Optimal High-Frequency Market Making

Takahiro Fushimi, Christian González Rojas
and Molly Herman

Stanford University

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Model for price

Optimal Bid and Ask Model

We will use the framework developed by Avellaneda and Stoikov (2008), which obtains optimal bid and ask:

- Agent optimizes its value function:

$$v(x, s, q, t) = \mathbb{E}_t[-e^{-\gamma(x+qS_T)}]$$

to obtain the market-maker's indifference price:

$$r(s, t) = s - q\gamma\sigma^2(T - t)$$

- Which allows the market-maker to obtain the optimal spread:

$$\delta^a + \delta^b = \gamma\sigma^2(T - t) + \frac{2}{\gamma}\ln\left(1 + \frac{\gamma}{\kappa}\right)$$

Problem: Inventory risk associated to order size is not addressed.

Model for inventory control

Dynamic Order Size

We will use a decaying function to model order size, unlike Guéant, Lehalle and Fernandez-Tapia (2012), who cap trading at a maximum inventory level.

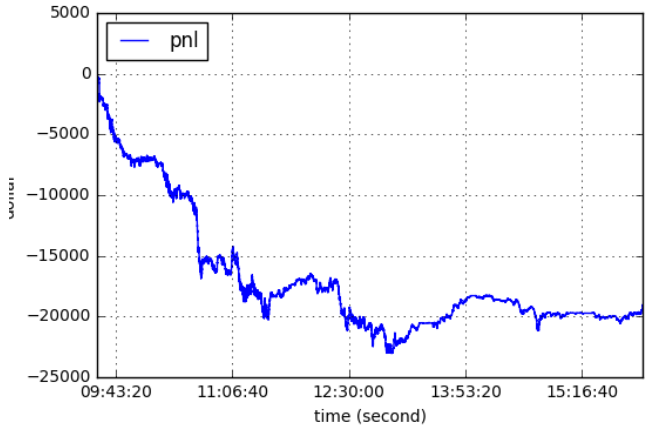
This allows us to keep trading and profit from rebates.

Trading Algorithm

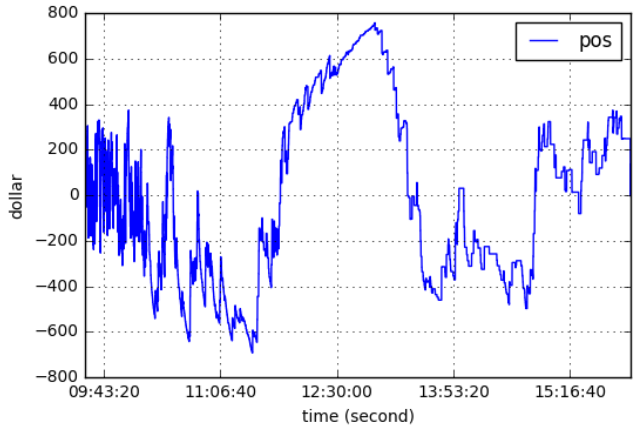
```
while current time < 16:00 do
    if no orders in the book then
        Quote bid and ask prices ;
    else if 1 order in the book then
        if current time - execution time > waiting time then
            Cancel the outstanding order;
            Quote new bid and ask prices;
        else
            Wait
        end
    else if 2 orders in the book then
        if current time - quote time > update time then
            Cancel both order;
            Quote new bid and ask prices;
        else
            Wait
        end
    end
end
```

