Cross-sectional Mean Reversion

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Final presentation, 06/05/18

Context

- Goal: developing a statistical arbitrage strategy
- Universe: most traded US equity; trading on a daily basis
- Method:
  - producing a trading signal;
  - trading according to the signal, while market/factor neutral;
  - evaluating with Sharpe ratio and holding period.

Overview

I. Signal generation on Cross-Sectional stocks correlation

II. Integration of factor risks and beta-residuals

III. Algorithm implementation and evaluation

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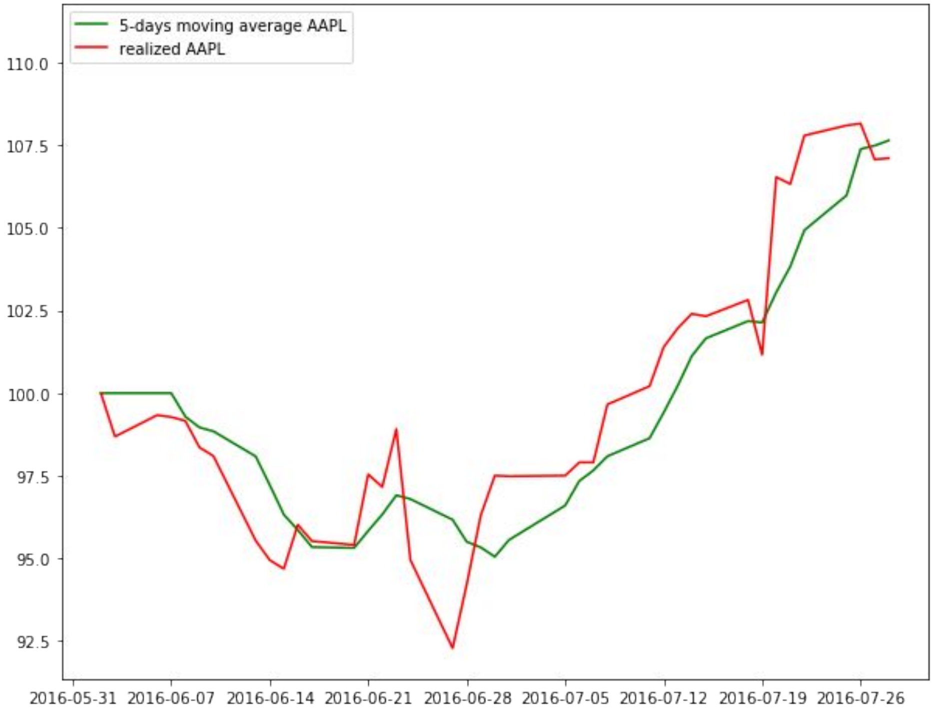
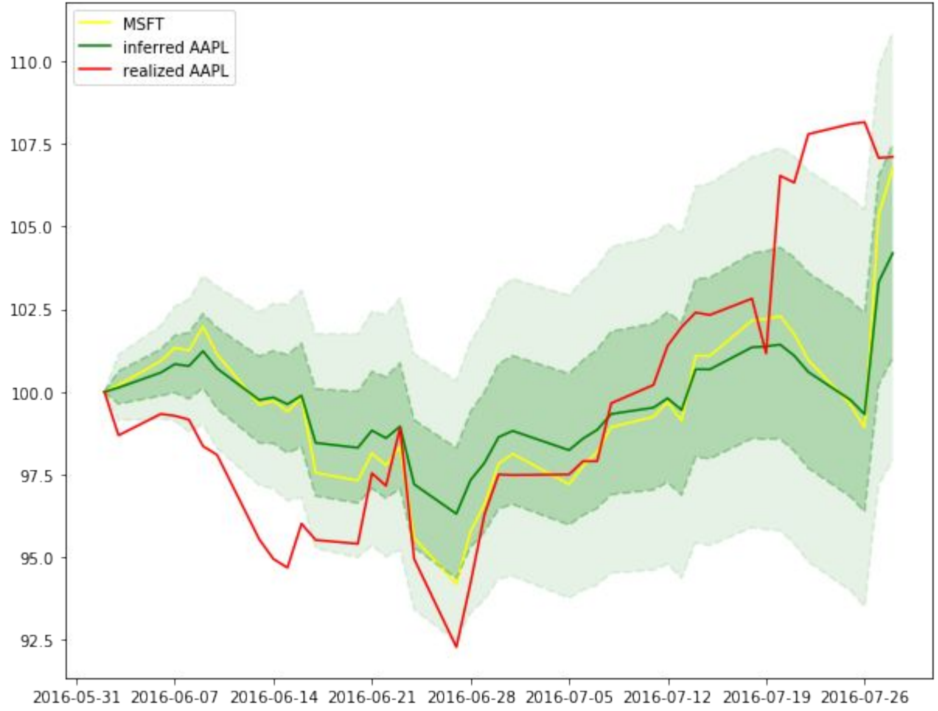
Mean Reversion

