

# Game Theory

Kevin: So according to game theory, it is not advantageous for either of us to work on this lecture

Tom: I didn't need game theory to tell me that I'm lazy; I already knew that

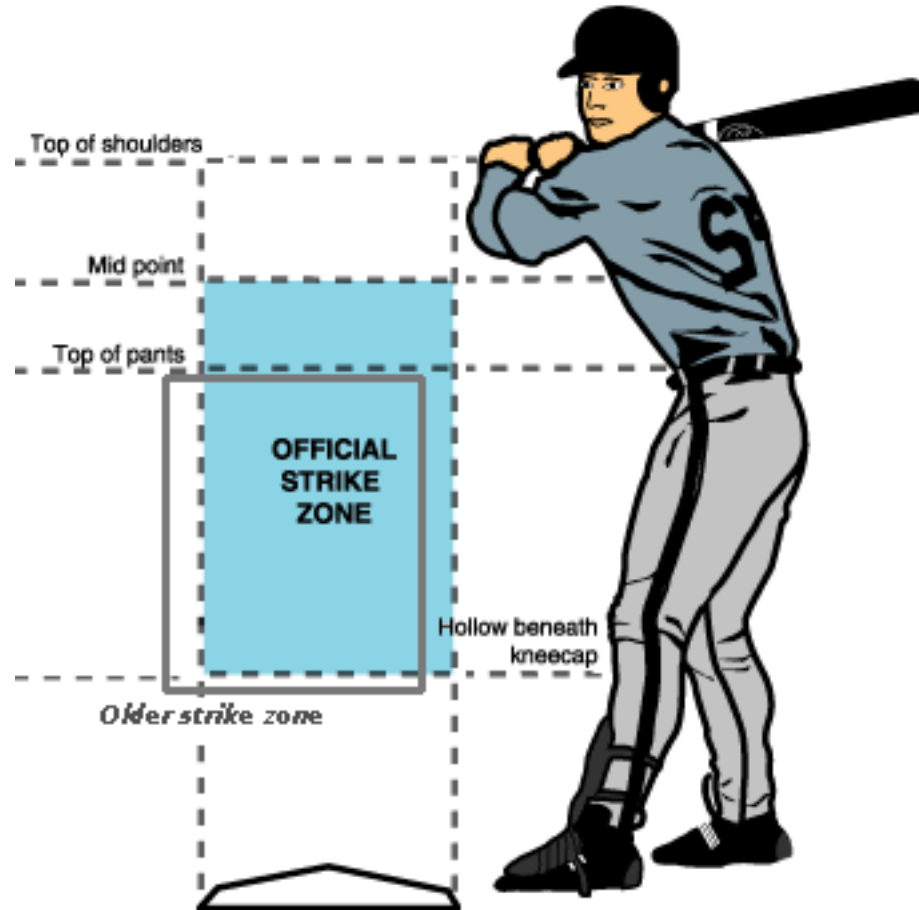
# What is Game Theory?

- Game theory is a mathy formalization of strategic situations
- A strategic situation is any situation where your success depends on what other people do
- This includes

