7 Action

We can now apply beliefs, preferences, and confidence to choices between different possible actions or options.

7.1 Choice

DEFINITION 7.1.1. *Action*. Let I be a set of agents and O be a set of *options* in a context C. Then the *action function* $a{:}IxC \rightarrow O$ describes the *action* performed by each agent i in I. We can denote this *action structure* (or *choice structure*) by $\langle I,C,O,a \rangle$.

DEFINITION 7.1.2. *Choice*. For all agents i in $\langle I,C,O,a \rangle$, the action $a(i,C)$ is a *choice* iff $|O|>1$. In this case, we may refer to a as a *choice function*.

DEFINITION 7.1.3. *Revealed preference*. For an agent i, $o_1RP_io_2$ is a *revealed preference* for option o_1 over option o_2 iff there exists a choice structure $\langle I,C,O,a \rangle$ in which $i \in I$, $o_1,o_2 \in O$, and $a(i,C)= o_1$.

AXIOM 7.1.4. *Option-set consistency*. If $o_1RP_io_2$, then in every choice structure $\langle I,O,C \rangle$ such that $o_1,o_2 \in O$, $a(i,c) \neq o_2$.

The above axiom is an application of the regularity or context-independence principle for preferences, from definition 3.3.1, to revealed preferences. It is a version of what is known as the *weak axiom of revealed preference (WARP)* (originally proposed by Samuelson, 1938), for action/choice functions. Revealed preference theory in its modern form is usually developed for what are called *choice correspondences*, which fits better with weak preference than with strict preference. A correspondence is a mapping into a subset, in this case a subset of the set of options, whereas a function is a mapping into an element of a set. We develop definitions and axioms for choice and revealed preference somewhat differently here because doing so leads to somewhat easier results, and serves adequately to set up the key experimental results in this area.

THEOREM 7.1.5. If RP_i is an asymmetric relation for an agent i over option set O, then it satisfies option-set consistency for all $o_1,o_2 \in O$.
Proof. Assume $o_1RP_io_2$. We will prove this implies the consequent side of 7.1.4, by contradiction. Suppose there is some choice structure $\langle I',O',C',a \rangle$ such that $o_1,o_2 \in O'$, $a(i,c)=o_2$. Therefore, $o_2RP_io_1$, by the definition of revealed preference (7.1.3). But this violates our assumption that RP_i is an asymmetric.

COROLLARY 7.1.6. If RP_i is a preference relation for an agent i over option set O, then it

satisfies option-set consistency for all $o_1, o_2 \in O$.
Proof. If RP_i is a preference relation, then it is asymmetric, by definition 3.1.3. Thus, it meets the antecedent criterion for theorem 7.1.5.

Several experiments, including the one described in 3.3.3, have demonstrated violations of option-set consistency in people's choices. The following experiments provide two more such cases, accounted for by two different psychological principles: *tradeoff contrast* and *extremeness aversion*.

EXPERIMENT 7.1.7. *Tradeoff contrast*. When two options differ from each other on two dimensions (e.g. price and quality), option-set consistency can be violated by adding another option that contrasts with the amount of tradeoff between the dimensions exhibited by one of the original options. Experiment 3.3.3 demonstrated this effect with microwave ovens that differed in price and quality, showing that "the tendency to prefer [a higher quality option] over [a less expensive option] is enhanced if the decision maker encounters other choices in which a comparable improvement in quality is associated with a larger difference in price" (Tverksy and Simonson, 1993). They go on to say: "Another implication of the tradeoff contrast hypothesis is that the 'market share' of x can be increased by adding to { x, y } a third alternative z that is clearly inferior to x but not to y. Violations of regularity based on this pattern were first demonstrated by Huber et al. (1982). The following example is taken from Simonson and Tversky (1992). One group (n = 106) was offered a choice between $6 and an elegant Cross pen. The pen was selected by 36% of the subjects and the remaining 64% chose the cash. A second group (n = 115) was given a choice among three options: $6 in cash, the same Cross pen, and a second less attractive pen. The second pen, we suggest, is dominated by the first pen but not by the cash. Indeed, only 2% of the subjects chose the less attractive pen, but its presence increased the percentage of subjects who chose the Cross pen from 36% to 46%, contrary to regularity." This result indicates that, under the assumption that subjects in the second group would have shown a similar pattern if they had been offered only the $6 and the cross pen, a significant percentage of subjects violated open-set consistency by "switching" their choice from the $6 to the Cross pen when another option was added.