- Mean Reversion Hypothesis: *Prices will go back to their average.*
- Other formulation: *Divergence from the model followed by return to the model.*



Cross-Sectional mean reversion

- Cross-Sectional Hypothesis: *Stocks behave like the stocks they are historically correlated to.*



Signal generation

- At each date :
  - Fit a GARCH model as of $n_{predict}$ days in the past
  - For each stock :
    - Lock the performance of the others since the fit
    - Compute predicted mean and variance based on the others
    - Compare realization with those mean and variance
- Mean reversion on this criterion

Cross-Sectional mean reversion

- Underlying single-stock model: *Generalized AutoRegressive Conditional Heteroskedasticity*
- Cross-Sectional model: *Innovations are correlated by a matrix* R

$$
\begin{aligned}
\text{returns}: &\quad r_{t,i} = \mu_i + \epsilon_{t,i} \\
\text{innovations}: &\quad \epsilon_t \sim \sigma_t \,.\, N(0, R) \\
\text{volatilities}: &\quad \sigma^2_{t,i} = w_i + \alpha_i \epsilon^2_{t-1,i} + \beta_i \sigma^2_{t-1,i}
\end{aligned}
$$

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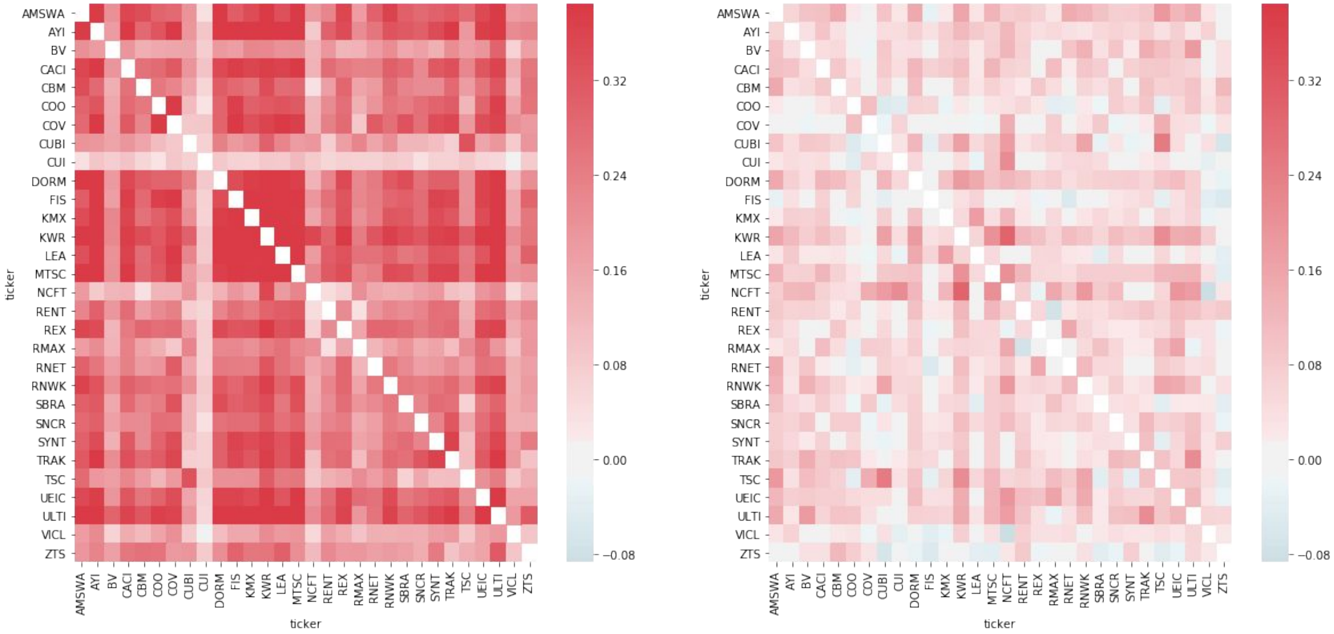
Market beta-residuals

