honors Thesis**
*1-6 units, Aut (Staff), Win (Staff), Spr (Staff)*

**MATH 199. Independent Work**
Undergraduates pursue a reading program; topics limited to those not in regular department course offerings. Credit can fulfill the elective requirement for math majors. Approval of Undergraduate Affairs Committee is required to use credit for honors majors area requirement.
*1-3 units, Aut (Staff), Win (Staff), Spr (Staff)*

## GRADUATE COURSES IN MATHEMATICS

Primarily for graduate students; undergraduates may enroll with consent of instructor.

**MATH 205A. Real Analysis**
Basic measure theory and the theory of Lebesgue integration. Prerequisite: 171 or equivalent.
*3 units, Aut (Ryzhik, L)*

**MATH 205B. Real Analysis**
Point set topology, basic functional analysis, Fourier series, and Fourier transform. Prerequisites: 171 and 205A or equivalent.
*3 units, Win (Vasy, A)*

**MATH 205C. Real Analysis**
Continuation of 205B.
*3 units, Spr (Katznelson, Y)*

**MATH 210A. Modern Algebra**
Groups, rings, and fields; introduction to Galois theory. Prerequisite: 120 or equivalent.
*3 units, Aut (Milgram, R)*

**MATH 210B. Modern Algebra**
Galois theory. Ideal theory, introduction to algebraic geometry and algebraic number theory. Prerequisite: 210A.
*3 units, Win (Brumfiel, G)*

**MATH 210C. Modern Algebra**
Continuation of 210B. Representations of groups and noncommutative algebras, multilinear algebra.
*3 units, Spr (Bump, D)*

**MATH 215A. Complex Analysis, Geometry, and Topology**
Analytic functions, complex integration, Cauchy's theorem, residue theorem, argument principle, conformal mappings, Riemann mapping theorem, Picard's theorem, elliptic functions, analytic continuation and Riemann surfaces.
*3 units, Aut (Li, J)*

**MATH 215B. Complex Analysis, Geometry, and Topology**
Topics: fundamental group and covering spaces, homology, cohomology, products, basic homotopy theory, and applications. Prerequisites: 113, 120, and 171, or equivalent; 215A is not a prerequisite for 215B.
*3 units, Win (Galatius, S)*

**MATH 215C. Complex Analysis, Geometry, and Topology**
Differentiable manifolds, transversality, degree of a mapping, vector fields, intersection theory, and Poincare duality. Differential forms and the DeRham theorem. Prerequisite: 215B or equivalent.
*3 units, Spr (Cohen, R)*

**MATH 216A. Introduction to Algebraic Geometry**
Algebraic curves, algebraic varieties, sheaves, cohomology, Riemann-Roch theorem. Classification of algebraic surfaces, moduli spaces, deformation theory and obstruction theory, the notion of schemes. May be repeated for credit.
*3 units, not given this year*

**MATH 216B. Introduction to Algebraic Geometry**
Continuation of 216A. May be repeated for credit.
*3 units, not given this year*

**MATH 217A. Differential Geometry**
Smooth manifolds and submanifolds, tensors and forms, Lie and exterior derivative, DeRham cohomology, distributions and the Frobenius theorem, vector bundles, connection theory, parallel transport and curvature, affine connections, geodesics and the exponential map, connections on the principal frame bundle. Prerequisite: 215C or equivalent.
*3 units, Win (Schoen, R)*

**MATH 217B. Differential Geometry**
Riemannian manifolds, Levi-Civita connection, Riemann curvature tensor, Riemannian exponential map and geodesic normal coordinates, Jacobi fields, completeness, spaces of constant curvature, bi-invariant metrics on compact Lie groups, symmetric and locally symmetric spaces, equations for Riemannian submanifolds and Riemannian submersions. Prerequisite: 217A.
*3 units, Spr (Brendle, S)*

**MATH 220. Partial Differential Equations of Applied Mathematics**
(Same as CME 303.) First-order partial differential equations; method of characteristics; weak solutions; elliptic, parabolic, and hyperbolic equations; Fourier transform; Fourier series; and eigenvalue problems. Prerequisite: foundation in multivariable calculus and ordinary differential equations.
*3 units, Aut (Nolen, J)*

