Optimal High-Frequency Market Making

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Fundamentals of Market Making

Example Order Book

Source of profits:

1. Repeatedly capture bid-ask spread
2. Obtain rebate for providing liquidity

Risk factors:

1. Inventory risk
2. Relatively uninformed trading

Model for price

Optimal Bid and Ask Model

We will use the framework developed by Avellaneda and Stoikov (2008), which obtains optimal bid and ask:

- Agent optimizes its value function:

$$v(x, s, q, t) = \mathbb{E}_t[-e^{-\gamma(x+qS_T)}]$$

  to obtain the market-maker's indifference price:

$$r(s, t) = s - q\gamma\sigma^2(T - t)$$

- Which allows the market-maker to obtain the optimal spread:

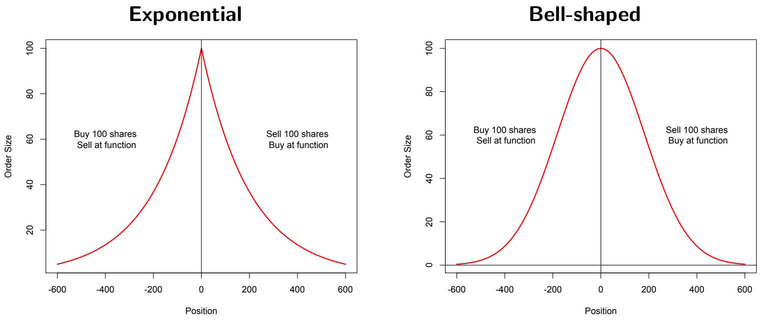
$$\delta^a + \delta^b = \gamma\sigma^2(T - t) + \frac{2}{\gamma}\ln\left(1 + \frac{\gamma}{\kappa}\right)$$

  Problem: Does not address inventory risk properly. Symmetric spread is an issue.

Model for inventory control

Dynamic Order Size

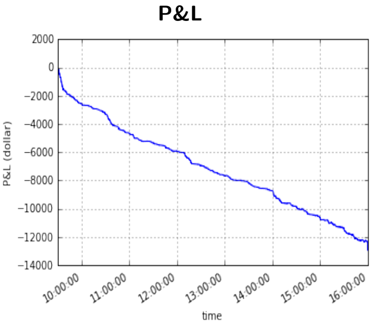
We will use a decaying function to model the size of our orders, unlike Guéant, Lehalle and Fernandez-Tapia (2012), who cap trading at a maximum inventory level.

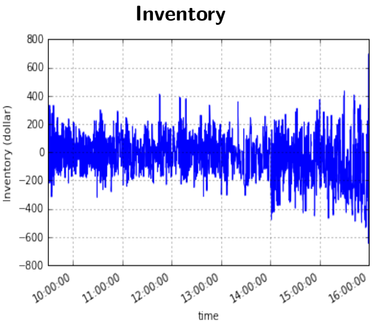


This allows us to keep trading and profit from rebates.

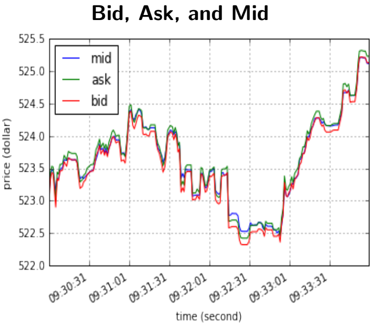
Preliminary results

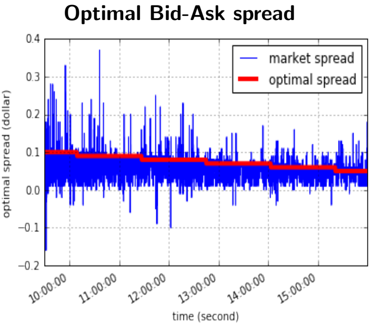
We simulated our strategy on AAPL on 2/24/2014 following a previous group's report of parameters. Start trading at 9:30am and end at 4:00pm.





1. Negative P&L $\Rightarrow -12,908.762$ at 4pm
2. Inventory risk is controlled





1. We buy high and sell low $\Rightarrow$ Negative spread
2. The optimal spread is a function of time

$$\delta^a + \delta^b = \underbrace{\gamma\sigma^2}_{constant} (T - t) + \underbrace{\frac{2}{\gamma} \ln\left(1 + \frac{\gamma}{\kappa}\right)}_{constant}$$

Ideally, the spread changes in a more dynamic fashion, depending on other parameters such as time-varying volatility

Future Work

1. Dynamic spread
   - Volatility modeling
   - Asymmetric spread
2. Parameter estimation
   - Estimate σ and κ from historical data
   - Calibrate γ
3. A different pricing model
   - Olivier Guéant, Charles-Albert Lehalle & Joaquín Fernandez-Tapia (2013) Dealing with the inventory risk: a solution to the market making problem

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References

1. Marco Avellaneda & Sasha Stoikov (2008) High-frequency trading in a limit order book, *Quantitative Finance*, 8:3, 217-224, DOI: 10.1080/14697680701381228
2. Olivier Guéant, Charles-Albert Lehalle & Joaquín Fernandez-Tapia (2013) Dealing with the inventory risk: a solution to the market making problem, *Mathematics and Financial Economics* 7:477. https://doi.org/10.1007/s11579-012-0087-0
3. Xinyu Fan, Zheyuan Fan, Xiongfeng Li, Yao Li & Jingyuan Mo (2014) High Frequency Trading in Limit Order Book, *MS&E448 Stanford University* https://web.stanford.edu/class/msande448/2014/final/group3.pdf