8 Projection

Our treatment of action in section 7 neglected the fact that outcomes can occur at different times for an agent, and there can be a separation in time (forward or backward) between the act of choice and the realization of one or more outcomes. Projection is the process by which an agent applies preferences at one time to a choice involving outcomes, at least one of which may occur at a different time.

8.1 Choice and Time

DEFINITION 8.1.1. The *choice time* t in a set $\mathcal{T}$ (representing a time line or set of times) of an action in which an agent chooses an option o, is the time at which the agent chooses o. Leaving out the agent and the context, we can write the *time-dependent action function* as $a:\mathcal{T}\rightarrow X$, and the *time-dependent action* at choice time t as $a(t)$.

DEFINITION 81.2. A an option $o=<x,t>$ is a *time-dependent option* for an agent at choice time t iff the outcome x is in the range X of the agent's time-dependent action function $a:\mathcal{T}\rightarrow X$. We can also refer to the set of options O as a set of "outcomes" X instead, and to the chosen option o as an outcome x, per definition 7.2.1. We can also say that a defines revealed preferences $<x,t>RP<y,t>$ analogous to definition 7.1.3.

DEFINITION 8.1.3. The *outcome time* $T\in\mathcal{T}$ of a time-dependent action $a(t)=x_T$, in which an agent chooses an option/outcome x_T is the time T when x_T occurs. If $T>t$, we say the outcome is *delayed* by the *duration* T-t from the choice time t. We can refer to the *time-dependent outcome set* $X\times\mathcal{T}$ as $X_\mathcal{T}$.

DEFINITION 8.1.4. An outcome set X, time set $\mathcal{T}$, and action function a form a *temporal choice structure* $<X,\mathcal{T},a>$ for an agent iff $X_\mathcal{T}$ is the time-dependent outcome set $X\times\mathcal{T}$ and $a:\mathcal{T}\rightarrow X_\mathcal{T}$ is a time-dependent action function.

DEFINITION 8.1.5. The temporal choice structure $<X,\mathcal{T},a>$ of an agent satisties *dynamic consistency* with respect to an outcome set X iff for all choice times t and t', for all outcome times T, and for all outcomes x in X, $a(t)=x_T$ iff $a(t')=x_T$.

Dynamic consistency means that an agent chooses the same outcome across choice times for a given outcome time. This assumption seems normative if we hold the information context fixed across choice times. Someone who violates dynamic consistency may, for example, "plan to behave a certain way in the future, but later, in the absence of new information, revise this plan" (Loewenstein, O'Donoghue, and Rabin, 2003). One can construct a variant of the above definition for preferences, which allows for cases in which an agent must make a binding choice at time t, but later reverses his/her preference at time t' and regrets the decision made at t.

EXERCISE 8.1.6. Define a variant of dynamic consistency for preferences instead of choices.

Psychological studies involving choice and time have looked primarily at three classes of choice situations: choices between outcomes that will occur at different times (intertemporal choice), choices between outcomes in the future that will occur at the same time (prospective choice), and choices or preferences between outcomes that have occurred in the past (retrospective choice). We will develop the theory and review experiments in each of these areas.

8.2 Intertemporal Choice

Intertemporal choice involves choices between outcomes that will occur at different times. Choices are made at a give choice time, but the resulting time-dependent revealed preferences can be compared for consistency across different choice times.

DEFINITION 8.2.1. A *time-dependent utility function* $u: X_T \times \mathcal{T} \rightarrow \mathbb{R}$ represents an agent's temporal choice structure $<X,\mathcal{T},a>$ iff the revealed preferences RP defined by $<X,\mathcal{T},a>$ are such that for all times $t,T,T' \in \mathcal{T}$ and outcomes $x_T, y_{T'} \in \mathcal{T}$ (denoting x_T occuring at outcome time T and $y_{T'}$ occuring at outcome time T'), $u(x_T,t) > u(y_{T'},t)$ iff $<x_T,t> RP <y_{T'},t>$.