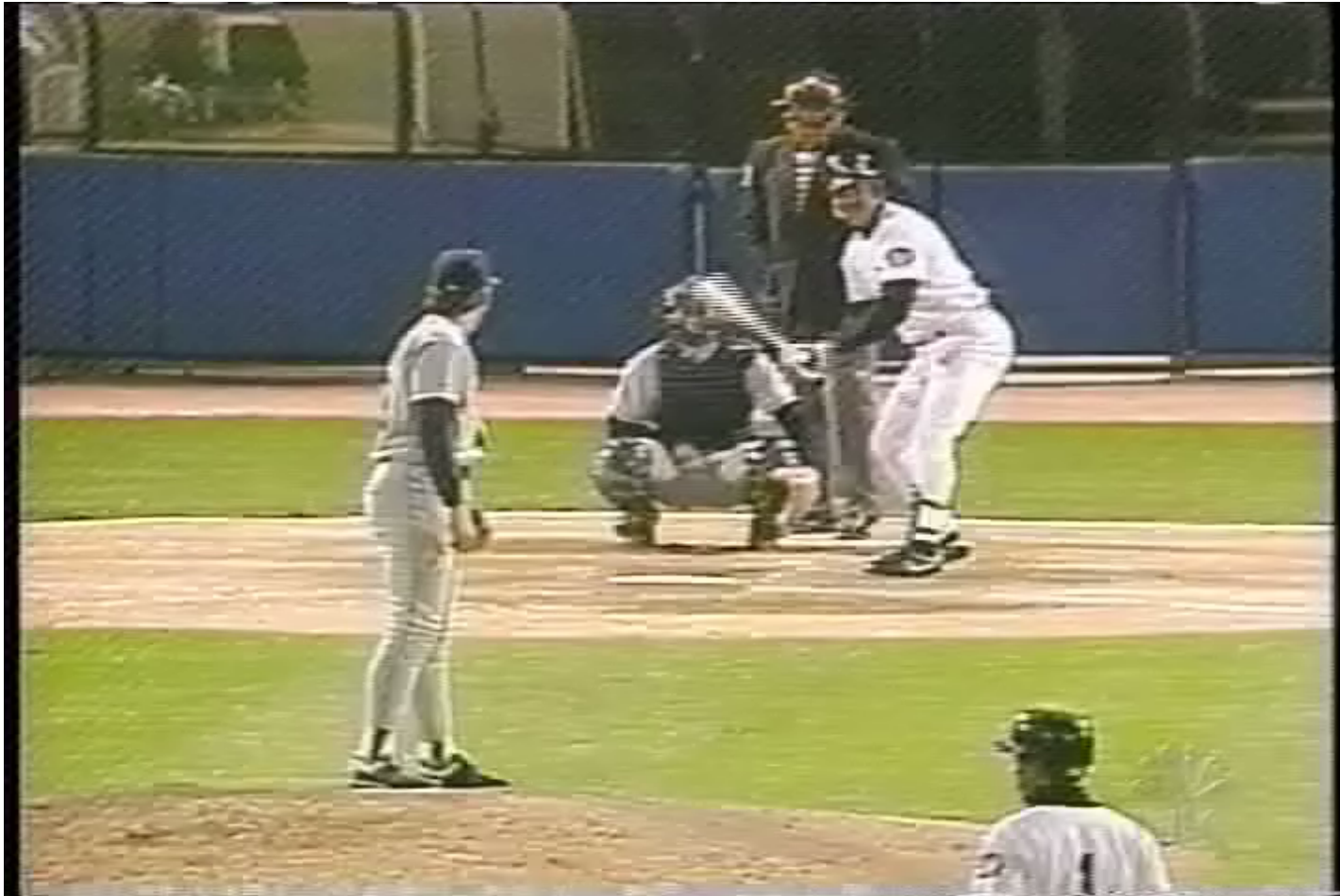
# Chess



# Baseball – ball or strike



# Baseball – pitching in general

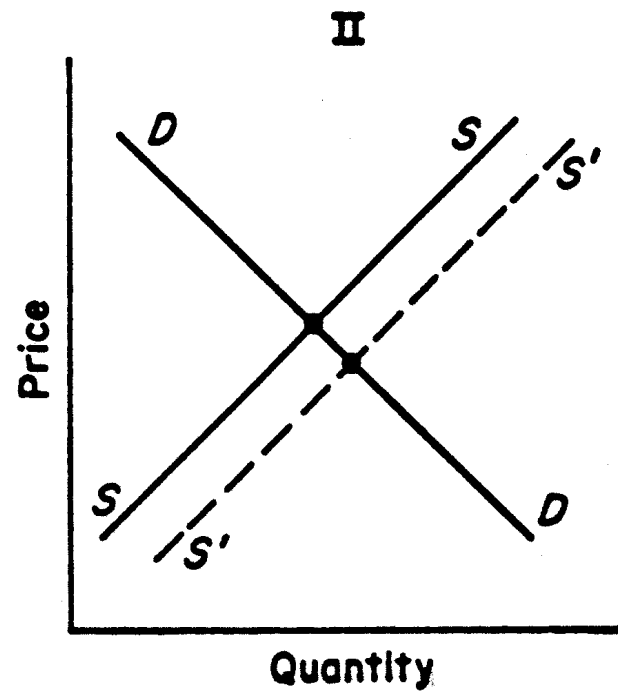
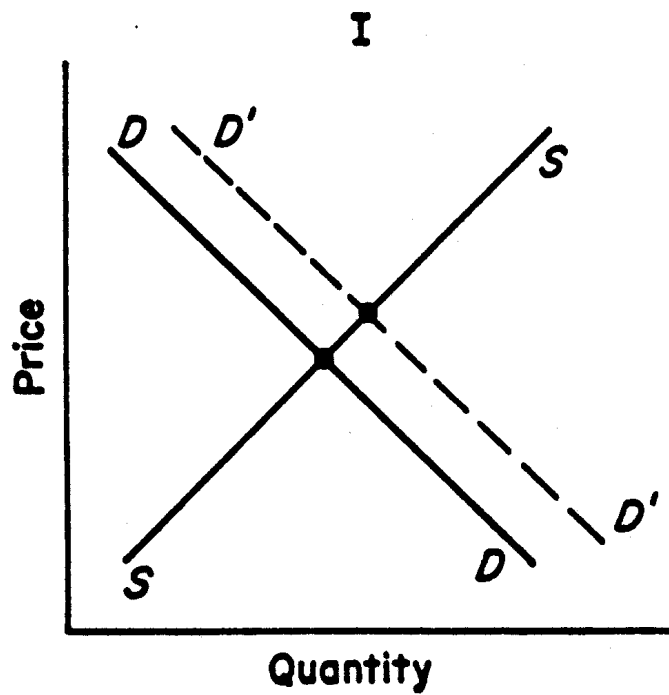