Thesys Results: AMZN on June 12, 2017

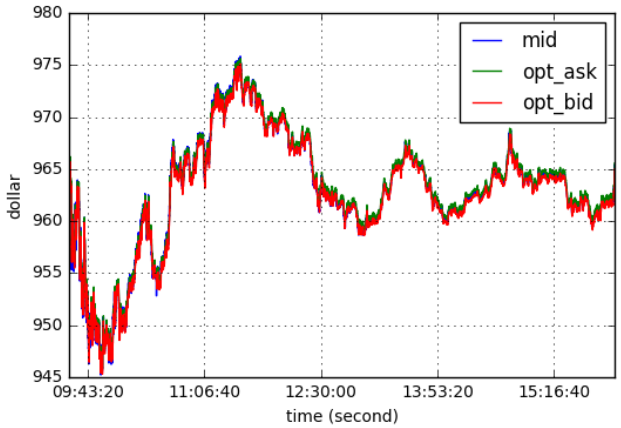
P&L



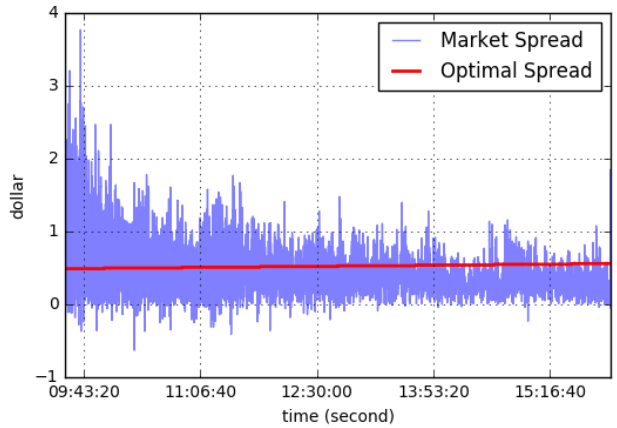
Inventory



Quoted Prices



Optimal Bid-Ask Spread



Trading Simulator

The Simulator

1. **Market order dynamics**:
   - Let ξ be the depth of our quote, we model the number of arrivals as a Time-Inhomogeneous Poisson Process:

$$N_t \sim Pois\left(\int_0^t \lambda(s,\xi)ds\right) \text{ where } \lambda(t,\xi) = \alpha_t e^{-\mu\xi}$$

2. **Execution criteria**:
   - Assume execution occurs if $X = 1$ for $X \sim Ber(\lambda(t,\xi)\cdot\Delta)$.
   - We allow for partial order execution by modeling the size of market orders as a $Gamma(k,\theta)$.
3. **Other assumptions**:
   - Time interval is 1 second $\Rightarrow$ No latency
   - No price impact
   - No competition with other market makers

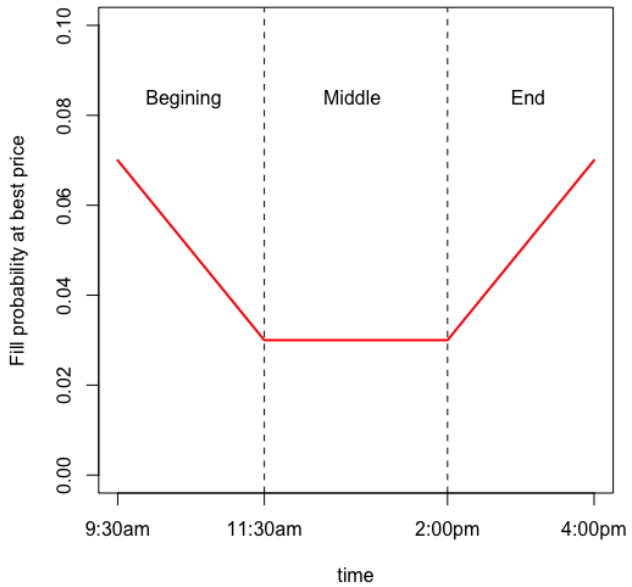
Model for Market order dynamics

Intensity Modeling

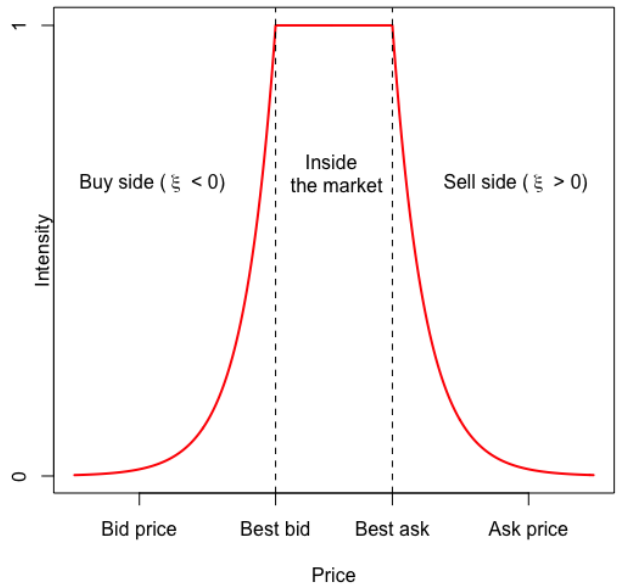
We assume that the intensity of the Poisson Process is a product of time and depth components.

