VinU Mini-Lecture
Introduction to Optimization
Homework 5
Course Instructor: Yinyu Ye

Problem 1

True or False: In the LP standard form, for any basis, a basic variable set is optimal, i.e., the corresponding basic solution is optimal, if and only if the corresponding reduced cost is non-negative.

Problem 2

Using the simplex procedure, solve

$$\begin{aligned} \text{maximize} \quad & -x_1 + x_2 \\ \text{subject to} \quad & x_1 - x_2 \leq 2 \\ & x_1 + x_2 \leq 6 \\ & x_1 \geq 0, \qquad x_2 \geq 0. \end{aligned}$$

(Hint: Recall Page 3-4 of Lecture Note # 5 to transfer this LP to the standard form, and Page 10-11 to find a improving direction and a new basic variable set.)

Problem 3

Consider a linear program in standard form: where $\mathbf{A}$ has 3 rows and 6 columns. Suppose, we are using the primal simplex method to solve this linear program. Let $\mathbf{x}$ be the current basic feasible solution, with $(x_1,\ x_2,\ x_3)$ as the basic variables and $(x_4,\ x_5,\ x_6)$ as the non-basic variables. Let $\mathbf{B}$ denote the current basis and the basic variable index set, let $\mathbf{D}$ denote the rest of the columns and the non-basic variable index set, and let $\mathbf{r}$ denote the reduced cost vector. Assume $\mathbf{x_B} > \mathbf{0}$, and suppose:

$$\mathbf{B}^{-1}\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 & \gamma & 1 & -1 \\ 0 & 1 & 0 & -3 & 2 & -2 \\ 0 & 0 & 1 & 0 & 2 & 3 \end{pmatrix}$$

Suppose $\mathbf{r}_\mathbf{D}^T = (r_4,\ r_5,\ r_6) = (1, 2, -1)$, and suppose $\mathbf{x_B} = (1,\ 2,\ 3)$:

1. Which variable is the incoming variable?

2. Which variable is the outgoing variable to make the matrix corresponding to the new basic variable set to be non-singular?

Problem 4

Write down the optimality conditions for the following linear programming problem:

$$\begin{aligned} &\min\ \boldsymbol{c}^T\boldsymbol{x} \\ &s.t.\ \boldsymbol{A}\boldsymbol{x} \le \boldsymbol{b} \\ &\qquad \boldsymbol{x} \ge \boldsymbol{0}. \end{aligned}$$

Problem 5 Transportation Simplex Method

Recall the transportation simplex method and the optimal transport problem on Page 12-34 of Lecture Note #5.

1. Based on the reduced cost computed on Page 33, specify which unused cell will be the incoming variable at this step. Then, determine the new resource allocation with a chain-reaction cycle as Step 3 on Page 33 of Lecture Note #5.

2. Is the new allocation optimal? If so, provide a short reason; if not, specify the new incoming unused cell.