- We want stock returns independently of the market performance
- Having flat exposure to common factors reduces risk and systemic exposure

$$
\begin{aligned}
\text{market returns}: \quad & r_t^M = \text{returns of the S\&P500} \\
\text{single stock returns}: \quad & r_{t,i} = \beta_{i,t} \,.\, r_t^M + \epsilon_{i,t} \\
\text{beta coefficients}: \quad & \beta_{i,t} \text{ as backward rolling OLS}
\end{aligned}
$$

Stock returns correlations

- Using market beta-residuals yields more relevant correlations



Risk factor-residuals

- Having flat exposure to common factors (volatility, momentum, sectors…) reduces risk and systemic exposure
- Same method as market residual, but we build the factors ourselves

$$\text{single stock returns}: \quad r_{t,i} = \beta^1_{i,t} . F^1_t + \beta^2_{i,t} . F^2_t + \epsilon_{i,t}$$

$$\text{beta coefficients}: \quad \beta^k_{i,t} \text{ as backward rolling OLS}$$

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Numerical considerations

- Impact of bad conditioning of the correlation matrix on numerical stability
- Moore-Penrose pseudo-inverse for matrix inversion stabilization
- Woodbury formula for efficient perturbed matrix inversion
- Stabilization of the GARCH procedure by catching diverging cases

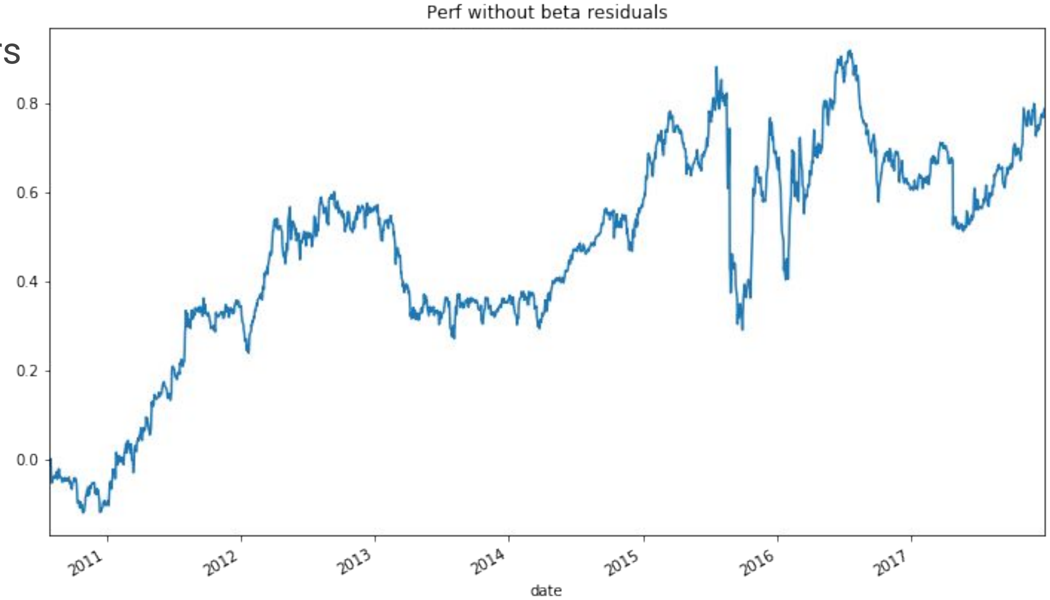
Results

- Difficult to get a stable and clean signal due to outliers in the data, and numerical instabilities when using too many stocks
- We present results obtained on trading on the largest 100 stocks in the universe
- We removed the market when fitting to get accurate correlation estimations
- We gradually zeroed our exposition to the risk factors to identify their impact.