$$\lambda(t, \xi) = \underbrace{\alpha_t}_{time} \cdot \underbrace{e^{-\mu\xi}}_{depth} \quad \text{for } t \in [9{:}30,\ 16{:}00],\ \xi \geq 0$$

Time component α_t



Depth component $e^{-\mu\xi}$



Setting

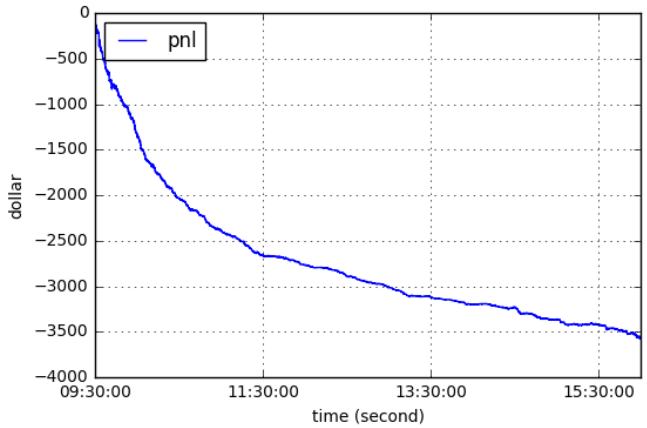
- Date: June 12, 2017, 9:30am - 4pm
- Calibrate parameters using the data from the previous week
- P&L = Cash + Book Value
  - Cash = shares sold $*$ price - shares bought $*$ price (and rebate)
  - Book Value = Current Position $*$ Market Mid Price
- Rebate: 0.003 per unit share

Stocks to trade

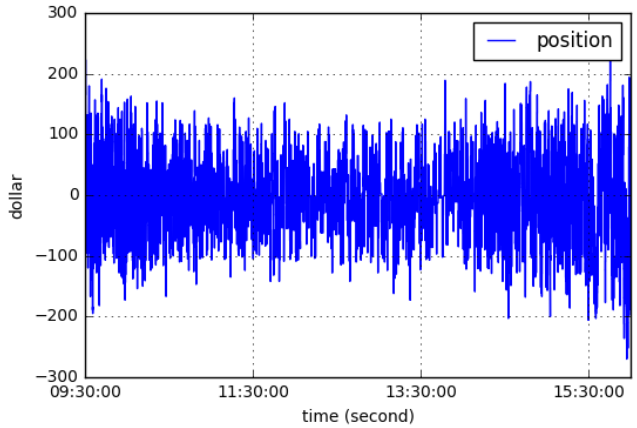
	Volume	Performance	Open Spread	Close Spread
AAPL	high	high	0.05	0.01
AMZN	low	high	0.49	0.56
GE	high	low	0.04	0.01
IVV	low	high	0.03	0.01
M	low	low	0.09	0.01

