2.Belief

2.1Propositions, Truth, and Falsity

A *proposition* (e.g., *p*) is the content of a statement. Assume the existence, in some context *C*, of a *truth-value function* V_t which maps propositions and contexts into {T,F}, where $V_t(p,C)$=T if *p* is true in *C*, and $V_t(p,C)$=F if *p* is false in *C*.

EXAMPLE:

Consider the following statements uttered in context *D*:

- "John called Mary yesterday."
- "Yesterday, Mary was called by John."
- "John llamó a Mary ayer."

All three statements express the proposition *r*: *John called Mary yesterday.* The context *D* defines the meanings of the terms (e.g. *John, called, Mary,* and *yesterday*). $V_t(r,D)$=T iff John called Mary yesterday in the context *D*. Otherwise, $V_t(r,D)$=F.

2.2Beliefs

Operationally, an agent *A* may be taken to *believe* a proposition *p* in a context *C* if *A* asserts the truth of a statement whose content is *p* in *C*. This is a sufficient but not a necessary condition for inferring that *A* believes *p*. We write $B_A(p,C)$. More strongly we might assume that (in *C)* if *A* belives *p*, then *A* will asset the truth of a statement whose content is *p* when given an opportunity.

EXAMPLES:

Lisa is asked in context *D*, "Did John call Mary yesterday?" Lisa answers, "Yes." We infer that $B_{lisa}(r,D)$. We call this a "recognition" statement because Lisa is prompted with the proposition and recognizes it to be true.

Sam is asked in context *D,* "What happened yesterday?" Sam answers, "John called Mary." We infer that $B_{Sam}(r,D)$. We call this a "recall" statement because Sam is not prompted with the proposition itself, but instead produces it when asked what happened generally.

2.3Veridicality

A belief $B_A(p,C)$ is *veridical* iff $V_t(p,C)$=T. We also call this "true belief". Unveridical beliefs are "false beliefs".

EXAMPLE:

$B_{lisa}(r,D)$. John called Mary yesterday in context *D.* Therefore, Lisa's belief that *r* is true in *D* is veridical, or a true belief.

2.4 Models of Veridicality

Applying the different goals for analyzing rationality mentioned in the notes for “Rationality”, we can define the following models for the veridicality of beliefs.

2.4.1 Normative Veridicality

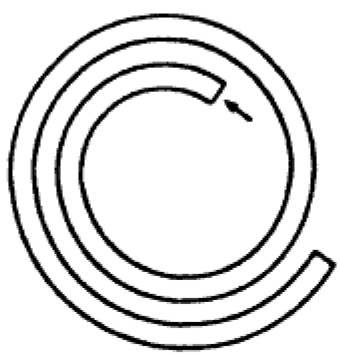
An agents beliefs should be veridical. More weakly, beliefs should be true when the truth is available in the agent's environment or context. Beliefs should not be false when the truth is available.

2.4.2 Descriptive Veridicality

People's beliefs are often predictably and systematically unveridical even when they have regular exposure to the truth.

EXAMPLES

Naive physics.



Consider the diagram above (McCloskey, 1983; reproduced in Zbikowski, 1997). A metal ball is shot at high speed through a curved tube lying on a flat surface. Your task is to predict the trajectory of the ball when it exits the tube.

Draw your answer now on the diagram, then turn over the page.

Slightly over 50% of undergraduates in McCloskey's study, including some who had studied physics, gave an answer like that shown in diagram a below, although the correct answer is shown in b.

The students whose answers were like diagram a exhibited a false belief. The “impetus” theory underlying these students' false beliefs was a widely held theory of motion in medieval times.

In the above case, one could argue that subjects may not have had adequate prior exposure to the truth. But other examples of naïve motion beliefs in McCloskey's studies included cases with which any college student would be familiar from experience. For example, students mispredicted the trajectory of a ball rolled across a table as it falls to the ground, predicting that it would fall much closer to the table than it does. False beliefs about kinematics are widespread and were often parts of received science before Newton's time, e.g. the belief that heavy objects fall faster than lighter ones.

2.5 Logic

Beliefs can give rise to other beliefs via *inferences*, following accepted principles of logic. There are many different systems of logic. The basics are covered in the article “Classical Logic” by Stewart Shapiro in the *Stanford Encyclopedia of Philosophy* (http://plato.stanford.edu/entries/logic-classical/).

2.5.1 Normative Analysis of Logical Inference

A standard normative view of logical inference is that conclusions drawn from premises should be *valid*. A valid conclusion is one that is necessarily true if the premises are true (i.e. that is logically implied by the premises). Furthermore, inference should reliably produce valid conclusions. This property is known as *soundness*. In the ideal case, a reasoner should be able to infer all possible valid conclusions in a given logical system, a property of derivation known as *completeness.*

Logic is often assumed to be a facet of natural law which is never disobeyed in reality. Thus, agents might be assumed to have extensive exposure to its principles and to be able to apply them correctly, at least to simple problems involving few premises and terms.

2.5.2Descriptive Analysis of Logical Inference

Human reasoners, when confronted with logic problems, virtually never satisfy completeness. Only those trained in logic who take the time to construct proofs carefully approach soundness. And a high percentage of subjects, including virtually all of those without special training in logic, draw invalid inferences even in quite simple problems. Propositional logic generally leads to fewer errors than categorical (syllogistic logic), and people do much better with problems whose content is of a type that is familiar from their experience than they do with abstract or unfamiliar problems (Baron, 2000).

EXAMPLES:

Belief bias.

Evans, Barston, and Pollard (1983) studied subjects' acceptance of conclusions in syllogisms that varied along two dimensions: valid-invalid and believable-unbelievable. The general findings were: (1) People endorsed more valid than invalid conclusions; (2) they endorsed more believable than unbelievable conclusions—the "belief bias" effect; (3) the extent of the belief bias is more marked on invalid than on valid syllogisms.