**MATH 221. Mathematical Methods of Imaging**
Mathematical methods of imaging: array imaging using Kirchhoff migration and beamforming, resolution theory for broad and narrow band array imaging in homogeneous media, topics in high-frequency, variable background imaging with velocity estimation, interferometric imaging methods, the role of noise and inhomogeneities, and variational problems that arise in optimizing the performance of imaging algorithms and the deblurring of images. Prerequisite: 220.
*3 units, not given this year*

**MATH 222. Computational Methods for Fronts, Interfaces, and Waves**
High-order methods for multidimensional systems of conservation laws and Hamilton-Jacobi equations (central schemes, discontinuous Galerkin methods, relaxation methods). Level set methods and fast marching methods. Computation of multi-valued solutions. Multi-scale analysis, including wavelet-based methods. Boundary schemes (perfectly matched layers). Examples from (but not limited to) geometrical optics, transport equations, reaction-diffusion equations, imaging, and signal processing.
*3 units, not given this year*

**MATH 224. Topics in Mathematical Biology**
Mathematical models for biological processes based on ordinary and partial differential equations. Topics: population and infectious diseases dynamics, biological oscillators, reaction diffusion models, biological waves, and pattern formation. Prerequisites: 53 and 131, or equivalents.
*3 units, not given this year*

**MATH 227. Partial Differential Equations and Diffusion Processes**
Parabolic and elliptic partial differential equations and their relation to diffusion processes. First order equations and optimal control. Emphasis is on applications to mathematical finance. Prerequisites: MATH 131 and MATH 136/STATS 219, or equivalents.
*3 units, Win (Ryzhik, L)*

**MATH 228A. Ergodic Theory**
Measure preserving transformations and flows, ergodic theorems, mixing properties, spectrum, Kolmogorov automorphisms, entropy theory. Examples. Classical dynamical systems, mostly geodesic and horocycle forms on homogeneous spaces of SL(2,R). May be repeated for credit. Prerequisites: 205A,B.
*3 units, not given this year*

**MATH 230A. Theory of Probability**
(Same as STATS 310A.) Mathematical tools: asymptotics, metric spaces; measure and integration; Lp spaces; some Hilbert spaces theory. Probability: independence, Borel-Cantelli lemmas, almost sure and Lp convergence, weak and strong laws of large numbers. Weak convergence and characteristic functions; central limit theorems; local limit theorems; Poisson convergence. Prerequisites: 116, MATH 171.
*2-4 units, Aut (Diaconis, P)*

**MATH 230B. Theory of Probability**
(Same as STATS 310B.) Stopping times, 0-1 laws, Kolmogorov consistency theorem. Uniform integrability. Radon-Nikodym theorem, branching processes, conditional expectation, discrete time martingales. Exchangeability. Large deviations. Laws of the iterated logarithm. Birkhoff's and Kingman's ergodic theorems. Recurrence, entropy. Prerequisite: 310A or MATH 230A.
*2-4 units, Win (Dembo, A)*

**MATH 230C. Theory of Probability**
(Same as STATS 310C.) Infinitely divisible laws. Continuous time martingales, random walks and Brownian motion. Invariance principle. Markov and strong Markov property. Processes with stationary independent increments. Prerequisite: 310B or MATH 230B.
*2-4 units, Spr (Dembo, A)*

**MATH 231A. An Introduction to Random Matrix Theory**
(Same as STATS 351A.) Patterns in the eigenvalue distribution of typical large matrices, which also show up in physics (energy distribution in scattering experiments), combinatorics (length of longest increasing subsequence), first passage percolation and number theory (zeros of the zeta function). Classical compact ensembles (random orthogonal matrices). The tools of determinental point processes.
*3 units, Aut (Diaconis, P)*

**MATH 231B. The Spectrum of Large Random Matrices**
Asymptotics of eigenvalues of large random matrices, focusing on Wigner matrices and the Gaussian unitary ensemble: the combinatorics of non-crossing partitions and word graphs, concentration inequalities, Cauchy-Stieltjes transform, Hermite polynomials, Fredholm determinants, Laplace asymptotic method, special functions (Airy, Painleve), and stochastic calculus. Prerequisities: STATS 310A or MATH 205A.
*3 units, Win (Dembo, A)*