Results: AAPL, June 12, 2017

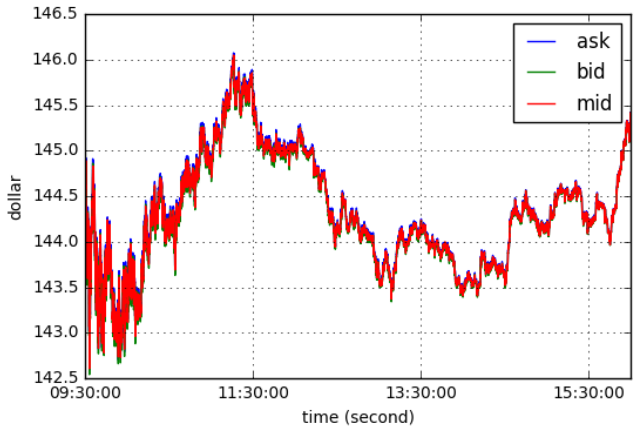
P&L



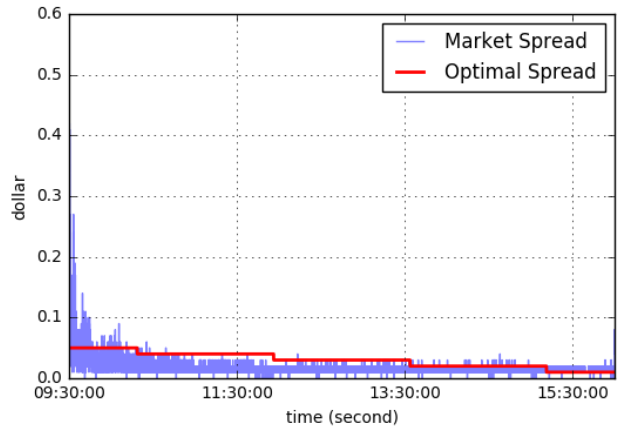
Inventory



Quoted Prices

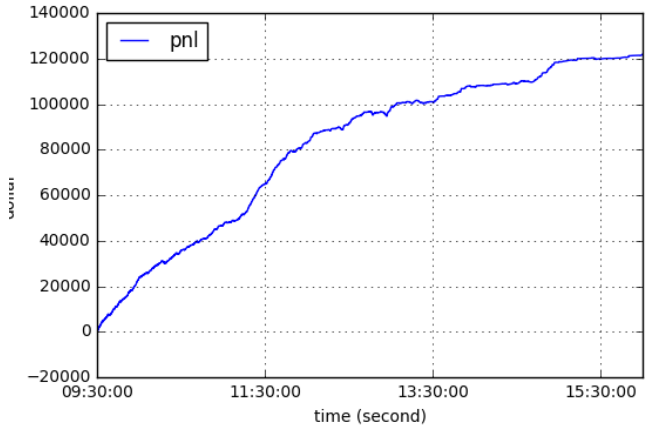


Optimal Bid-Ask Spread



Results: AMZN, June 12, 2017

P&L



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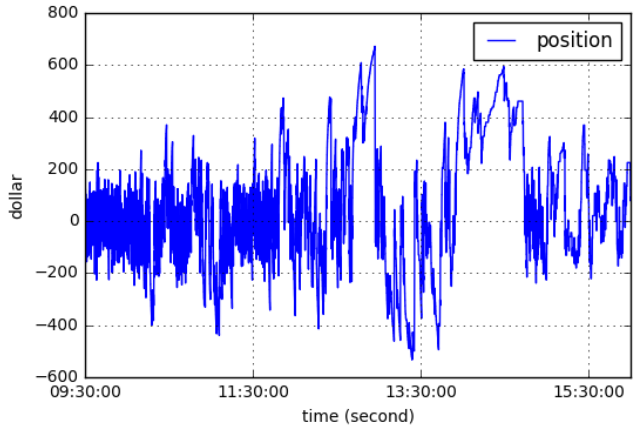

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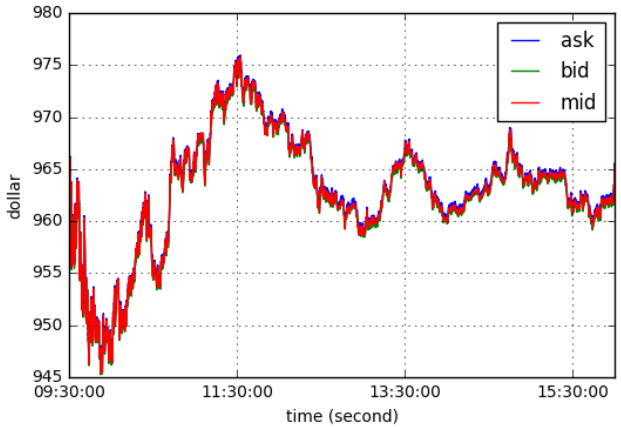
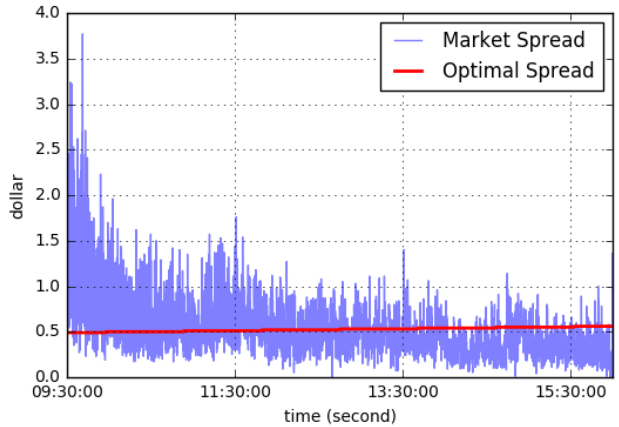
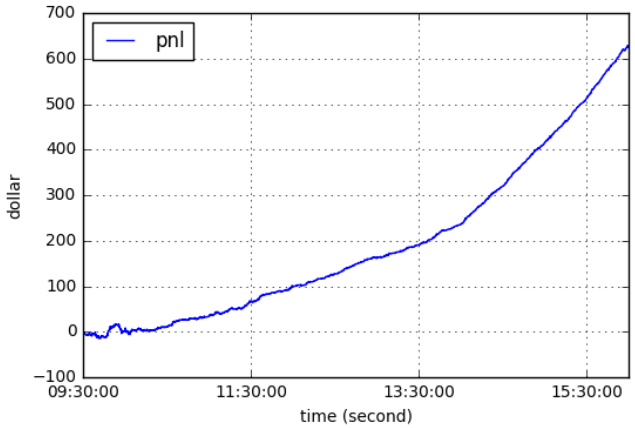