EXPERIMENT 7.1.8. *Extremeness aversion - compromise*. Tversky and Simonson (1993) suggest that "options with extreme values within an offered set will be relatively less attractive than options with intermediate values." This principle is a consequence of loss amplification (see below). They describe an experiment in which "subjects were asked to choose among 35 mm cameras varying in quality and price. One group (n = 106) was given a choice between a Minolta X-370 priced at $170 and a Minolta 3000i priced at $240. A second group (n = 115) was given an additional option, the Minolta 7000i priced at $470. Subjects in the first group were split evenly between the two options, yet 57% of the subjects in the second group chose the mid-dle option (Minolta 3000i), with the remaining divided about equally between the two extreme options." Thus, "the introduction of an extreme option reduced the market share of the other extreme option, but not of the middle option." This experiment demonstrates one of the consequences of extremeness aversion: a principle of compromise. But it violates open-set consistency under the assumption that we can equate behavior in the first group with the choices that the second group would have made had they been in that condition. This is a reasonable assumption when the groups have been randomly assigned, but it calls for follow-up studies that show a consistent pattern. The existence of multiple between-subjects experiments showing similar results bolsters this reasoning.

AXIOM 7.1.9. *Transitivity of revealed preference*. In binary choices involving $n \geq 2$ options, if $o_1 RP_i o_2$, $o_2 RP_i o_3$, and so on through $o_{n-1} RP_i o_n$, then $\neg o_n RP_i o_1$.

The above axiom is a version of what is sometimes called the *strong axiom of revealed preference (SARP)*.

THEOREM 7.1.10. If RP_i is a preference relation for an agent *i* over option set *O*, then it satisfies transitivity of revealed preference for all $o_1, o_2 \in O$.
Proof. Assume $o_1 RP_i o_2$ and $o_2 RP_i o_3$, but $o_3 RP_i o_1$. RP_i is a preference relation by hypothesis, so by 3.1.4 it is transitive. But this contradicts the assumption.

EXERCISE 7.1.11. Does 7.1.9 imply 7.1.4? Why or why not?

Experiment 3.1.5 effectively demonstrated violations of weak transitivity for revealed preferences in the context of evaluating applicants for college admission. A strong normative argument against such violations is the so-called *money-pump argument*. Someone who violates transitivity will presumably pay some amount of money for the option they prefer. But if you prefer A over B, B over C, C over D, and D over E, but then also prefer E over A, and if your choices match your preferences, then a would-be exploiter can start by giving you E, then offer you D in exchange for E and some amount of money, then C in exchange for D and some money, and so on, and in theory you will continue to be willing to pay ad infinitum through the cycle in which you pay to exchange A for E and the process repeats.

EXERCISE 7.1.12. Design an experiment in which you would expect people to have intransitive revealed preferences for a good, which could be exchanged along with money for another good, in order to test the money pump argument. What do you expect to happen as the process cycles?

We conclude this review of the basic theory of choice with a principle that is fundamentally assumed in most normative theories.

DEFINITION 7.1.13. *Descriptive invariance*. For all agents *i* in a choice structure $\langle I,C,O,a \rangle$, the action $a(i,C)$ should not vary depending on how one or more options in *O* are described, provided the agent agrees that the option set is the same under both descriptions.

We will apply this principle in the experimental study of risky decision making.

7.2 Risk

In most real-world choices, the outcome that results if a given option is chosen is not known with certainty. When different outcomes may result, with different *known* probabilities, from the same option being chosen, we say that the option carries *risk*. Much of the study of decision making, both in theory and in experiments, involves risky choice.

DEFINITION 7.2.1. An option o in an option set O is a *gamble* (also known as a *lottery*) iff there is a set X (with $|X|=n\geq 1$) of outcomes $x_1,x_2,\ldots,x_n$, such that for all $j\in\{1,\ldots,n\}$, $0\leq P(x_j|o)\leq 1$ and $\sum_X P(x_j|o)=1$. We can write such a gamble L in the form $(\langle x_1,p_1\rangle,\langle x_2,p_2\rangle,\ldots\langle x_n,p_n\rangle)$, where $p_j=P(x_j|o)$, and $\langle x_j,p_j\rangle$ means that outcome x_j occurs with probability p_j. We can refer to the space of gambles $\mathscr{L}$ as the set of all gambles defined over an outcome set X. (Note: We can also refer to a gamble itself as an "outcome", although it is not in the set X. In fact the set $\mathscr{L}$ of such outcomes is uncountable.)

