Statistics

Kevin: I'm coming up with 32.33%, uh, repeating of course, percentage of survival

Tom: Well that's a lot better than we usually do

Where do stats matter?

- Statistics and combinatorics has a deep history in gambling
- Magic cards are just like playing cards as far as ordering
- Deal mostly with combinatorics

Important questions in Magic

- Why do most decks have exactly 60 cards?
- Why do most decks stay around 36 spells and 24 lands?
- How many lands in your opening hand are correct not to mulligan?

Why do most decks have exactly 60 cards?

- you want to draw you best spells
- x copies of Baneslayer Angel, y cards
- trying to maximize x/y
- That's easy

Why do most decks stay around 36 spells and 24 lands?

- true: you can only play 1 land a turn
- generally true: more expensive spells are better
- generally true: you want to draw just enough lands to play all of your spells

I want a deck named after me…

2 Dwarven Trader
2 Goblin of the Flarg
4 Ironclaw Orcs
3 Dwarven Lieutenant
2 Orcish Librarian
2 Brothers of Fire
2 Orcish Artillery
2 Orcish Cannoneers

2 Dragon Whelp

4 Lightning Bolt
4 Incinerate
1 Fireball
1 Immolation
1 Shatter
1 Detonate

4 Brass Man
1 Black Vise

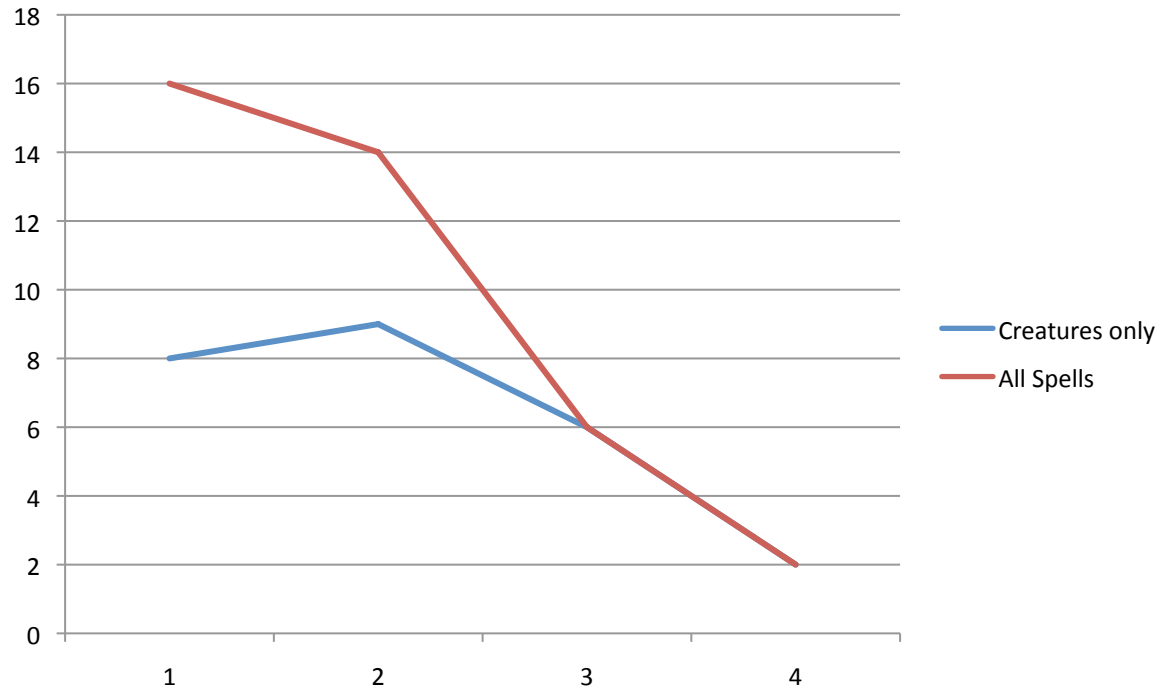
4 Strip Mine
4 Mishra's Factory
2 Dwarven Ruins
13 Mountain

- Piloted by Paul Sligh at PT Atlanta '96
- Made Semis
- During the Black Summer
- But this deck is bad