The above definition provides a way to move from time-dependent actions/choices (interpreted as revealed preferences) to time-dependent utilities. This is important because the theory of intertemporal choice is generally developed in terms of utilities, which can then appear in equations.

DEFINITION 8.2.2. A *time-dependent utility function* $u: X_T \times \mathcal{T} \rightarrow \mathbb{R}$ representing an agent's temporal choice structure $<X,\mathcal{T},a>$ satisfies *(temporal) stationarity* (or *delay independence*) iff for all times $t_1,t_2,T,T' \in \mathcal{T}$, where both T and T' are greater than (occur after) both t_1 and t_2, and for all outcomes $x_T, y_{T'} \in \mathcal{T}$ (denoting x_T occuring at T and $y_{T'}$ occuring at T'), $u(x_T,t_1) > u(y_{T'},t_1)$ iff $u(x_T,t_2) > u(y_{T'},t_2)$.

THEOREM 8.2.3. If an agent's temporal choice structure $<X,\mathcal{T},a>$ satisfies dynamic consistency, then any time-dependent utility function representing $<X,\mathcal{T},a>$ satisfies stationarity.

EXERCISE 8.2.4. Prove 8.2.3.

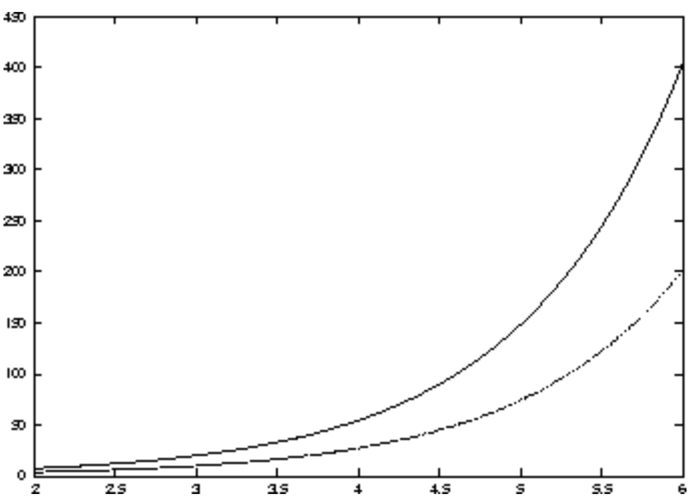
EXERCISE 8.2.5. Does stationarity imply dynamic consistency?

DEFINITION 8.2.6. A time-dependent, positive utility function $u: X_T \times \mathcal{T} \rightarrow \mathbb{R}$ defined for time-dependent outcomes $x_T \in X_T$ and choice and outcome times $t<T$ by $u(x_T,t) = d(t,T)u(x_T,T)$ is *time discounting* iff $d(t,T) < 1$ for all values t and T. If an agent has a time discounting utility function, we say the agent is *impatient*.

DEFINITION 8.2.7. A time discounting utility function $u(x_T,t) = d(t,T)u(x_T,T)$ is an *exponential discounting* function iff $d(t,T) = r^{T-t}$ for some constant $r < 1$.

Exponential discounting has some desirable properties, which we will not prove here, e.g. (a) it ensures that a stream of outcomes at different points in the future will add up to a finite utility, thus allowing comparisons between such streams and a complete preference relation between them (see Mas-Collel, Whinston, and Green, 1995).

EXAMPLE 8.2.8. An example of an exponential discounting function is shown in the diagram below, with time on the horizontal axis and utility on the verticle axis. The two curves represent values of the same exponential discounting function for two two different outcomes, one of which has higher utility than the other. As you can see from the graph, the two curves never cross each other. This is a feature of exponential discounting.