EXPERIMENT 7.2.3. *Framing effect - "Asian disease".* A framing effect is a violation of descriptive invariance. A famous example of such an effect was demonstrated in the following experiment involving a choice between gambles. The experiment is described by Kahneman and Tversky (1984) as follows: "The following pair of problems illustrates a violation of this requirement. The total number of respondents in each problem is denoted by *N,* and the percentage who chose each option is indicated in parentheses.

> Problem 1 (N = 152): Imagine that the U.S. is preparing for the outbreak of an unusual Asian disease, which is expected to kill 600 people. Two alternative programs to combat the disease have been proposed. Assume that the exact scientific estimates of the consequences of the programs are as follows:
> If Program A is adopted, 200 people will be saved. (72%)
> If Program B is adopted, there is a one-third probability that 600 people will be saved and a two-thirds probability that no people will be saved. (28%)

Which of the two programs would you favor? The formulation of Problem 1 implicitly adopts as a reference point a state of affairs in which the disease is allowed to take its toll of 600 lives. The outcomes of the programs include the reference state and two possible gains, measured by the number of lives saved. As expected, preferences are risk averse: A clear majority of respondents prefer saving 200 lives for sure over a gamble that offers a one-third chance of saving 600 lives. Now consider another problem in which the same cover story is followed by a different description of the prospects associated with the two programs:

> Problem 2 (N = 155):
> If Program C is adopted, 400 people will die. (22%)
> If Program D is adopted, there is a one-third probability that nobody will die and a two-thirds probability that 600 people will die. (78%)

It is easy to verify that options C and D in Problem 2 are undistinguishable in real terms from options A and B in Problem 1, respectively. The second version, however, assumes a reference state in which no one dies of the disease. The best outcome is the maintenance of this state and the alternatives are losses measured by the number of people that will die of the disease. People who evaluate options in these terms are expected to show a risk seeking preference for the gamble (option D) over the sure loss of 400 lives, indeed, there is more risk seeking in the second version of the problem than there is risk aversion in the first."

We now turn to the theory of risky choice as developed by von Neumann and Morgenstern (1944), culminating in the expected utility theorem. We will develop the theory for *weak preference relations* over outcomes (which can include gambles), following the proof in Mas-Collel, Whinston, and Green (1995).

DEFINITION 7.2.4. An outcome x is *weakly preferred* to an outcome y (written as xP^*y) iff xPy or xIy.

In words, the above definition says that an agent weakly prefers one outcome over another if and only if the agent either (a) strictly prefers the first outcome over the second, or (b) is indifferent to them.

COROLLARY 7.2.5. xP^*y iff $\neg yPx$.
Proof. If xP^*y then by 7.2.4, either xPy or xIy. If xPy then $\neg yPx$ by the asymmetry property of 3.1.3(a). If xIy then $\neg yPx$ by definition 3.1.8.

COROLLARY 7.2.6 xIy iff xP^*y and yP^*x.

EXERCISE 7.2.7. Prove 7.2.6.

DEFINITION 7.2.8. $P^* \subseteq X \times X$ is a *weak preference relation* iff
(a) $\forall x,y \in X$ xP^*y or yP^*x (*completeness*),
and
(b) $\forall x,y,z \in X$ xP^*y & $yP^*z \Rightarrow xP^*z$ (*transitivity*).

COROLLARY 7.2.9. If P is a strict preference relation, then the relation P^* defined by P is a weak preference relation.

EXERCISE 7.2.10. Prove 7.2.9.

DEFINITION 7.2.11. A weak preference relation P^* satisfies the *independence* or *substitution* axiom for risky choice over an outcome set X iff $\forall x,y,z$ in X and probability $p \in (0,1]$, if xP^*y then $(\langle x,p\rangle,\langle z,1-p\rangle)\ P^*\ (\langle y,p\rangle,\langle z,1-p\rangle)$.

The independence axiom is the cornerstone of expected utility theory. It says that if one (weakly) prefers outcome x to outcome y when the outcome is known with certainty, then one should (weakly) prefer a gamble in which outcome x happens with probability p over a gamble yielding outcome y with probability p, provided that the outcome is the same (z) if neither x nor y happens.