It’s really bad

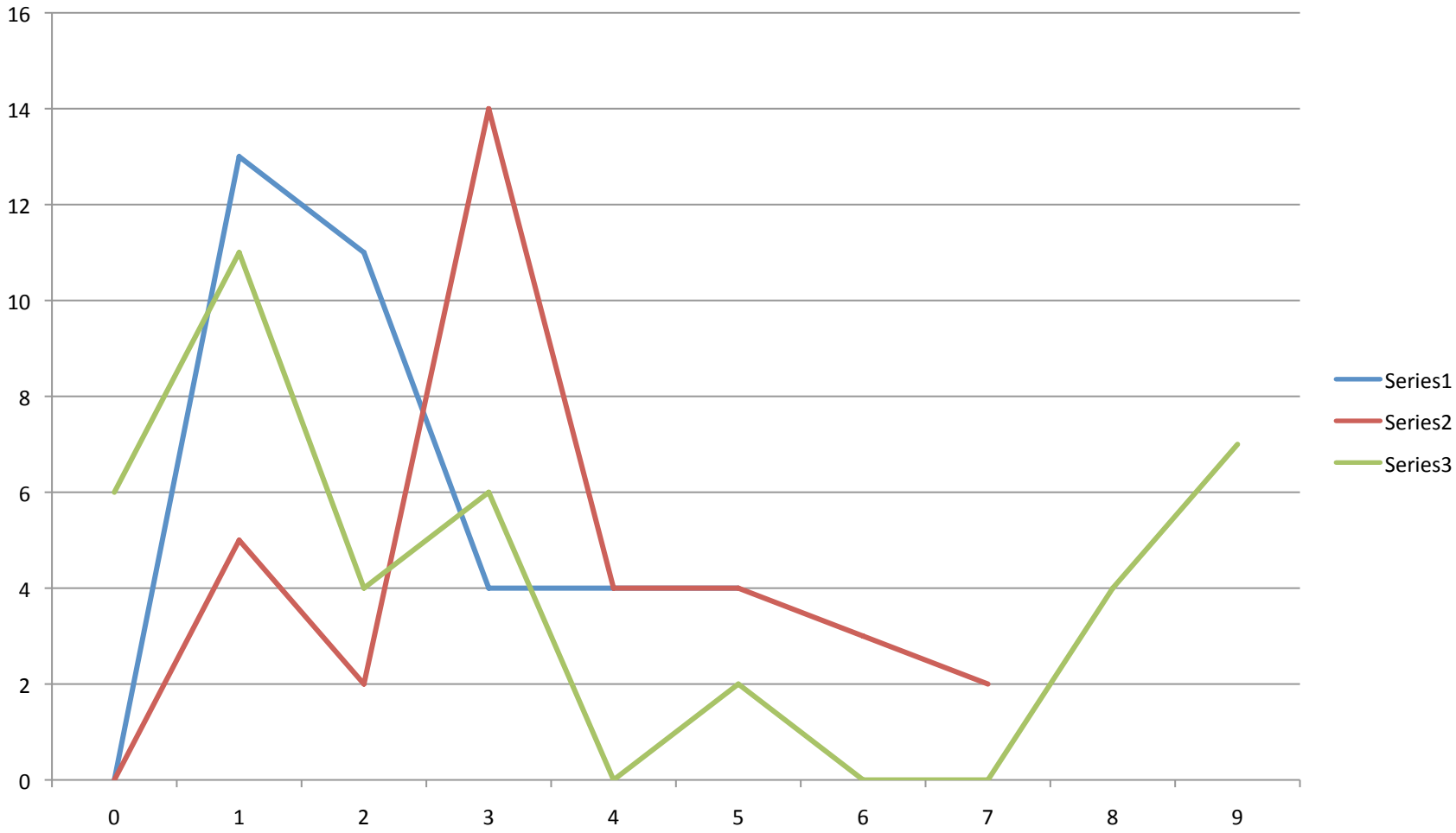
The mana curve*

1 mana slot: 9-13
2 mana slot: 6-8
3 mana slot: 3-5
4 mana slot: 1-3
X spell: 2-3
Lightning bolt (critter kills): 8-10
mana 23-26 15-17 of color



*according to Jay Schneider

Guess the deck



So how many lands do you need?