# Asking someone on a date

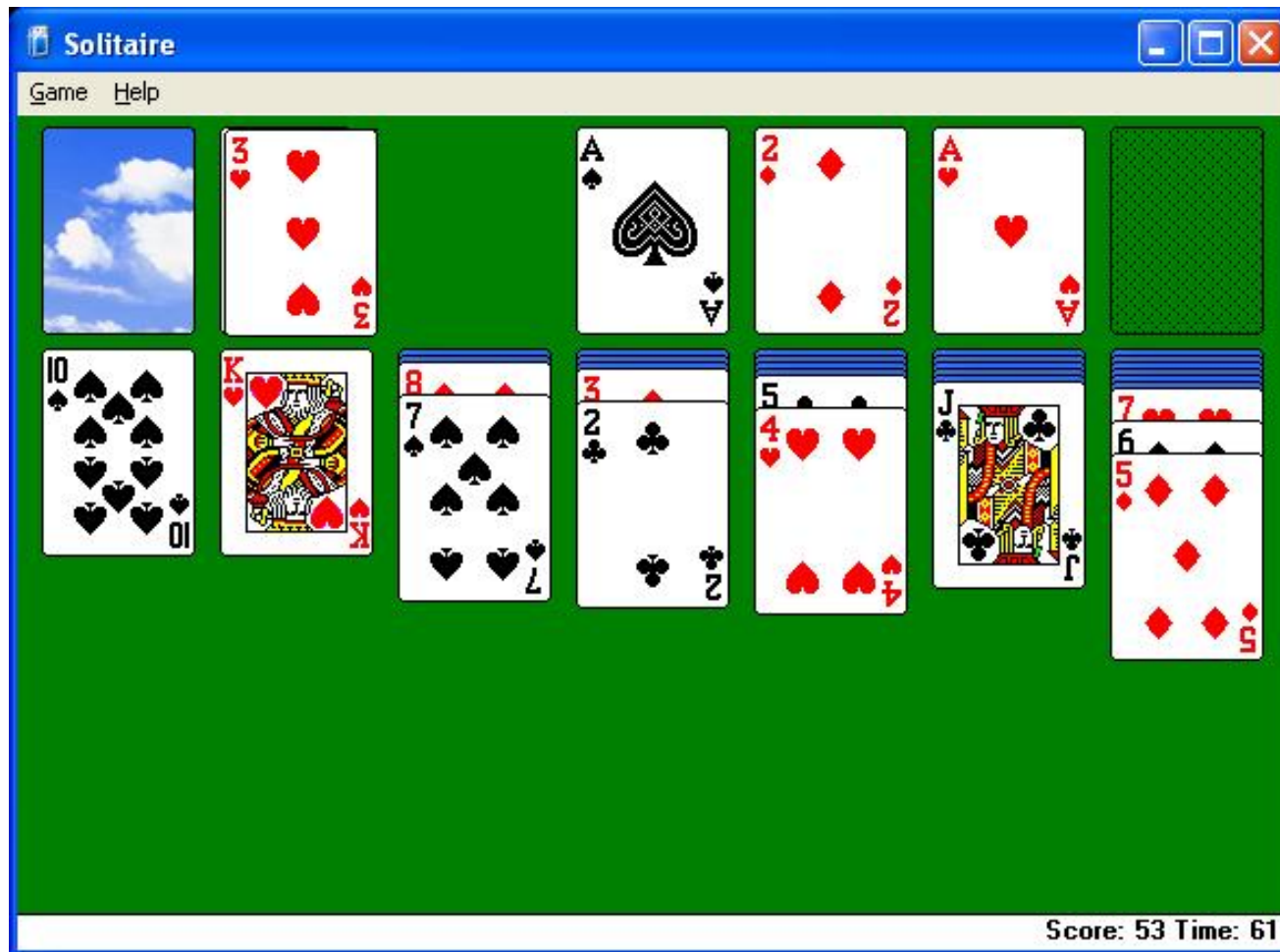


\*Likely not important in this class

# Production levels



# NOT - Solitaire





# NOT – iTunes choices

The screenshot shows the iTunes application interface. The top menu bar includes 'iTunes', 'File', 'Edit', 'View', 'Controls', 'Store', 'Advanced', 'Window', and 'Help'. The status bar at the top right shows the system temperature (45°C), time (Sun 12:42 PM), and search icon. The main window title is 'iTunes'. Below the title bar is a playback control bar showing the current song 'Monday Night Magic #174 - Push Button, Receive Bacon' by Tom Gustafson, with a progress bar from 29:37 to -42:17. A search bar is located on the right side of the playback bar.

The left sidebar contains the following sections:

- LIBRARY: Music (30 items), Movies, TV Shows, Podcasts, Radio
- STORE: iTunes Store, Purchased
- ▶ SHARED
- ▼ PLAYLISTS: iTunes DJ, Yale University, ECON 159: Game The..., 90's Music, Classical Music, Music Videos, My Top Rated, Recently Added, Recently Played, Top 25 Most Played, Cedrol Fall 2007: Vol. I, Cedrol Fall 2007: Vol. II, Christmas Music, **Disney Stuff** (selected), Favorites, Kevin Leung's iPod Selec..., LAME Import, Pop, Stands Tunes, Stuff I've Played, Trumpet Tunes, Tuba Orchestral Pieces

The main playlist table is as follows:

	Name	Time	Artist	Composer	Album	Pla...	Rating
1	<input checked="" type="checkbox"/> Part Of Your World	3:16	Jodi Benson	Alan Menken & Howar...	The Little Mermaid	32	
2	<input checked="" type="checkbox"/> Under The Sea	3:16	Samuel E. Wright	Alan Menken & Howar...	The Little Mermaid	33	
3	<input checked="" type="checkbox"/> Les Poissons	1:36	Rene Aberjonois	Alan Menken & Howar...	The Little Mermaid	28	
4	<input checked="" type="checkbox"/> Kiss The Girl	2:44	Samuel E. Wright	Alan Menken & Howar...	The Little Mermaid	27	
5	<input checked="" type="checkbox"/> Belle	5:10	Walt Disney Music Company w/P...	Menken, Alan and Ho...	Beauty And The Beast	13	
6	<input checked="" type="checkbox"/> Belle (Reprise)	1:05	Walt Disney Music Company w/P...	Menken, Alan and Ho...	Beauty And The Beast	13	
7	<input checked="" type="checkbox"/> Gaston	3:40	Walt Disney Music Company w/R...	Menken, Alan and Ho...	Beauty And The Beast	12	
8	<input checked="" type="checkbox"/> Gaston (Reprise)	2:04	Walt Disney Music Company w/R...	Menken, Alan and Ho...	Beauty And The Beast	7	