**MATH 231C. Free Probability**
Background from operator theory, addition and multiplication theorems for operators, spectral properties of infinite-dimensional operators, the free additive and multiplicative convolutions of probability measures and their classical counterparts, asymptotic freeness of large random matrices, and free entropy and free dimension. Prerequisite: STATS 310B or equivalent.
*3 units, Spr (Staff)*

**MATH 232. Topics in Probability: Malliavin Calculus, Fractional Brownian Motion and Applications**
Malliavin calculus: derivative and divergence operators, Skorohod integral. Fractional Brownian motion: relavance for financial mathematics, Ito and Tanaka formula, driving force for the heat equation. Ito formula for irregular Gaussian processes and other applications of Malliavin calculus. May be repeated for credit. Prerequisites: MATH 236, STATS 310C or equivalent.
*3 units, Win (Staff)*

**MATH 233. Probabilistic Methods in Analysis**
Proofs and constructions in analysis obtained from basic results in Probability Theory and a 'probabilistic way of thinking.' Topics: Rademacher functions, Gaussian processes, entropy.
*3 units, Win (Katznelson, Y)*

**MATH 236. Introduction to Stochastic Differential Equations**
Brownian motion, stochastic integrals, and diffusions as solutions of stochastic differential equations. Functionals of diffusions and their connection with partial differential equations. Random walk approximation of diffusions. Prerequisite: 136 or equivalent and differential equations.
*3 units, Win (Papanicolaou, G)*

**MATH 238. Mathematical Finance**
(Same as STATS 250.) Stochastic models of financial markets. Forward and futures contracts. European options and equivalent martingale measures. Hedging strategies and management of risk. Term structure models and interest rate derivatives. Optimal stopping and American options. Corequisites: MATH 236 and 227 or equivalent.
*3 units, Win (Papanicolaou, G)*

**MATH 239. Computation and Simulation in Finance**
Monte Carlo, finite difference, tree, and transform methods for the numerical solution of partial differential equations in finance. Emphasis is on derivative security pricing. Prerequisite: 238 or equivalent.
*3 units, Spr (Toussaint, A)*

**MATH 240. Topics in Financial Mathematics: Fixed Income Models**
Introduction to continuous time models for arbitrage-free pricing of interest rate derivatives. Bonds, yields, and the construction of yield curves. Caps, floors, swaps, swaptions, and bond options. Short rate models. Yield curve models. Forward measures. Forward and futures. LIBOR and swap market models. Prerequisite: MATH 238.
*3 units, Spr (Toussaint, A)*

**MATH 244. Riemann Surfaces**
Compact Riemann surfaces and algebraic curves; cohomology of sheaves; Serre duality; Riemann-Roch theorem and application; Jacobians; Abel's theorem. May be repeated for credit.
*3 units, Spr (Kerckhoff, S)*

**MATH 245A. Topics in Algebraic Geometry: Moduli Theory**
Intersection theory on the moduli spaces of stable curves, stable maps, and stable vector bundles. May be repeated for credit.
*3 units, not given this year*

**MATH 245B. Topics in Algebraic Geometry: Dessin d'Enfants**
Grothendieck's theory of dessin d'enfants, a study of graphs on surfaces and their connection with the absolute Galois group of the rational numbers. Belyi's theorem, representations of the absolute Galois group as automorphisms of profinite groups, Grothendieck-Teichmuller theory, quadratic differentials, and the combinatorics of moduli spaces of surfaces. May be repeated for credit.
*3 units, not given this year*

**MATH 247. Topics in Group Theory**
Topics include the Burnside basis theorem, classification of p-groups, regular and powerful groups, Sylow theorems, the Frattini argument, nilpotent groups, solvable groups, theorems of P. Hall, group cohomology, and the Schur-Zassenhaus theorem. The classical groups and introduction to the classification of finite simple groups and its applications. May be repeated for credit.
*3 units, Win (Diaconis, P)*