- 24 is standard
  - 24 * (10 / 60) = 4 lands by turn 4
- Might scale downward as low as 20 for aggro
  - 20 * (10 / 60) = 3 lands by turn 4
- Might go up to 27 for control decks
  - card drawing nets you extra cards
  - 7 card opening and 2 draw spells
  - 27 * (17 / 60) = 7.65 lands by turn 7

How to search for creatures?

The Setup

- Y creatures left in your deck
- Z cards left in your deck (assume 50 from now on)
- Expected number of creatures revealed by both?

Commune with Nature

- Regardless of creatures revealed, you only get 1
  - $P(X > 0)$
- Equivalent to $1 – P(X > 0^c)$
  - or $1 – P(X = 0)$

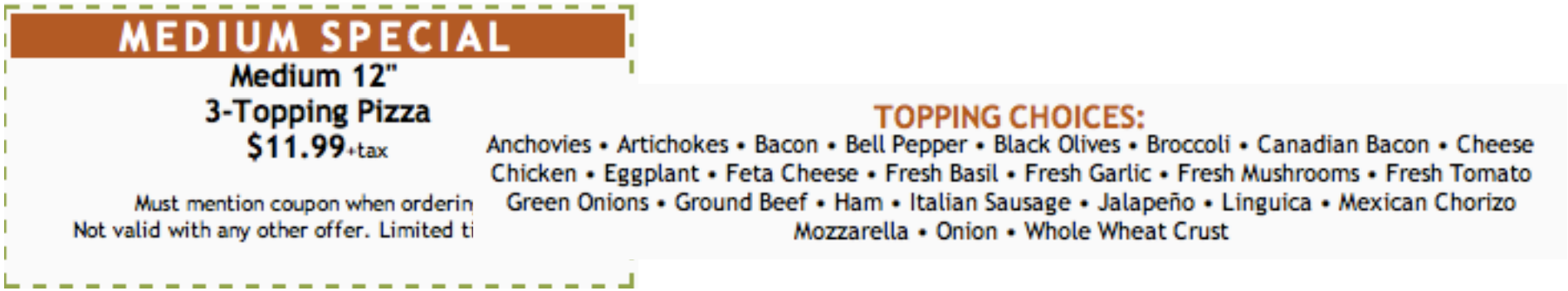
Combinations

- Combinations are the number of ways that k elements can be taken from a set of n elements
- Combinations are unordered

$$\mathbf{C}(n,k) = \mathbf{C}_n^k = \mathbf{C}_k^n = {}_nC_k = \binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

How combinations work

- I'm a cheap-ass when it comes to ordering pizza
- 25 (n) different toppings, choose 3 (k) of them
- 25 * 24 * 23 different ways to choose an ordered set of them, or n! / (n-k)!
- don't care about order, and k! ways to order
- So 2300 different pizzas



The math

P(X > 0) = $1 - \frac{\binom{50-Y}{5}}{\binom{50}{5}}$ Number of combinations of picking non-creature

Total combinations

So if you have, say, 20 creatures left

P(X > 0) = $1 - \frac{\binom{30}{5}}{\binom{50}{5}}$ = 0.93274084842077441522399894277785119598255583454473371217 12...

$\mathrm{E}(X) = \sum_i x_i p(x_i)$ so we have an expected value of about .93 with 20 creatures