EXPERIMENT 7.2.12. *Certainty effect*. Kahneman and Tversky (1979) gave subjects two problems. In problem 1, they were asked to choose between (A) an 80% chance to win $4000 or (B) $3000 for certain. In problem 2, subjects were asked to choose between (C) a 20% chance to win $4000, or (D) a 25% chance to win $3000. Out of 95 subjects, in a within subjects study, 80% chose B over A, and 65% chose C over D. Over half of the subjects violated the independence axiom of risky choice, because the axiom says that if gamble B is preferred to gamble A, then a gamble in which gamble B is received with probability .25, and otherwise nothing is received (i.e. gamble D), should be preferred to a gamble in which A is received with probability .25 and otherwise nothing is received (i.e. gamble C). But subjects do not obey this latter choice pattern.

EXPERIMENT 7.2.13. Kahneman and Tversky (1979) also showed violations of the independence axiom for risky choice in which none of the gambles being compared involve a certain outcome. They gave 66 subjects two problems:

(1) Choose between (A) a 45% chance to win $6000 or (B) a 90% chance to win $3000.
(2) Choose between (C) a .1% chance to win $6000 or (D) a .2% chance to win $3000.

In the first problem, 86% of subjects chose B, while in the second problem, 73% of subjects chose C.

EXERICISE 7.2.14. Show that the results of experiment 7.2.13 violate the independence axiom.

DEFINITION 7.2.15. A weak preference relation P^* satisfies the *reducibility* or *distribution* (or *consequentialist*) axiom for risky choice over an outcome set X iff $\forall x,y$ in X and probabilities $p,q \in (0,1]$, $(\langle(\langle x,p\rangle,\langle y,1-p\rangle),q\rangle,\langle y,1-q\rangle)\ I\ (\langle x,pq\rangle,\langle y,1-pq\rangle)$, where I is the indifference relation defined by P^*.

The reducibility axiom says that a *compound gamble/lottery* of the kind given on the left hand side of the I relation in this definition is indifferent to a *simple* gamble of the kind given on the right hand side.

EXPERIMENT 7.2.16. *Isolation* or *pseudo-certainty effect*. Kahneman and Tversky (1979) told subjects:

> "Consider the following two-stage game. In the first stage, there is a probability of .75 to end the game without winning anything, and probability of .25 to move into the second stage. If you reach the second stage you have a choice between [an 80% to win $4000], or [$3000 for certain]. Your choice must be made before the game starts, i.e. before the outcome of the first stage is known."

They explain for readers of the results: "Note that in this game, one has a choice between .25 x.80= .20 chance to win 4,000, and a .25 x 1.0 = .25 chance to win 3,000. Thus, in terms of final outcomes and probabilities one faces a choice between (4,000, .20) and (3,000, .25) …" For the two stage game problem, 78% of subjects chose the latter prospect ($3000 for certain if you move to the second stage). But when subjects are asked to choose between the reduced gambles (.20 chance to win $4000 versus .25 chance to win $3000), as noted in experiment 7.2.12 above (problem 2), 65% choose the former option. This pattern of preferences contradicts the reducibility axiom because the axiom implies that subjects should be indifferent between a 25% chance to get a 80% chance to win $4000, and a 20% chance to win $4000. The indifference relation implies that if the reduced form is preferred to a 25% to win $3000, then the two-stage form should also be preferred to a 25% chance to win $3000.

EXERCISE 7.2.17. Prove using the definitions of preference and indifference that the isolation effect in 7.2.16 violates the reducibility axiom (7.2.15).

DEFINITION 7.2.18. A weak preference relation P^* satisfies the *continuity* or *archimedean* axiom for risky choice over an outcome set X iff $\forall x,y,z$ in X, if xP^*y and yP^*z, then there exist probabilities $p,q \in (0,1)$ such that $(<x,p>,<z,1-p>)\ P^*\ y$ and $y\ P^*\ (<x,q>,<z,1-q>)$.

The continuity axiom says that there is no outcome so good or so bad that an arbitrary nonzero probability of receiving that good (or "bad") can completely determine one's preference between gambles.

EXAMPLE 7.2.19. *Willingness to risk dying*. Kreps (1988) provides the following example to argue for the intuitive plausibility of the continuity axiom [with notation modified for consistency with the above]: "*x* is a gamble in which you get $1000 for sure; *y* is a gamble in which you get $10 for sure; and *z* is a gamble in which you are killed for sure. Most people would express the preferences xPy and yPz. And so, the axiom holds, there must exist a probability $p \in (0,1)$, presumably close to 1, such that $(<x,p>,<z,1-p>)Py$. That is, you are willing to risk a small (but nonzero) chance of your death to trade up from $10 to $1000." Does this seem reasonable to you?