9	<input checked="" type="checkbox"/> Be Our Guest	3:45	Walt Disney Music Company w/J...	Menken, Alan and Ho...	Beauty And The Beast	10	
10	<input checked="" type="checkbox"/> Something There	2:19	Walt Disney Music Company w/R...	Menken, Alan and Ho...	Beauty And The Beast	11	
11	<input checked="" type="checkbox"/> The Mob Song	3:31	Walt Disney Music Company	Menken, Alan and Ho...	Beauty And The Beast	9	
12	<input checked="" type="checkbox"/> Beauty And The Beast	2:47	Walt Disney Music Company w/A...	Menken, Alan and Ho...	Beauty And The Beast	9	
13	<input checked="" type="checkbox"/> One Jump Ahead	2:23	Brad Kane	Alan Menken	Aladdin OST	35	
14	<input checked="" type="checkbox"/> Friend Like Me	2:26	Robin Williams	Alan Menken	Aladdin OST	32	
15	<input checked="" type="checkbox"/> Prince Ali	2:52	Robin Williams	Alan Menken	Aladdin OST	28	
16	<input checked="" type="checkbox"/> A Whole New World	2:41	Brad Kane, Lea Salonga	Alan Menken	Aladdin OST	31	
17	<input checked="" type="checkbox"/> Circle Of Life	3:59	Carmen Twillie & Lebo M.	Elton John/Tim Rice	The Lion King	27	
18	<input checked="" type="checkbox"/> I Just Can't Wait To Be King	2:51	Jason Weaver, Rowan Atkinson &...	Elton John/Tim Rice	The Lion King	25	
19	<input checked="" type="checkbox"/> Hakuna Matata	3:34	Nathan Lane, Ernie Sabella, Jaso...	Elton John/Tim Rice	The Lion King	33	
20	<input checked="" type="checkbox"/> Can You Feel The Love Tonight	2:58	Joseph Williams, Sally Dworsky, ...	Elton John/Tim Rice	The Lion King	25	
21	<input checked="" type="checkbox"/> The Virginia Company	1:30	Alan Menken & Stephen Schwartz	Alan Menken & Stephe...	Pocahontas	8	
22	<input checked="" type="checkbox"/> Steady As The Beating Drum (Main Title)	1:46	Alan Menken & Stephen Schwartz	Alan Menken & Stephe...	Pocahontas	8	
23	<input checked="" type="checkbox"/> Just Around The Riverbend	2:28	Alan Menken & Stephen Schwartz	Alan Menken & Stephe...	Pocahontas	8	
24	<input checked="" type="checkbox"/> Mine, Mine, Mine	3:06	Alan Menken & Stephen Schwartz	Alan Menken & Stephe...	Pocahontas	8	
25	<input checked="" type="checkbox"/> Colors Of The Wind	3:35	Alan Menken & Stephen Schwartz	Alan Menken & Stephe...	Pocahontas	7	
26	<input checked="" type="checkbox"/> Perfect World	2:21	Tom Jones		The Emperor's New Groove	22	