Beast Hunt

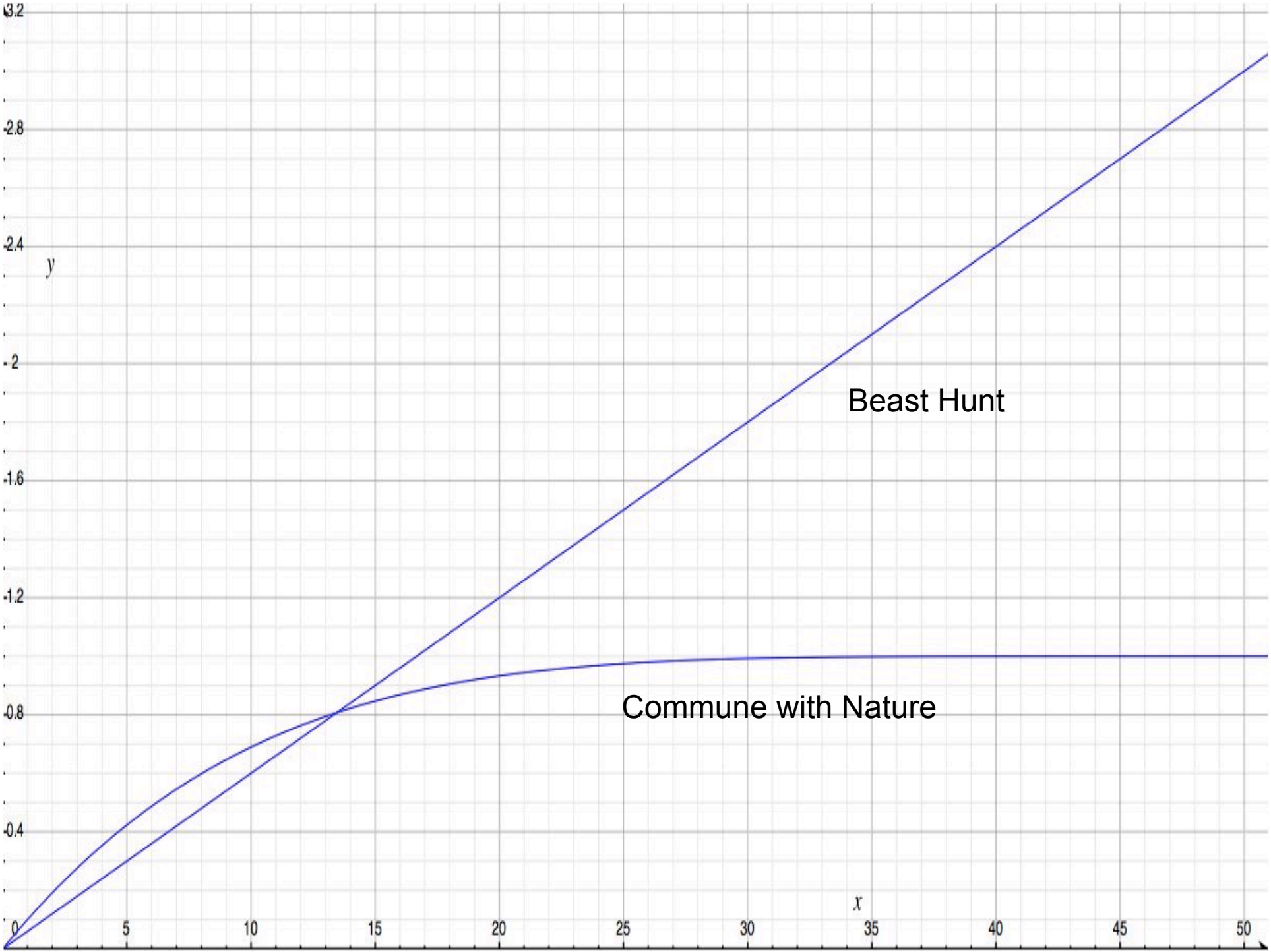
- Scales from 0 to 3 creatures, each of which has a different probability of happening

$$P(X = i) = \frac{\binom{Y}{i}\binom{50-Y}{3-i}}{\binom{50}{3}}$$

$$E(X) = P(X = 0) * 0 + P(X = 1) * 1 + P(X = 2) * 2 + P(X = 3) * 3$$

For 20 creatures left

$$E(X) = 29/140 * 0 + 87/196 * 1 + 57/196 * 2 + 57/980 * 3 = 1.2$$


3.2
2.8
2.4
y
2
1.6
1.2
0.8
0.4
0
5
10
15
20
25
30
35
40
45
50
x
Beast Hunt
Commune with Nature

The distribution

- Drawing cards is a hypergeometric distribution

$$P(X = k) = \frac{\binom{m}{k}\binom{N-m}{n-k}}{\binom{N}{n}}.$$

- k = successes observed
  - creatures drawn
- m = successes possible
  - creatures in deck
- N = size of the population
  - deck size
- n = draws

Do you mulligan 1-landers?