**MATH 248. Algebraic Number Theory**
Introduction to modular forms and L-functions. May be repeated for credit.
*1-3 units, not given this year*

**MATH 248A. Algebraic Number Theory**
Structure theory and Galois theory of local and global fields, finiteness theorems for class numbers and units, adelic techniques. Prerequisites: MATH 210A,B.
*3 units, Aut (Conrad, B)*

**MATH 249A. Introduction to Modular Forms**
The analytic theory of holomorphic and non-holomorphic modular forms and associated L-functions. Topics include Hecke operators, L-functions, Weil's converse theorem, trace formulas, sub-convexity for L-functions and applications, and Selberg's eigenvalue conjecture. May be repeated for credit. Prerequisites: 205A,B,C, or comparable knowledge of analysis.
*3 units, Aut (Soundararajan, K)*

**MATH 249B. Topics in Number Theory: Class Field Theory**
Classification of abelian extensions of local and global fields; classical, adelic, and cohomological formulations; applications to L-functions. May be repeated for credit.
*3 units, Win (Conrad, B)*

**MATH 249C. Topics in Number Theory: Class Field Theory and the Langlands Conjectures**
*3 units, Spr (Staff)*

**MATH 254. Geometric Methods in the Theory of Ordinary Differential Equations**
Topics may include: structural stability and perturbation theory of dynamical systems; hyperbolic theory; first order PDE; normal forms, bifurcation theory; Hamiltonian systems, their geometry and applications. May be repeated for credit.
*3 units, not given this year*

**MATH 256A. Partial Differential Equations**
The theory of linear and nonlinear partial differential equations, beginning with linear theory involving use of Fourier transform and Sobolev spaces. Topics: Schauder and L2 estimates for elliptic and parabolic equations; De Giorgi-Nash-Moser theory for elliptic equations; nonlinear equations such as the minimal surface equation, geometric flow problems, and nonlinear hyperbolic equations.
*3 units, Spr (Vasy, A)*

**MATH 256B. Partial Differential Equations**
Continuation of 256A.
*3 units, Win (Liu, T)*

**MATH 257A. Symplectic Geometry and Topology**
Linear symplectic geometry and linear Hamiltonian systems. Symplectic manifolds and their Lagrangian submanifolds, local properties. Symplectic geometry and mechanics. Contact geometry and contact manifolds. Relations between symplectic and contact manifolds. Hamiltonian systems with symmetries. Momentum map and its properties. May be repeated for credit.
*3 units, Aut (Ionel, E)*

**MATH 257B. Symplectic Geometry and Topology**
Continuation of 257A. May be repeated for credit.
*3 units, Win (Ionel, E)*

**MATH 258. Topics in Geometric Analysis**
May be repeated for credit.
*3 units, Win (White, B)*

**MATH 261A. Functional Analysis**
Geometry of linear topological spaces. Linear operators and functionals. Spectral theory. Calculus for vector-valued functions. Operational calculus. Banach algebras. Special topics in functional analysis. May be repeated for credit.
*3 units, not given this year*

**MATH 263A. Lie Groups and Lie Algebras**
Definitions, examples, properties. Semi-simple Lie algebras, their structure and classification. Cartan decomposition: real Lie algebras. Representation theory: Cartan-Stiefel diagram, weights. Weyl character formula. Orthogonal and symplectic representations. May be repeated for credit. Prerequisite: 210 or equivalent.
*3 units, Win (Bump, D)*

**MATH 263B. Lie Groups and Lie Algebras**
Continuation of 263A. May be repeated for credit.
*3 units, Spr (Staff)*

**MATH 264. Matrix Valued Spherical Functions and Orthogonal Polynomials**
Theory of spherical functions on locally compact groups and on Lie groups. Families of orthogonal polynomials with respect to a weight matrix function on the real line, and the corresponding algebra of differential operators. Spherical functions associated to the complex projective space as orthogonal polynomials. Topics may include some applications to quasi birth and death processes. My be repeated for credit. Prerequisities: 114, 205A, and 217A.
*3 units, Aut (Staff)*

**MATH 266. Computational Signal Processing and Wavelets**
Theoretical and computational aspects of signal processing. Time-frequency transforms; wavelet bases and wavelet packets; linear and nonlinear multiresolution approximations; estimation and restoration of signals; signal compression. May be repeated for credit.
*3 units, not given this year*