THEOREM 7.2.20. *Expected utility theorem* (von Neumann and Morgenstern, 1944). P^* is a weak preference relation satisfying the reducibility, independence, and continuity axioms for risky choice over a space of lotteries/gambles $\mathscr{L}$ defined over an outcome set X iff there exists a function $U: \mathscr{L} \rightarrow \mathbb{R}$ such that for all lotteries L and L' in $\mathscr{L}$,
(a) U represents P^* over $\mathscr{L}$, that is $L\ P^*\ L'$ iff $U(L) \geq U(L')$, where $U(L) = U(<x_1,p_1>,<x_2,p_2>,\ldots<x_n,p_n>) = \sum U(x_j)p_j$ and $U(L')$ is defined analogously.
(b) U is an interval scale, that is, if V is any other function satisfying 1 and 2, then there exist real numbers b and $a > 0$, such that $V(x) = aU(x)+b$ also represents P^* over $\mathscr{L}$.
Proof. See Mas-Collel, Whinston, and Green (1995) (handout).

EXAMPLE 7.2.21. *Allais paradox.* (M. Allais, *Econometrica, 21*:503-546, 1953) [updated version]. This famous paradox showed how combining independence and reducibility leads to a counterintuitive preference pattern for many, perhaps most people. It was one of the earliest challenges to expected utility theory not just as a descriptive theory but also as a normative theory. Compare the following two situations:

Situation 1	*Situation 2*
Choose between:	Choose between:
Gamble 1: $5000	Gamble 3: <$5000,.11>,<$0,.89>
Gamble 2: (<$7500,.10>,<$5000,.89>,<$0,.01>)	Gamble 4: <$7500,.10>,<$0,.90>

Most people prefer gamble 1 to gamble 2, but prefer Gamble 4 to Gamble 3, even though this pattern is inconsistent with the independence and reducibility axioms. In particular:

gamble 1 *P* gamble 2 can be rewritten as
(<$5000,.11>,<$5000,.89>) *P* [<(<$0,1/11>,<$7500,10/11>),.11>,<$5000,.89>]; and
gamble 4 *P* gamble 3 can be rewritten as
[<(<0,1/11>,<$7500,10/11>),.11>,<$0,.89>] *P* (<$5000,.11>,<$0,.89>) (cf independence).

Since expected utility theory requires an ordering consistent with the interval function of utility, this pattern of preferences cannot be accommodated. In particular, the preference for gamble 1 over gamble 2 implies that u(gamble 1) > u(gamble 2), and hence that u($5000) > .10u($7500)+.89u($5000)+.01u(0), so .11u($5000) > .10u($7500)+.01u($0). But the preference in situation 2 implies that u(gamble 4) > u(gamble 3); hence .10u($7500)+.90u($0) > .11u($5000)+.89u($0), implying .10u($7500)+.01u($0) > .11u($5000), contradicting the inequality derived from the most common preference in situation 1.

7.3 Uncertainty

The expected utility theorem asssumes that probabilities are known. But often in a choice situation, there is uncertainty about outcomes without known probabilities. In such situations, we may apply the concept of *subjective probability*, or what we believe the probability to be. This is essentially the same *confidence*, defined in 4.2.1.

PRINCIPLE 7.3.1. (Ramsey, 1927). If an agent has confidence *D(x)=d* that an event or outcome *x* will occur, then for a given prize $, that agent should be indifferent between a lottery that (a) pays $ if x happens, and 0 otherwise, and (b) pays $ if an event with known probability *d* happens, and 0 otherwise.

EXAMPLE 7.3.2. *Ellsberg Paradox*. Ellsberg (1961) proposed the following scenario. An urn has balls of three colors (red, blue, and yellow), with 1/3 red balls and unknown proportions of blue and yellow balls. He suggested (and subsequent tests have verified) that most people prefer (are not indifferent between) a bet that pays some amount of money $ only if a red ball is drawn and a bet that pays the same amount ($) if a yellow ball is drawn. The most common pattern is a preference for the “red” bet. But this violates 7.3.1, because the subjective probability estimate given by those who prefer the sure bet is 1/3 for yellow. A number of experiments have demonstrated that subjects in large numbers prefer the “unambiguous” bet (red) over the “ambiguous” one (blue or yellow) (see Camerer and Weber, 1992, for a review).