- Let's say that a victory is drawing 3-4 lands in your first 4 turns
  - Less and you're screwed
  - More and you're flooded
- Assume we have a standard 24 land, 60 card deck that is perfectly shuffled

Assume you're going first

Victory with the 1 lander

- P(X = 3) + P(X = 4) where X is the # of lands
- you draw 3 more cards by turn 4
- k = 2, m = 23, N = 53, n = 3
- $P(X = 3) = \frac{\binom{23}{2}\binom{53-23}{3-2}}{\binom{53}{3}} = .324$

so about 40% of drawing out of it

- k = 3, m = 23, N = 53, n = 3
- $P(X = 4) = \frac{\binom{23}{3}\binom{53-23}{3-3}}{\binom{53}{3}} = .076$

Victory with a mulligan

- P(X = 3) + P(X = 4)
- You draw 6 + 3 more cards by turn 4
- k = 3, m = 24, N = 60, n = 9
- $P(X = 3) = \frac{\binom{24}{3}\binom{60-24}{9-3}}{\binom{60}{9}} = .267$

- k = 4, m = 24, N = 60, n = 9
- $P(X = 4) = \frac{\binom{24}{4}\binom{60-24}{9-4}}{\binom{60}{9}} = .271$

so about 55% of a good hand

That's better

Assume you’re going second

Victory with the 1 lander

- P(X = 3) + P(X = 4)
- You draw 4 cards by turn 4
- k = 2, m = 23, N = 53, n = 4
- $P(X = 3) = \frac{\binom{23}{2}\binom{53-23}{4-2}}{\binom{53}{4}} = .375$

so about 55% of drawing out

- k = 3, m = 23, N = 53, n = 4
- $P(X = 4) = \frac{\binom{23}{3}\binom{53-23}{4-3}}{\binom{53}{4}} = .181$

Victory with a mulligan

- P(X = 3) + P(X = 4)
- You draw 6 + 4 more cards by turn 4
- k = 3, m = 24, N = 60, n = 10
- $P(X = 3) = \frac{\binom{24}{3}\binom{60-24}{10-3}}{\binom{60}{10}} = .224$
- k = 4, m = 24, N = 60, n = 10
- $P(X = 4) = \frac{\binom{24}{4}\binom{60-24}{10-4}}{\binom{60}{10}} = .274$

so about 49.8% chance of getting enough lands

Monte Carlo Simulations

- Monte Carlo simulations approximate something by repeated sampling
- Often used for problems
  - without closed forms
  - with absurdly complicated closed forms
- Magic games are definitely the latter
- How can we use the Monte Carlo method?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How do you goldfish?

- Goldfishing is just drawing hands to see how a deck does
  - Good way to see how a deck curves
- Also tells you how fast a deck is
  - Hulk Flash could win turn .5
  - Goblins can win turn 3
- Playtesting is essential
- Can we quantify that?

How do these go together?

- Monte Carlo simulations can run millions of games in seconds
- Can be scaled upwards to determine how games go
- Or can be as simple as finding combo pieces