**MATH 269A. Affine Complex Manifolds and Symplectic Geometry**
Plurisubharmonic functions and pseudoconvexity: geometric theory. Construction of pseudoconvex shapes. Complex analysis on Stein manifolds. Symplectic geometry of Stein manifolds. Existence theorem for Stein complex manifolds. May be repeated for credit.
*3 units, Aut (Eliashberg, Y)*

**MATH 269B. Affine Complex Manifolds and Symplectic Geometry**
Symplectic convexity and Weinstein manifolds. Symplectic topology of subcritical Weinstein manifolds. From Weinstein to Stein structure. Morse-Smale theory for plurisubharmonic functions on Stein manifolds. Deformation theory for Stein complex structures. Symplectic field theory of Weinstein manifolds. May be repeated for credit.
*3 units, Win (Eliashberg, Y)*

**MATH 270. Geometry and Topology of Complex Manifolds**
Complex manifolds, Kahler manifolds, curvature, Hodge theory, Lefschetz theorem, Kahler-Einstein equation, Hermitian-Einstein equations, deformation of complex structures. May be repeated for credit.
*3 units, Win (Li, J)*

**MATH 271. The H-Principle**
The language of jets. Thom transversality theorem. Holonomic approximation theorem. Applications: immersion theory and its generaliazations. Differential relations and Gromov's h-principle for open manifolds. Applications to symplectic geometry. Microflexibility. Mappings with simple singularities and their applications. Method of convex integration. Nash-Kuiper C^1-isometric embedding theorem.
*3 units, Spr (Eliashberg, Y)*

**MATH 272A. Topics in Partial Differential Equations**
*3 units, Aut (Tzou, L)*

**MATH 282A. Low Dimensional Topology**
The theory of surfaces and 3-manifolds. Curves on surfaces, the classification of diffeomorphisms of surfaces, and Teichmuller space. The mapping class group and the braid group. Knot theory, including knot invariants. Decomposition of 3-manifolds: triangulations, Heegaard splittings, Dehn surgery. Loop theorem, sphere theorem, incompressible surfaces. Geometric structures, particularly hyperbolic structures on surfaces and 3-manifolds.
*3 units, Aut (Kerckhoff, S)*

**MATH 282B. Homotopy Theory**
Homotopy groups, fibrations, spectral sequences, simplicial methods, Dold-Thom theorem, models for loop spaces, homotopy limits and colimits, stable homotopy theory.
*3 units, Win (Carlsson, G)*

**MATH 282C. Fiber Bundles and Cobordism**
Possible topics: principal bundles, vector bundles, classifying spaces. Connections on bundles, curvature. Topology of gauge groups and gauge equivalence classes of connections. Characteristic classes and K-theory, including Bott periodicity, algebraic K-theory, and indices of elliptic operators. Spectral sequences of Atiyah-Hirzebruch, Serre, and Adams. Cobordism theory, Pontryagin-Thom theorem, calculation of unoriented and complex cobordism. May be repeated for credit.
*3 units, Spr (Milgram, R)*

**MATH 284A. Geometry and Topology in Dimension 3**
The Poincare conjecture and the uniformization of 3-manifolds. May be repeated for credit.
*3 units, Win (Staff)*

**MATH 284B. Geometry and Topology in Dimension 3**
The Poincare conjecture and the uniformization of 3-manifolds. May be repeated for credit.
*3 units, Spr (Staff)*

**MATH 286. Topics in Differential Geometry**
May be repeated for credit.
*3 units, Win (Mazzeo, R), Spr (Schoen, R)*

**MATH 290B. Finite Model Theory**
(Same as PHIL 350B.) Classical model theory deals with the relationship between formal languages and their interpretation in finite or infinite structures; its applications to mathematics using first-order languages. The recent development of the model theory of finite structures in connection with complexity classes as measures of computational difficulty; how these classes are defined within certain languages that go beyond first-order logic in expressiveness, such as fragments of higher order or infinitary languages, rather than in terms of models of computation.
*3 units, not given this year*