The table below from Evans, Handley, and Harper (2001) shows examples of the four kinds of syllogism used by Evans, Barston, and Pollard (1983) together with the overall acceptance rates for each type.

Category	Example	Acceptance rate
Valid-believable	No police dogs are vicious Some highly trained dogs are vicious Therefore, some highly trained dogs are not police dogs	89%
Valid-unbelievable	No nutritional things are inexpensive Some vitamin tablets are inexpensive Therefore, some vitamin tablets are not nutritional	56%
Invalid-believable	No addictive things are inexpensive Some cigarettes are inexpensive Therefore, some addictive things are not cigarettes	71%
Invalid-unbelievable	No millionaires are hard workers Some rich people are hard workers Therefore, some millionaires are not rich people	10%

Figural effect.

When given the syllogism:

Some A are B
All B are C

Therefore Some A are C

most subjects accept the conclusion, correctly. However, when the order of terms is reversed in the first premise to read "Some B are A", far fewer subjects accept the conclusion. This order-dependency, known as the figural effect, is summarized for all types of syllogisms in the following table (from Johnson-Laird, 1980):

The "Figural Effect" Observed in Syllogistic Inference
(from Johnson-Laird & Steedman, 1978)
The Percentages of A-C and C-A Conclusions as a Function of the Figure of the Premises

	Figure of Premises			
Form of Conclusion	A-B B-C	B-A C-B	A-B C-B	B-A B-C
A-C	51.2	4.7	21.2	31.9
C-A	6.2	48.1	20.6	17.8

Note: The table includes both valid and invalid conclusions; the effect is equally strong for both of them. The balance of the percentages corresponds almost entirely to responses of the form, "No valid conclusion can be drawn."

Multiple models.

Consider the following premises:

No X are Y
All Y are Z

What can be concluded? Most people say "No X are Z" (Johnson-Laird and Bara, 1984, referenced in Baron, 2000). But this answer is wrong.

Consider the following instantiation of the above premises:

No cats are dogs
All dogs are animals

This should make it easy to see that the conclusion "No cats are animals" is false. In this case, the difficulty of the X-Y-Z form of the syllogism is not caused by the figural effect or a belief bias (Think about it: Why not?), but rather, in the interpretation of Johnson-Laird and his colleagues, by the fact that the premises have multiple models, some in which none of the X are Z, some in which some of the X are Z, and some in which all of the X are Z. In Johnson-Laird's theory, logical inferences are harder as the premises admit more models, for example because people construct models to test the validity of conclusions and many stop prematurely, failing to consider all possible models. Patterns of incorrect answers generally support the theory.

2.6 Propositional Independence

A proposition *p* is *independent of (and consistent with)* another proposition *q* in a context *C* iff $V_t(q,C)$ does not determine $V_t(p,C)$.

2.6.1 Normative Analysis of Propositional Independence

If *p* is independent of *q,* then an agent's belief in *q* should not determine the agent's belief in *p*.

2.6.2 Descriptive Analysis of Propositional Independence

People's beliefs in one proposition often predict their beliefs in another independent proposition when both propositions are *consonant* with an strongly held third belief, and the negation of either independent proposition would be *dissonant* with the third belief. Beliefs are dissonant when one belief could be taken as supporting the negation of the other. Beliefs are consonant when they can be taken as supporting each other. Many experiments indicate that people generally try to avoid dissonant beliefs, even when they are independent of and consistent with each other.

EXAMPLE:

Belief overkill.

"People who favored a nuclear test-ban believed that testing created a serious medical danger, would not lead to major weapons improvements, and was a source of international tension. Those who opposed the treaty usually took the opposite position on all three issues.... choices are easier since all considerations are seen as pointing to the same conclusion. Nothing has to be sacrificed. But, since the real world is not as benign as these perceptions, values are indeed sacrificed." (Jervis, 1976, pp. 129-130)

2.7 Doxastic Logic

The basic principles of doxastic logic are covered in the article on "Modal Logic" by James Garson in the *Stanford Encyclopedia of Philosophy* (http://plato.stanford.edu/entries/logic-modal/).

2.7.1 Normative Analysis of Belief Inference

Among other possible normative principles of belief inference, one might reasonably ask of an ideal agent that the distribution axiom of modal logic, included in system K (for Kripke), be obeyed by agents' beliefs. Specifically:

$$B(p \Rightarrow q) \;\Rightarrow\; (Bp \Rightarrow Bq).$$

In words, if an agent believes that *p* implies *q*, then if the agent believes *p*, the agent should also believe *q*. The consequence of this axiom, applied to beliefs, is that an agent believes all the consequences of their beliefs as long as the relationships of consequence are themselves believed.

If we take the stronger operationalized version of belief defined in section 2.2, specifically assuming that if an agent believes a proposition then the agent will assert it if given an opportunity, then obedience of the distribution axiom of doxastic logic implies that the agent will be able to report all the beliefs that follow from other beliefs together with believed implication rules.

2.7.2 Descriptive Analysis of Belief Inference

In practice, people often fail to evince beliefs that follow from other beliefs they evince. A classic example, though perhaps fictionalized, is the inability of the slave boy in Plato's *Meno* to correctly report a statement about geometry that Socrates, in the dialogue, goes on to show is implicit in the boy's other beliefs. You can demonstrate this effect for yourself if you try to answer quickly whether the number 578,931 is prime or not. You probably know how to compute whether or not a number is prime, so the prime status of this number is in the consequential closure of your beliefs, but you do not have ready access to this belief and therefore cannot exhibit it without much calculation.