Cascade Swans

Parth Modi, Regionals 2009

42 Lands (of various types)

4 Bloodbraid Elf
4 Swans of Bryn Argoll

4 Seismic Assault
4 Bituminous Blast
2 Ad Nauseam

+

= massive card drawing

Other cards

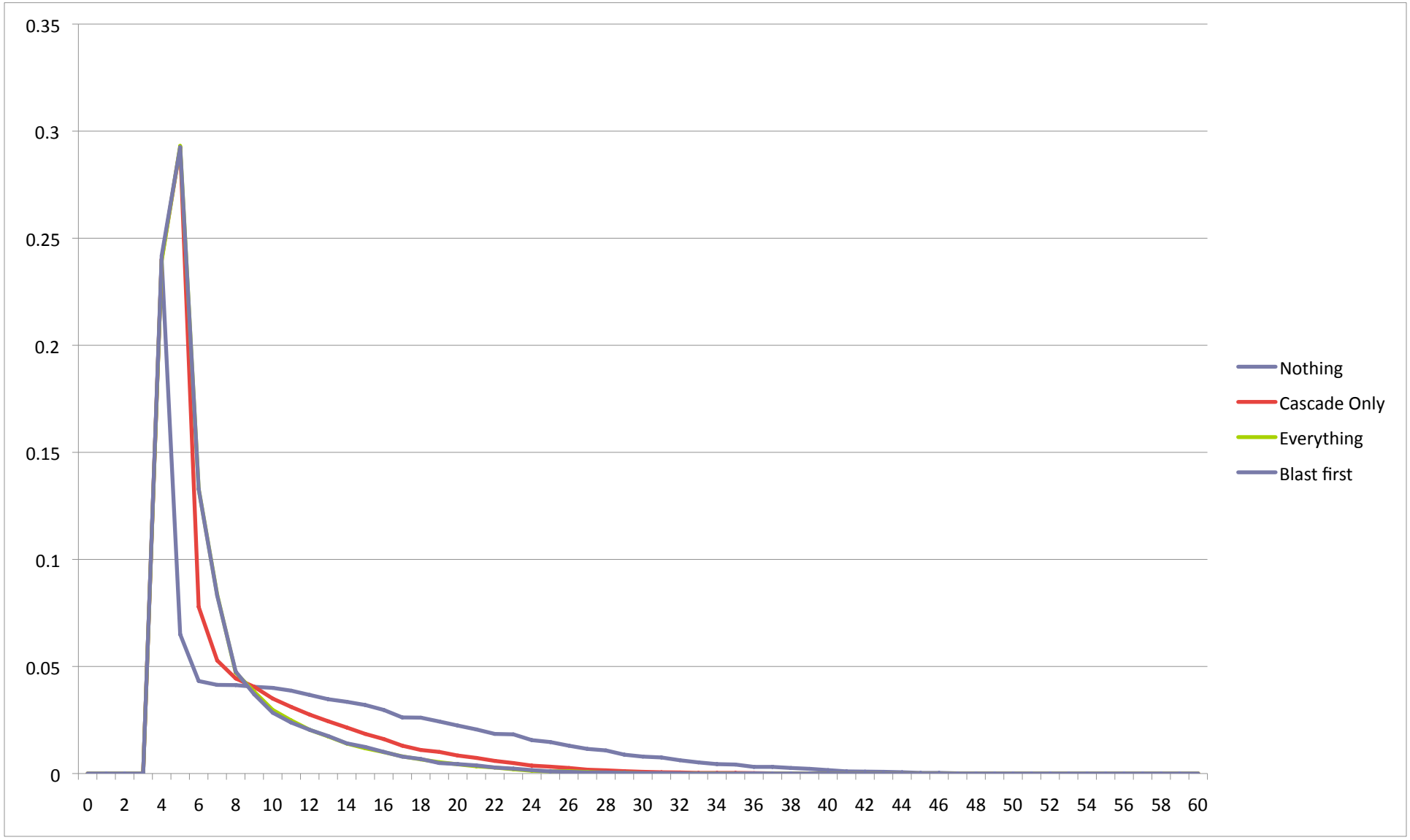

Bituminous Blast
Instant
Cascade (When you play this spell, remove cards from the top of your library from the game until you remove a nonland card that costs less. You may play it without paying its mana cost. Put the removed cards on the bottom in a random order.)
Bituminous Blast deals 4 damage to target creature.
Raymond Swanland


Bloodbraid Elf
Creature — Elf Berserker
Haste
Cascade (When you play this spell, remove cards from the top of your library from the game until you remove a nonland card that costs less. You may play it without paying its mana cost. Put the removed cards on the bottom in a random order.)
3/2
Dominick Domingo