Optimal Bid-Ask Spread

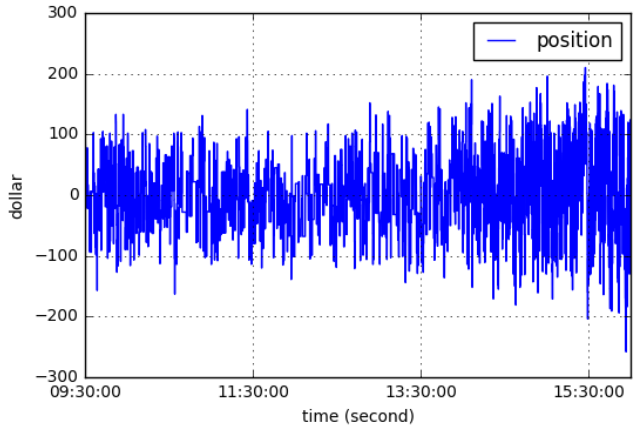


Results: GE, June 12, 2017

P&L



Inventory



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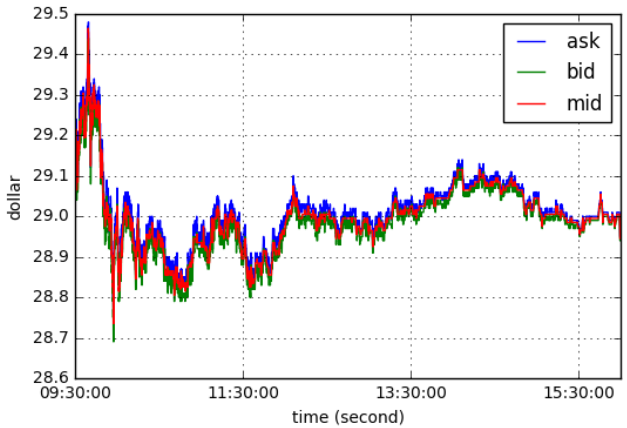
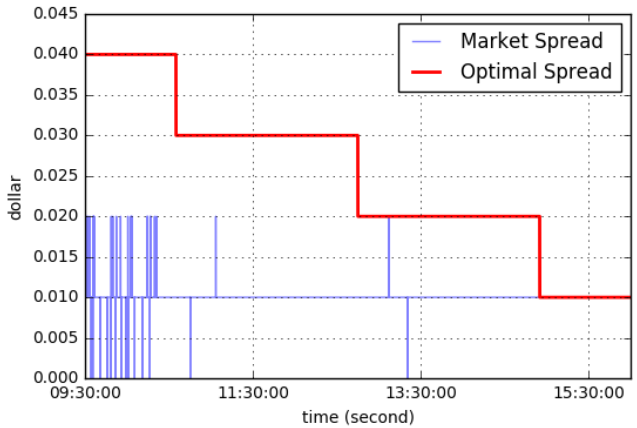
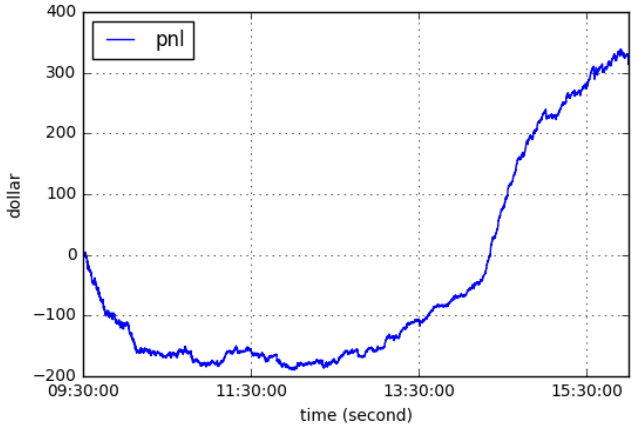