Results

Signal fitted with: plain returns

Portfolio built: without hedging risk factors



Performance :

- Sharpe : 0.410
- Return per trade : 0.014%
- Holding Period : 2.84

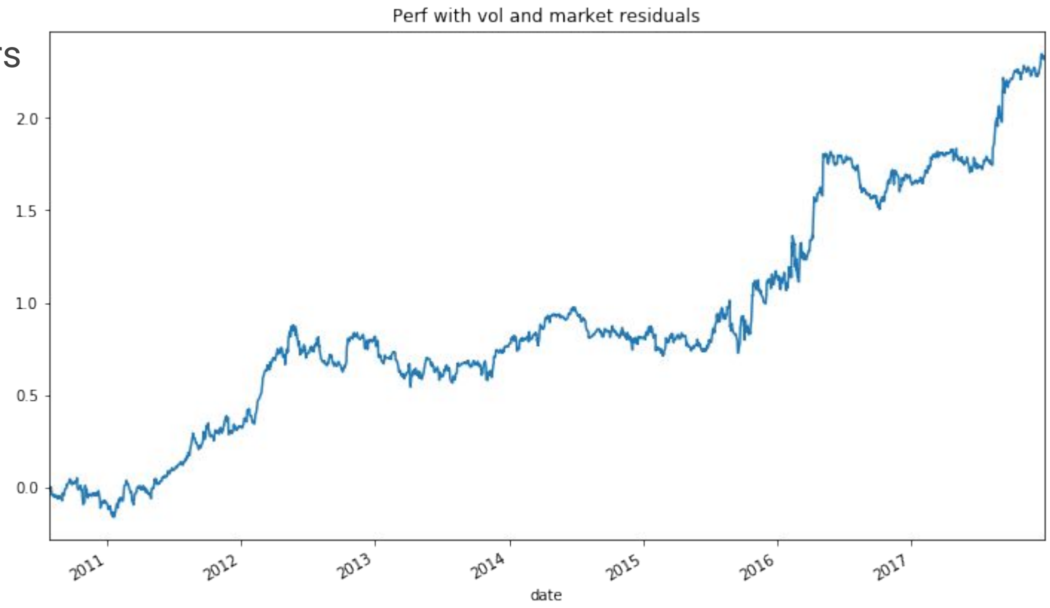
Results

Signal fitted with: market residuals

Portfolio built: without hedging risk factors

Performance :