The status bar at the bottom of the iTunes window shows: 26 songs, 1.2 hours, 94.7 MB. On the right side of the status bar are 'Computer' and 'Burn Disc' buttons.

The dock at the bottom of the screen contains various application icons including Finder, Mail, Safari, iTunes, System Preferences, and a trash can. The system tray on the right shows the date (OCT 4) and a watermark for 'www.CREStock.com'.

# Breakdown of the class

- 3 examples, 3 topics
- Let's go

# Example #1: Star

is the loneliest number...

# Star Format

- 5 players, each playing a different color
- Each player trying to defeat enemy colors before anyone else
  - G player tries to destroy B and U
- Interesting format because of ally pairings
- Friend of an friend is an enemy?
  - Friend of an enemy is a frenemy?

# Example situation – Star Format



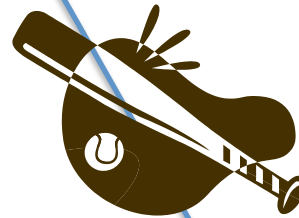
# Example situation – Star Format



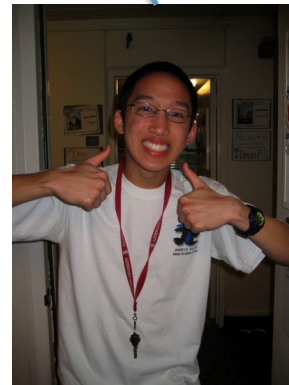
It's my turn  
Do I attack?



# I Swing, Akroma sits



20

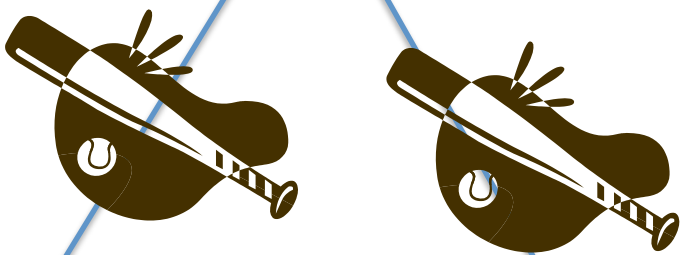


5





# Both Swing



15



15



# Neither swing



10



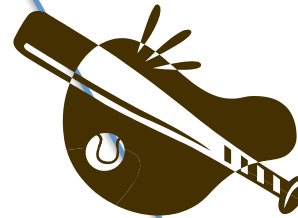
10

# What should I do?

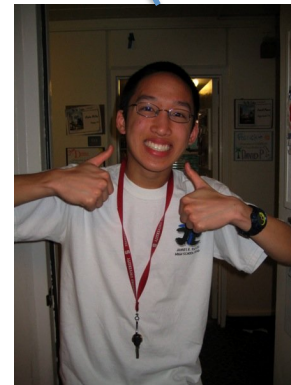


Assume ceteris paribus  
because it makes life  
easy

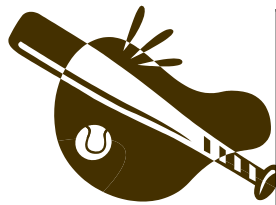
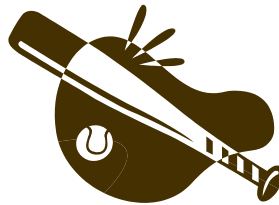
?