Optimal Bid-Ask Spread



Results: IVV, June 12, 2017

P&L



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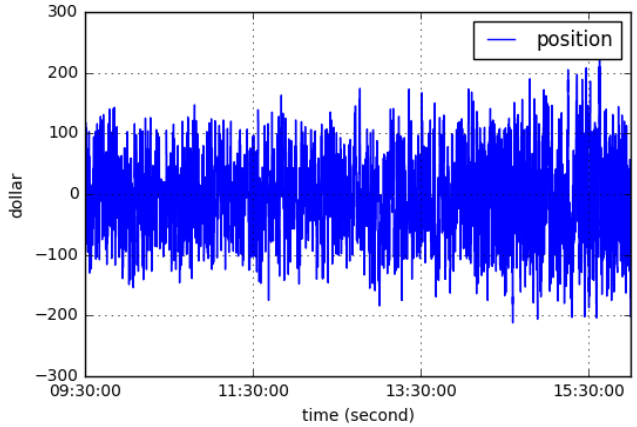
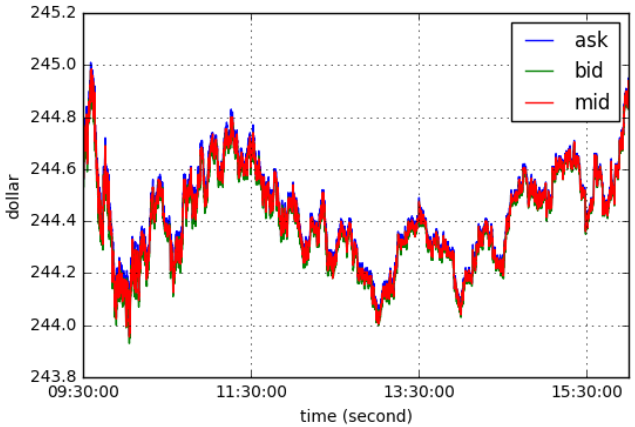
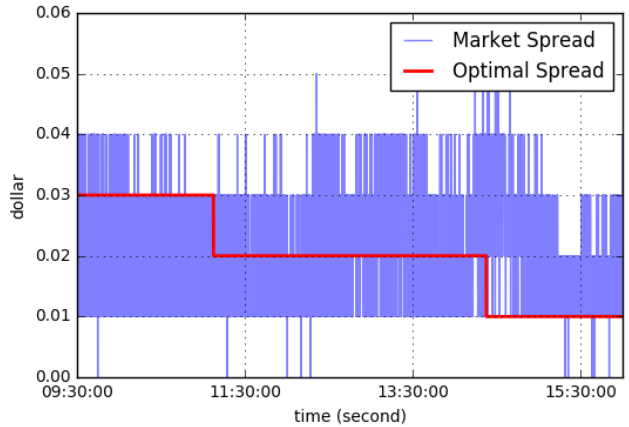