- Sharpe : 0.972
- Return per trade : 0.028%
- Holding Period : 2.57



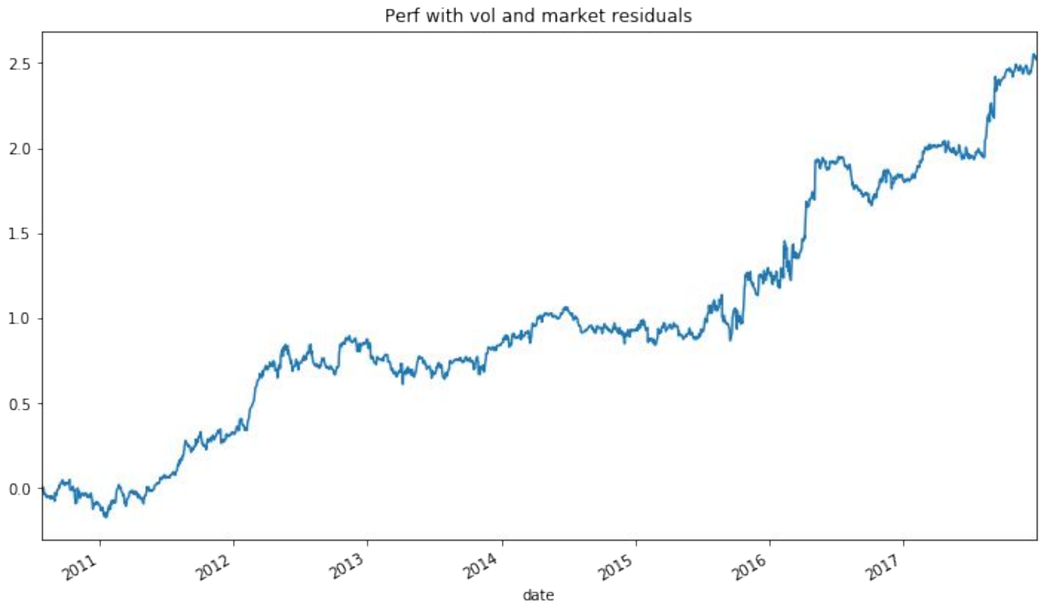
Results

Signal fitted with:market residuals

Portfolio built: hedging market

Performance :

- Sharpe : 1.067
- Return per trade : 0.031%
- Holding Period : 2.57



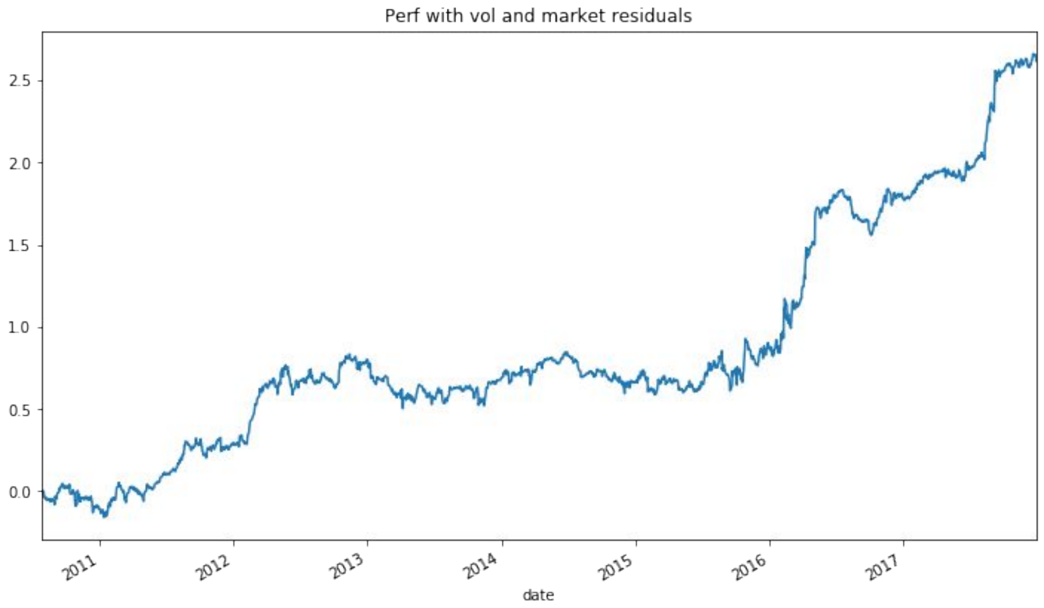
Results

Signal fitted with:market residuals

Portfolio built: hedging volatility

Performance :