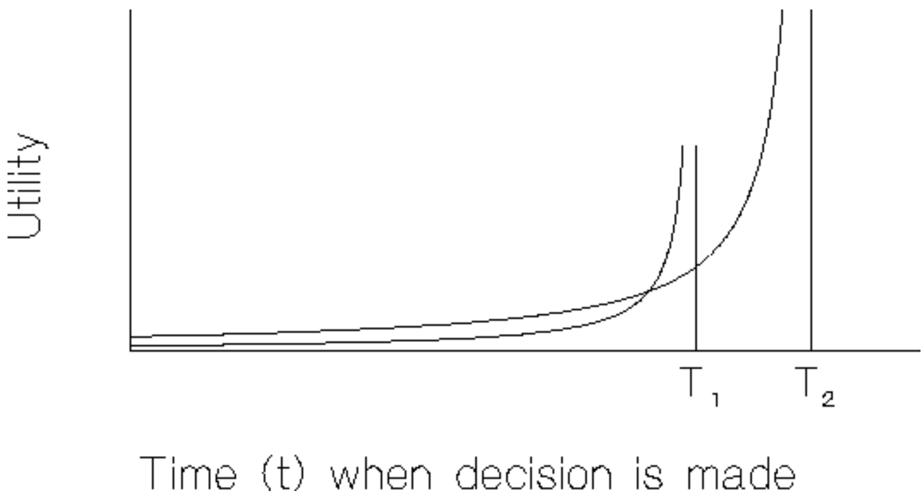
COROLLARY 8.2.9. An exponential discounting function satisfies stationarity.
Proof. Assume a time-dependent utility function $u{:}X_T \times \mathcal{T} \to \mathbb{R}$ representing an agent's temporal choice structure $\langle X, \mathcal{T}, a \rangle$. Stationarity requires (from 8.2.2) that for all times $t_1, t_2, T, T' \in \mathcal{T}$, where both T and T' are greater than (occur after) both t_1 and t_2, and for all outcomes $x_T, y_{T'} \in \mathcal{T}$ (denoting x_T occuring at T and $y_{T'}$ occuring at T'), $u(x_T,t_1) > u(y_{T'},t_1)$ iff $u(x_T,t_2) > u(y_{T'},t_2)$. To prove the forward direction, assume $u(x_T,t_1) > u(y_{T'},t_1)$. Applying 8.2.7, this means that $r^{T-t1}u(x_T,T) > r^{T-t1}u(y_T,T)$ which implies that $u(x_T,T) > u(y_{T'},T)$, so $r^{T-t2}u(x_T,T) > r^{T-t2}u(y_T,T)$ and therefore $u(x_T,t_2) > u(y_{T'},t_2)$. An analogous argument establishes the reverse direction and the corollary is proved.

EXPERIMENT 8.2.10. *Violation of stationarity/delay independence.* Ainslie and Haendel (1983) presented subjects with two choices between (1) \$50 today or \$100 in six months, and (2) \$50 in 12 months or \$100 in 18 months. Most subjects chose the \$50 today in problem 1, but

$100 in 18 months in problem 2. The result violates stationarity.

EXERCISE 8.2.11. Explain why the result of 8.2.10 violates stationarity (delay independence).

EXAMPLE 8.2.12. *Hyperbolic discounting*. An example of a time discounting utility function that violates stationarity is the *hyperbolic discounting function*, defined by $d(t,T) = 1/[1+k(T-t)]$, where k is a constant that governs the rate of discounting. A graph showing two hyperbolic discounting curves is shown below, for different outcome times T_1 and T_2.



As you can see from the graph, the two curves cross, which represents a reversal in preferences that violates stationarity. The hyperbolic discounter prefers the outcome which happens at T_1 if the choice time is just before T_1, but prefers the outcome which happens at T_2, If the choice is made earlier than that. Hyperbolic discount functions have been put forward as one model to account for results like that of experiment 8.2.10, but other models have also been proposed that also violate stationarity but assign a different form to the discounting function (see Green and Myerson, 2004).

Although the proof of the theorem below is beyond the scope of this course, we give it here to esablish the equivalence between the stationarity assumption and exponential discounting.

THEOREM 8.2.13. Temporal stationarity requires exponential discounting.
Proof. See Traeger (2007).

EXERCISE 8.2.14. In a paper called "Uncertainty as Wealth", Ainslie (2003) argues that modern civilization reduces people's overall emotional satisfaction by facilitating impulsive choices of outcomes which are chosen only because of impulsiveness - violations of stationarity. Give an example from your own life in which you think your overall happiness has been reduced because of impulsive choice, and make an argument that your choice violated stationarity. If you do not agree with Ainslie's assertion, provide a counter-argument.

