VinU Mini-Lecture
Introduction to Optimization
Homework 4
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Problem 1 Convex sets and convex functions

Identify whether the following sets or functions are convex

(a) $F := \{x \in R^n : \ Ax = b, \ x \geq 0\}$, where data matrix $A \in R^{m \times n}$ and vector $b \in R^m$.
(b) $\{x : x^2 \geq 1\}$.

(c) *Negative entropy*

$$h(p) = \sum_{i=1}^{n} p_i log(p_i)$$

On $\{p \in R^n, p_i > 0 \ \forall \ i, e^T p = 1\}$, where $e \in R^n$ is the vector of ones.

Problem 2

Consider the two-variable linear program with 6 inequaltiy constraints:

$$\max \ 3x_1 + 5x_2$$

$$s.t. \quad x_1 \geq 0$$

$$x_2 \geq 0$$

$$-x_1 + x_2 \leq 2.5$$

$$x_1 + 2x_2 \leq 9$$

$$x_1 \leq 4$$

$$x_2 \leq 3$$

(a) Plot the lines $x_j = 0$, $j = 1, \dots 6$ (all on the same two-dimensional graph of x_1 and x_2) where for $j = 3, 4, 5, 6$, x_j denotes the slack variable in j^{th} constraint in the LP standard form .

(b) Identify the extreme points of the feasible region as intersections of suitable lines $x_j = 0$.

(c) For each pair of adjacent extreme points of the feasible region, describe how each direction of the edge between them can be generated by increasing the value of a single variable chosen from $\{x_1, x_2, x_3, x_4, x_5, x_6\}$.

Problem 3

True or False: local optimality implies global optimality.

Problem 4

Recall Problem 3 in Homework 1. Write down the optimal solution and optimal value. Find the normal directions of hyperplanes associated with the optimal corner point. Then, decompose the objective vector into those directions.

This is an application of Theorem on Page 22 of Lecture Note #4, which can help us build some intuition of this theorem and the simplex method.