?

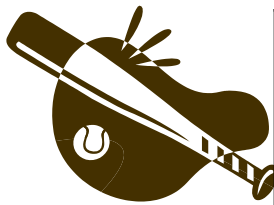
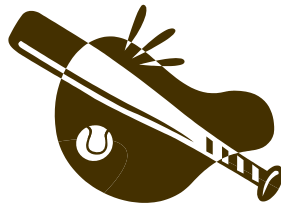


# A Better Way to Look at it



(15,15)	(5,20)
(20,5)	(10,10)

# If Akroma Swings...

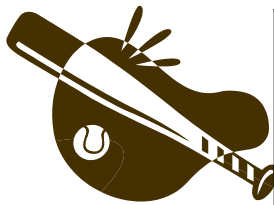
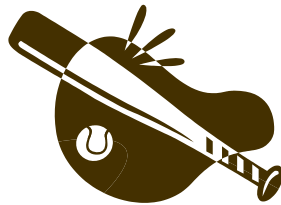


(15,15)



(20,5)

# We should sit

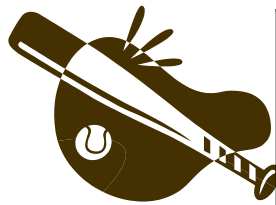
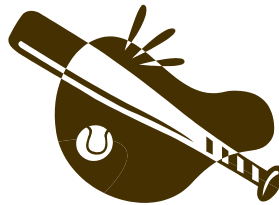


(15,15)



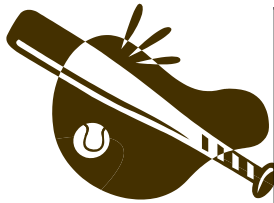
(20,5)

# If Akroma sits...

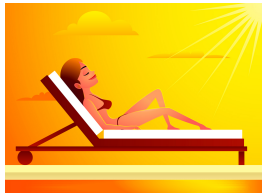


(15,15)	(5,20)
(20,5)	(10,10)

# We should sit



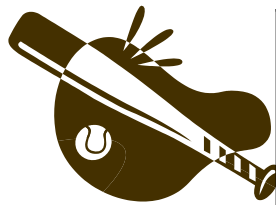
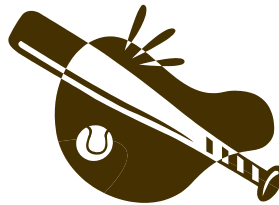
(5,20)

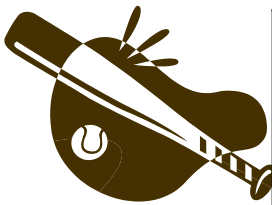



(10,10)



# We have a winner!

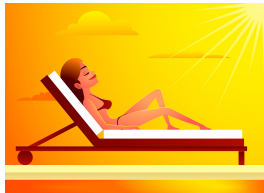
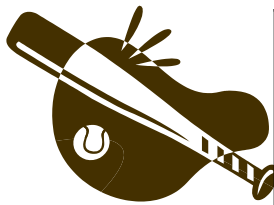
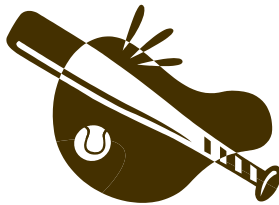


	(15,15)	(5,20)
	(20,5)	(10,10)

Our dominant strategy!

This is easy; so why is it known as the Prisoner's Dilemma?

# It's worse.



(15,15)	(5,20)
(20,5)	(10,10)

# How can we fix this?

- Why doesn't communication help?
- What did the 19<sup>th</sup> century railroad tycoons do?

# Why don't we play Magic like this?

- It's a repeated game
- Nothing is ceteris paribus
- Akroma knows what your choice is
- Life totals don't necessarily represent whether you're going to win
- More generally, you don't know what your opponents' goals are
  - This is why playing against Tom in a multiplayer match always gets me angry