8.3 Prospective Choice

Prospective choices are choices between two outcomes that share an outcome time. For example, you might be asked to choose today whether you would like to have fish or vegetable curry at a dinner to happen next week. In such situations, it is natural to ask whether the passage of time is likely to change our preferences, so that we might regret what we have chosen when the outcome actually happens.

DEFINITION 8.3.1. A time-dependent utility function $u:X_T\times\mathcal{T}\rightarrow\mathbb{R}$ representing an agent's temporal choice structure $\langle X,\mathcal{T},a\rangle$ exhibits *accurate projection* iff for all times $t,T\in\mathcal{T}$, where both T is greater than (occurs after) t, and for all outcomes $x_T,y_T\in X\times\mathcal{T}$ (denoting x_T occuring at T and y_T occuring at T), $u(x_T,t) > u(y_T,t)$ iff $u(x_T,T) > u(y_T,T)$.

Whereas stationarity compares preferences between outcomes at different choice points before different outcomes may occur, accurate projection compares a preference at a choice time with a preference at a common outcome time.

COROLLARY 8.3.2. Dynamic consistency implies accurate projection (and therefore a violation of accurate projection implies dynamic inconsistency).

EXERCISE 8.3.3. Prove 8.3.2.

EXPERIMENT 8.3.4. *Projection bias*. Read and Van Leeuwen (1998) asked office workers to choose between healthy and unhealthy snacks to be received in one week. Decision times and projected snack reception times occurred either when subjects were generally hungry (late in the afternoon) or satiated (right after lunch). The results are shown in the table below, with the percentage of subjects choosing the unhealthy snack shown in each cell.

		Hunger at outcome time	
		Hungry	Satiated
Hunger at choice time	Hungry	78.00%	42.00%
	Satiated	56.00%	26.00%

As can be seen from the results, subjects showed a bias in the direction of their hunger state at the choice time. If we accept that a random assignment of subjects to conditions in this between subjects design should produce choices that depend only on hunger at outcome time, the results indicate that many subjects would likely reverse their preferences from choice time to outcome time, violating unbiased projection.

EXPERIMENT 8.3.5. *Adaptation neglect*. People neglect effects of adaptation to surroundings in predicting future utility. In an experiment reported by Kahneman and Snell (1992), participants mispredicted, after initial (unpleasant) exposure, their (non)enjoyment of plain yogurt after 8 daily episodes of consumption, showing a projection bias in the direction of their initial reaction.

8.4 Retrospective Choice

Retrospective choice, which involves expressing a preference between outcomes that have already happened (e.g., Which of these experiences would you be more willing to repeat?), is closely connected with the notion of experienced utility, both moment to moment and over some length of time.

PRINCIPLE 8.4.1. *Time monotonicity for negative utility*. The concatenation of two experiences, both of which produce negative utility for an agent, should not be strictly preferred to either of the experiences unconcatenated with the other.

Kahneman, Wakker, and Sarin (1997) make a formal argument for the above principle.

EXPERIMENT 8.4.2. *Duration neglect*. Kahneman et al. (1993) tested people's hedonic memories. They gave subjects two unpleasant experiences:

> Short trial: Hold hand in 14◦C water for 60s
> Long trial: Hold hand in water for 90s; 14◦C for 60s, followed by gradual rise to 15◦C over next 30s

After the second trial, subjects were called in to repeat one of the two trials exactly.
Of these subjects, 65% chose to repeat the long trial. The subjects appeared to show "duration neglect". They remembered and overweighted the end of the experience (a gradual decline in pain).

APPLICATION 8.4.3. Redelmeier and Kahneman (1996) asked patients undergoing colonoscopy to report their intensity of pain every 60 seconds. Subjects later provided several measures of remembered utility for the whole experience. Remembered utility ratings reflected not total utility (addition of pain ratings) but a "peak and end" rule: the highest and last pain ratings dominated memory.