**MATH 292A. Set Theory**
(Same as PHIL 352A.) The basics of axiomatic set theory; the systems of Zermelo-Fraenkel and Bernays-Gödel. Topics: cardinal and ordinal numbers, the cumulative hierarchy and the role of the axiom of choice. Models of set theory, including the constructible sets and models constructed by the method of forcing. Consistency and independence results for the axiom of choice, the continuum hypothesis, and other unsettled mathematical and set-theoretical problems. Prerequisites: PHIL160A,B, and MATH 161, or equivalents.
*3 units, not given this year*

**MATH 292B. Set Theory**
(Same as PHIL 352B.) The basics of axiomatic set theory; the systems of Zermelo-Fraenkel and Bernays-Gödel. Topics: cardinal and ordinal numbers, the cumulative hierarchy and the role of the axiom of choice. Models of set theory, including the constructible sets and models constructed by the method of forcing. Consistency and independence results for the axiom of choice, the continuum hypothesis, and other unsettled mathematical and set-theoretical problems. Prerequisites: PHIL160A,B, and MATH 161, or equivalents.
*3 units, not given this year*

**MATH 293A. Proof Theory**
(Same as PHIL 353A.) Gentzen's natural deduction and sequential calculi for first-order propositional and predicate logics. Normalization and cut-elimination procedures. Relationships with computational lambda calculi and automated deduction. Prerequisites: 151, 152, and 161, or equivalents.
*3 units, not given this year*

**MATH 295. Computation and Algorithms in Mathematics**
Use of computer and algorithmic techniques in various areas of mathematics. Computational experiments. Topics may include polynomial manipulation, Groebner bases, computational geometry, and randomness. May be repeated for credit.
*3 units, not given this year*

**MATH 355. Graduate Teaching Seminar**
Required of and limited to first-year Mathematics graduate students.
*1 unit, Spr (Staff)*

**MATH 360. Advanced Reading and Research**
*1-9 units, Aut (Staff), Win (Staff), Spr (Staff), Sum (Staff)*

**MATH 361. Research Seminar Participation**
Participation in a faculty-led seminar which has no specific course number.
*1-3 units, Aut (Staff), Win (White, B), Spr (Kerckhoff, S), Sum (Staff)*

**MATH 380. Seminar in Applied Mathematics**
Guest speakers on recent advances in applied mathematics.May be repeated for credit.
*1 unit, Aut (Staff), Win (Staff), Spr (Staff)*

**MATH 381. Seminar in Analysis**
*1-3 units, by arrangement*

**MATH 384. Seminar in Geometry**
*1 unit, by arrangement*

**MATH 385. Seminar in Topology**
*1-3 units, by arrangement*

**MATH 386. Mathematics Colloquium**
Guest speakers on recent advances in mathematics. May be repeated for credit.
*1 unit, Aut (Staff), Win (Bump, D), Spr (Staff)*

**MATH 387. Seminar in Number Theory**
May be repeated for credit.
*1 unit, Aut (Staff), Win (Staff), Spr (Staff)*

**MATH 388. Seminar in Probability and Stochastic Processes**
*1-3 units, by arrangement*

**MATH 389. Seminar in Mathematical Biology**
*1-3 units, by arrangement*

**MATH 391. Research Seminar in Logic and the Foundations of Mathematics**
(Same as PHIL 391.) Contemporary work. May be repeated a total of three times for credit.
*1-3 units, Spr (Mints, G; Feferman, S)*

**MATH 395. Classics in Geometry and Topology**
Original papers in geometry and in algebraic and geometric topology. May be repeated for credit.
*3 units, Aut (Brumfiel, G), Win (Staff), Spr (Cohen, R)*

**MATH 396. Graduate Progress**
Results and current research of graduate and postdoctoral students. May be repeated for credit.
*1 unit, Aut (Staff), Win (Staff), Spr (Staff)*

**MATH 397. Physics for Mathematicians**
Topics from physics essential for students studying geometry and topology. Topics may include quantum mechanics, quantum field theory, path integral approach and renormalization, statistical mechanics, and string theory. May be repeated for credit.
*1 unit, Win (Staff)*