# Example #2: Kithkin

can be as bad as 1

\* if 2 is Hilary Clinton

# A Different Situation



# Deck Lists

You – Kithkin - Florian Michalec

## **Lands (23)**

16 Plains

3 Rustic Clachan

4 Windbrisk Heights

## **Creatures (20)**

2 Burrenton Forge-Tender

3 Cloudgoat Ranger

4 Figure of Destiny

4 Goldmeadow Stalwart

4 Knight of the White Orchid

3 Ranger of Eos

## **Spells (17)**

2 Ajani Goldmane

3 Harm's Way

4 Honor of the Pure

4 Path to Exile

4 Spectral Procession

Him – Jund Cascade – Pierre Simonnin

## **Lands (24)**

It's irrelevant

## **Creatures (22)**

4 Bloodbraid Elf

4 Boggart Ram-Gang

4 Demigod of Revenge

4 Kitchen Finks

4 Putrid Leech

2 Sygg, River Cutthroat

## **Other Spells (14)**

2 Doom Blade

3 Jund Charm

4 Lightning Bolt

4 Maelstrom Pulse

1 Volcanic Fallout



# So What Are Our Options?





# 1) Just Swing



# 2) HotP

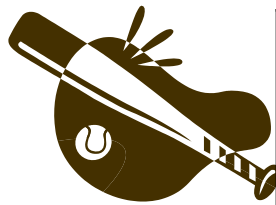




# 3) Tom



# Chart



4	5
3	7
2	2

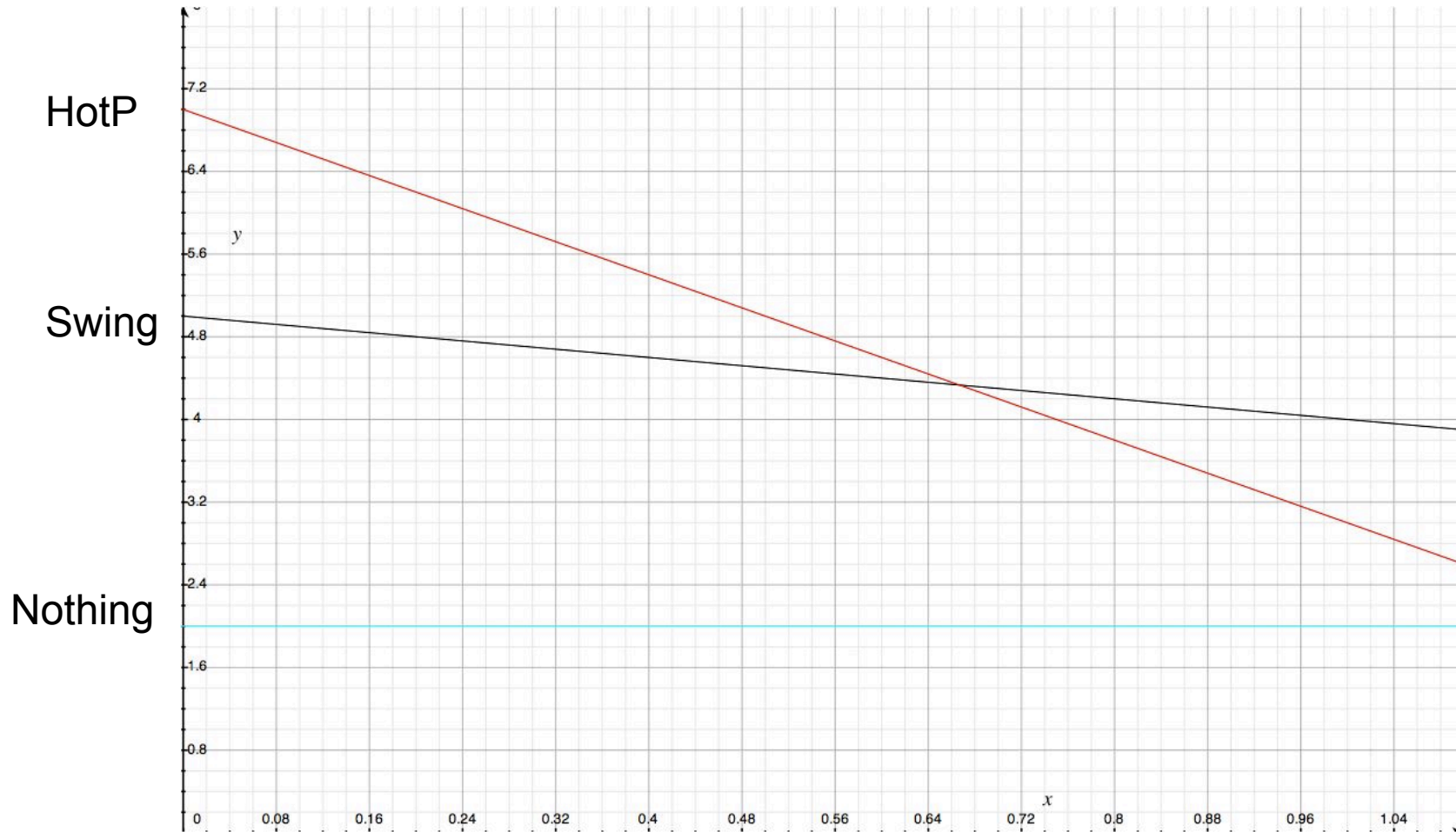
What did we learn from the last game?