Quoted Prices

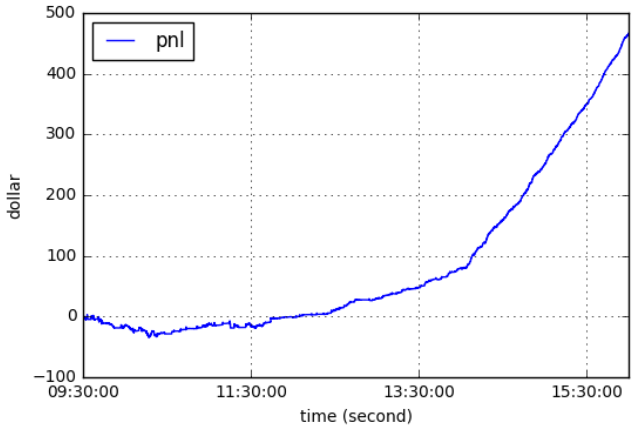


Optimal Bid-Ask Spread

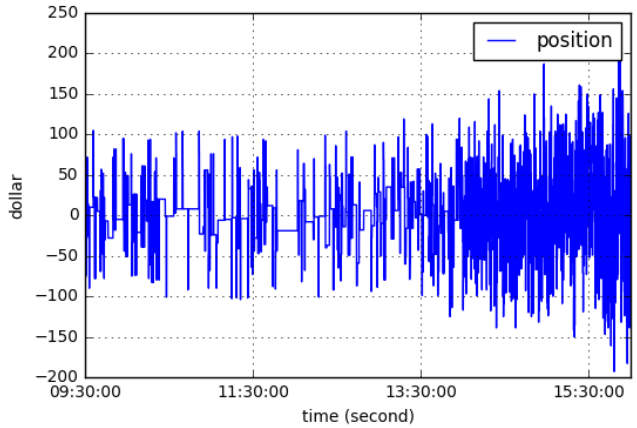


Results: M, June 12, 2017

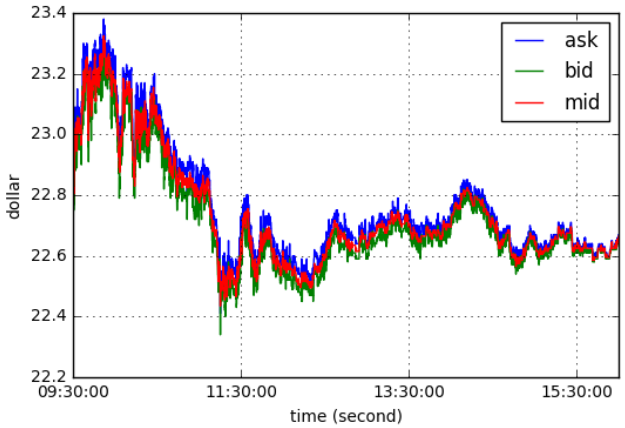
P&L



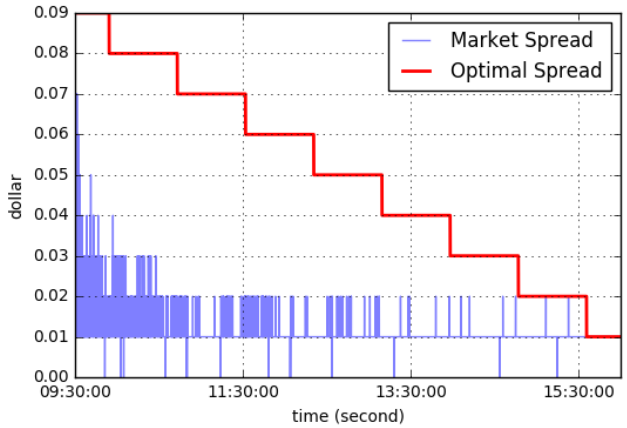
Inventory



Quoted Prices



Optimal Bid-Ask Spread



Results: Statistics

- Run the simulation from June 12, 2017 to June 16 2017

Main Results of the Trading Week

	Profits		Position	
	Mean	Stdev	Mean	Stdev
AAPL	-988.54	289.82	0.86	63.66
AMZN	32426.72	16157.0	48.52	438.33
GE	245.0	192.92	-2.41	60.92
IVV	23.14	129.9	-0.49	67.9
M	144.26	146.78	-0.83	46.14

Note: Strategy executed between June 12, 2017 and June 16, 2017

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References

1. Marco Avellaneda & Sasha Stoikov (2008) High-frequency trading in a limit order book, *Quantitative Finance*, 8:3, 217-224, DOI: 10.1080/14697680701381228
2. Olivier Guéant, Charles-Albert Lehalle & Joaquín Fernandez-Tapia (2013) Dealing with the inventory risk: a solution to the market making problem, *Mathematics and Financial Economics* 7:477. https://doi.org/10.1007/s11579-012-0087-0