VinU Mini-Lecture
Introduction to Optimization
Homework 6
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Problem 1

True or False: The dual of the dual of an LP is the primal.

Problem 2-4

Suppose there are 2 securities traded in a market. Security A will pay \$0.70 tomorrow if the weather is rainy and \$1.40 if the weather is not rainy. Security B will pay \$1.20 tomorrow no matter what the weather is like. The price of both securities today is \$1. A portfolio $\theta = (\theta_A, \theta_B)$ specifies the number of shares of Security A and Security B to buy. Note that a negative value for θ_A or θ_B corresponds to selling that security.

Consider the linear program:

$$\begin{aligned} \min \quad & \theta_A + \theta_B \\ \text{subject to} \quad & 0.7\theta_A + 1.2\theta_B \geq 0 \\ & 1.4\theta_A + 1.2\theta_B \geq 0 \\ & \theta_A, \theta_B \text{ free} \end{aligned}$$

Problem 2

Please answer the following questions about the above linear program:

- Argue that this linear program is feasible.
- Argue that if we can find a feasible solution with a negative objective function value, then the problem is unbounded.
- Explain why this situation corresponds to there being an arbitrage opportunity in the market.

Problem 3

Construct the dual problem of the above linear program. Answer the following questions related to dual feasibility:

- Argue that if the dual problem is infeasible, then there exists an arbitrage opportunity in the market.
- Argue that if the dual problem is feasible, then there is no portfolio with a negative price today, but has non-negative payoffs tomorrow.

Problem 4

Give an interpretation for the dual variables if they are strictly positive. Suppose we have a security that that pays \$3 tomorrow if the weather is rainy and nothing otherwise. How would you price this security in terms of the dual variables?