- Sharpe : 1.158
- Return per trade : 0.032%
- Holding Period : 2.57



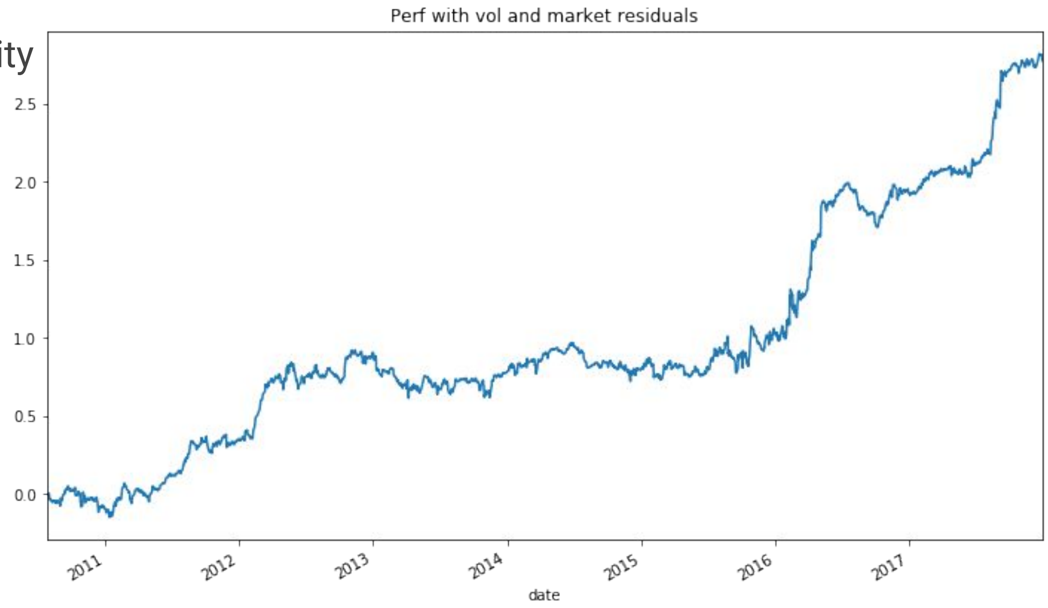
Results

Signal fitted with:market residuals.

Portfolio built: hedging market and volatility

Performance :

- Sharpe : 1.235
- Return per trade : 0.034%
- Holding Period : 2.57



Results

- Estimation of correlation gets better as we remove a common market driver in the stock movements;
- Hedging factors when building portfolio brings value;

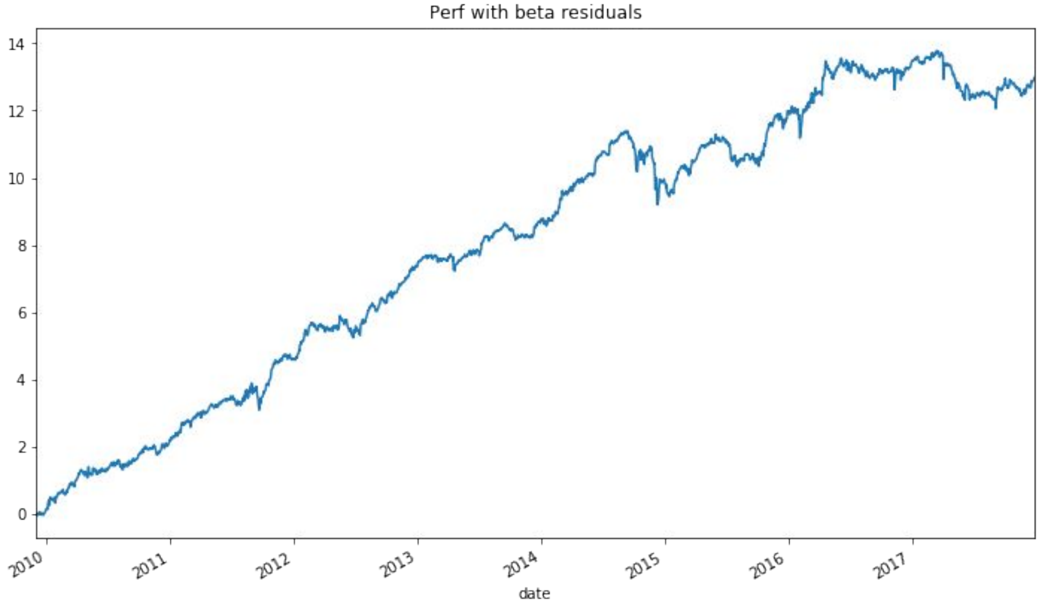
Remarks:

  - *Holding period is not accurate due to holes in the data*
  - *Hyperparameters have not been tuned yet*

Comparison with mean-reversion

Mean-reversion,

Portfolio built: hedging market and volatility



Performance :

- Sharpe : 1.425
- Return per trade : 0.087%
- Holding Period : 4.98

Comparison with mean-reversion

- Signals for mean-reversion and cross-correlation mean-reversion are uncorrelated;
- This strategy is not used by most hedge funds contrary to mean-reversion;
- With more work, the signal can be improved and stabilized.

Conclusion

- Cross-sectional correlation of returns can be used to design a "return-to-normal" trading signal;
- Including market factors leads to significant improvements in mean-reversion strategies performance;
- Numerical performance and stability, as well as quality of data are crucial to the evaluation of a trading signal.