Can we do better?  
How can we think of this problem differently?

# Analysis

- There is no dominant strategy
- Instead, we should choose based on our certainty
- If we're 50% sure they will bolt, what should we do?
- If we're 80% sure they will bolt, what should we do?

# Best Response Graph



# Some Math...

- $4x + 5(1 - x) = 3x + 7(1 - x)$
- $5 - x = 7 - 4x$
- $3x = 2$
- $x = 2/3$

# Example #3: Drafting\*

\* also a good way to catch up in MarioKart



# Drafting

- Drafting is just as much of a game as Magic itself
- 8 people sit in a circle, each opens a pack
- Takes 1, passes 14 left, takes 1, passes 13 left...
- Pack 2, pass right
- Pack 3, pass left again
- 40 card deck, usually ~17 lands
- Sideboard is everything else

# How Do I Draft?

- BREAD
- Bombs
- Removal
- Evasion
- Acceleration / Aggressive
- Dumb stuff? It kind of breaks down
- So what colors do I pick?
- Volunteers?

# Drafting buddies

- Drafting the same color as someone else is bad
- You both like white
  - That's common knowledge
- Both open good white and red cards
- What do you choose?

# You knew it was coming...




A 3x3 grid representing a game matrix. The top row and right column are empty. The middle row contains a red cell, a cell with the payoff  $(0,0)$ , and a cell with the payoff  $(1,2)$ . The bottom row contains a cell with the payoff  $(2,1)$  and a cell with the payoff  $(0,0)$ .

Make your choice...

So are there any dominated strategies?

What does best response tell you?

# Nash Equilibrium



When everyone is playing a best response to everyone else's strategy




A 3x3 grid representing a game matrix. The top row has three empty cells. The middle row has a red cell on the left, a white cell in the middle containing the payoff  $(0,0)$ , and a white cell on the right containing the payoff  $(1,2)$ . The bottom row has a white cell on the left, a white cell in the middle containing the payoff  $(2,1)$ , and a white cell on the right containing the payoff  $(0,0)$ .

# Nash Equilibrium



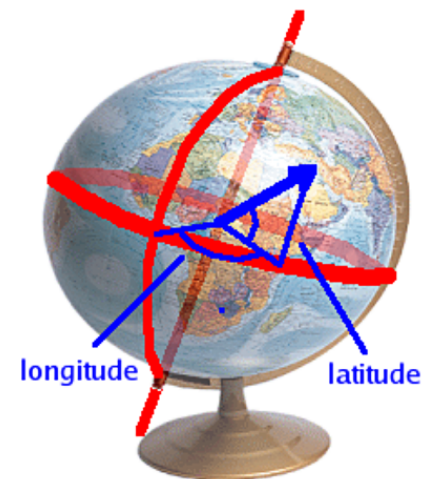
When everyone is playing a best response to everyone else's strategy



	(0,0)	(1,2)
	(2,1)	(0,0)

What can you do to get your best payoff?

Coordinate!



# Signaling in booster draft



- Avoid over-drafting a color
  - passing information is good
- 1<sup>st</sup> pack: doom blade, assassinate, pacifism
  - What do you take?
- Just as important to read signals

# Also Not Original

- Better known as the “Battle of the Sexes” based on picking a movie to see
  - Unsurprisingly, I’ve never had a problem like this
- Different than Prisoner’s Dilemma because it was just a coordination game
  - In Prisoner’s Dilemma, communication doesn’t matter
  - In this case, communication can help by playing a particular Nash Equilibrium
- Self-enforcing Nash Equilibrium



# How are you a better player?

- Do not play dominated strategies
  - Play close endgames like you get perfect topdecks
- Consider the expected payoff based on certainty
- Put yourself in their shoes
- Pay attention to signals and send signals
- Think about your games in terms of payoffs and strategies

# Want more?

- Mixed strategies use randomizations of pure strategies
  - Maybe a way to determine whether you should bluff attacks or not
- Iterative deletion towards NE in metagame shift

# Where Can You Learn More?

- Econ 160 (not offered; life sucks)
- [Ben Polak's lectures on Game Theory at Yale](#)
- [Intro to Game Theory – Frank Karsten](#)
- [Theory Games – Frank Karsten](#)