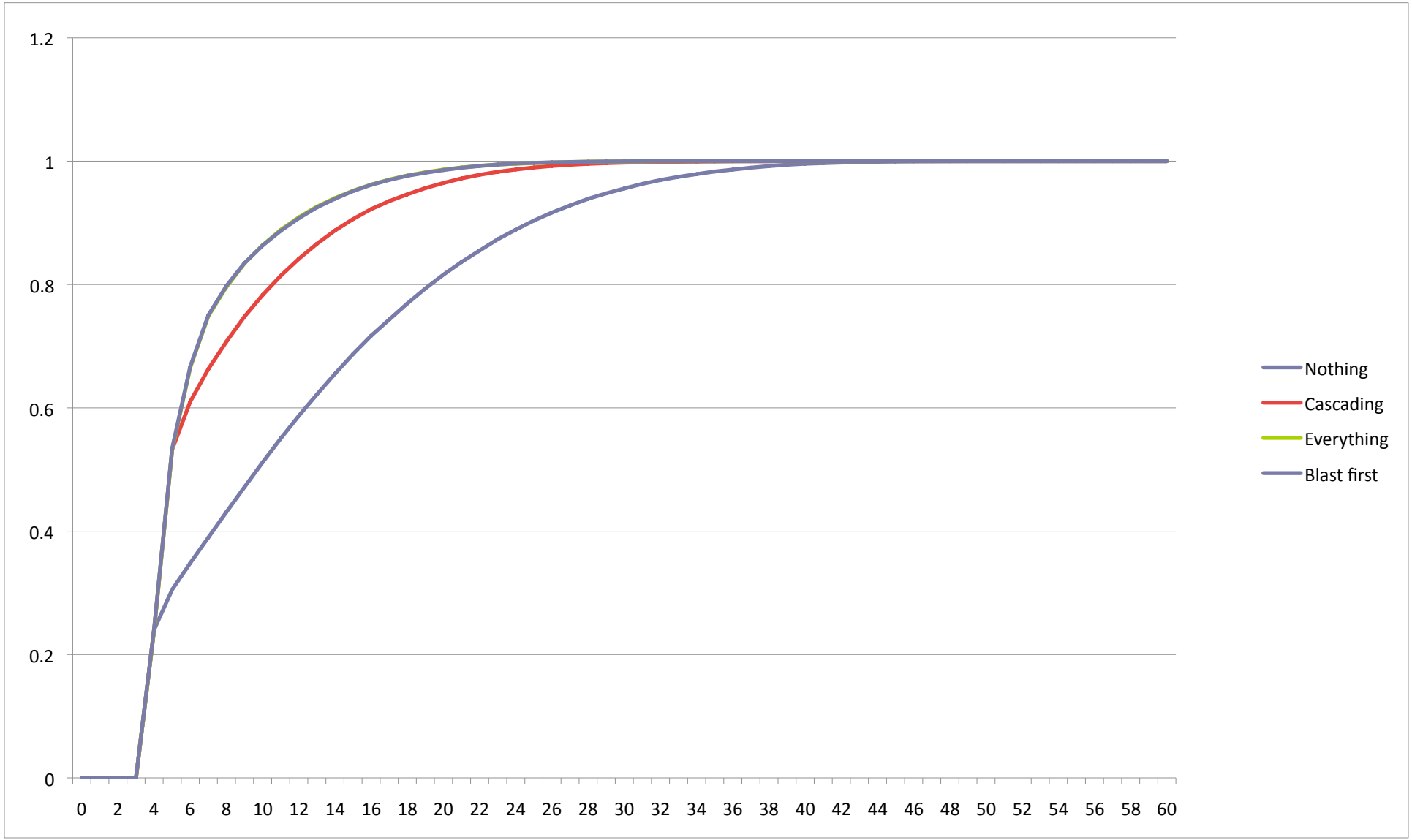

Ad Nauseam
Instant
Reveal the top card of your library and put that card into your hand. You lose life equal to its converted mana cost. You may repeat this process any number of times.
When the task spilled over into undeath, he stopped calling it his life's work.
Jeremy Jarvis

Let’s simulate

Some results



Seen differently



It’s not perfect

- Like any model, it’s missing a lot
- Just goldfishing
- Assume that swans + assault is a win
- No mulligans
- Ad Nauseam is done on life total
- Didn’t account for colors

How are you a better player?

- Play 60 cards in your deck
- Build decks with a number of lands based on how many you need
- Smooth out your mana curve to be efficient
- Know when to mulligan
- Understand the significance of playtesting in tuning
- Think about your game quantitatively

Want more?

- Probabilities of Gift of the Gargantuan
- Closed form for some combos
- Best way to shuffle
- Hypothesis testing for drafting

Where Can You Learn More?

- Stats 116, CS 109
- Basic Probability – MIT Opencourseware
- Mathemagics: Onslaught Fetch Lands Revisited – Garrett Johnson