VinU Mini-Lecture
Introduction to Optimization
Homework 2
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Problem 1

Recall the Supporting Vector Machine in Lecture 2. Let the red class of points contain three points

$$\boldsymbol{a}_1 = (0, 3), \boldsymbol{a}_2 = (1, 2), \boldsymbol{a}_3 = (2, 3),$$

and let the blue class of points contain three points

$$\boldsymbol{b}_1 = (1, 0), \boldsymbol{b}_2 = (2, 1), \boldsymbol{b}_3 = (1, 3),$$

which are illustrated in Figure 1.

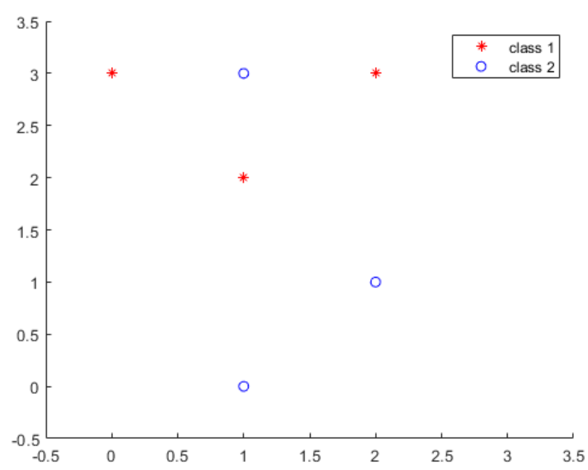

Figure 1: Scatter of all points

Is this problem strictly separable? If so, find a line to strictly separate blues and reds; if not, construct a "soft margin" SVM problem based on Page 5 of Lecture Note #2.

Problem 2

Recall the World Cup Winner Problem in Lecture Note #2. Suppose there are 5 securities available in the World Cup Assets market for open trading at fixed prices and pay-off's; see the table below. Here, for example, Security 1's pay-off is $1 if either Argentina, Brazil, or Italy wins. The Share Limit represent the maximum number of shares one can purchase, and Price is the current purchasing price per share of each security.

Security	Price π	Share Limit q	Argentina	Brazil	Italy	Germany	France
1	$ 0.75	10	1	1	1	0	0
2	$ 0.35	5	0	0	0	1	0
3	$0.40	10	1	0	1	0	1
4	$0.95	10	1	1	1	1	0
5	$0.75	5	0	1	0	1	0

1. Assume that shorting is not allowed, that is, one can only buy shares not sell. Formulate the problem as a linear program to decide how many shares of each security to purchase so as to maximize the worst-case (minimum) profit when the game is finally realized. Solve the LP and provide the optimal solution and value.

2. Now assume there is no share limit (can be ∞) and short is allowed, that is, the decision variable can be both positive (buy) and negative (sell). Reformulate the problem to decide how many shares of each security to purchase so as to maximize the worst-case (minimum) profit when the game is finally realized.

Problem 3

Convert the following problem to a linear program in standard form:

$$\begin{array}{rl} \text{minimize} & |x| + |y| + |z| \\ \text{subject to} & x + y \leq 1 \\ & 2x + z = 3. \end{array}$$

(Hint: See Page 10 of Lecture Note #2.)

Problem 4

A class of piecewise linear functions can be represented as $f(\mathbf{x}) =$ Maximum $(\mathbf{c}_1^T\mathbf{x}+d_1, \mathbf{c}_2^T\mathbf{x}+d_2, \ldots, \mathbf{c}_p^T\mathbf{x}+d_p)$. For such a function f, consider the problem

$$\begin{array}{rl} \text{minimize} & f(\mathbf{x}) \\ \text{subject to} & \mathbf{A}\mathbf{x} = \mathbf{b} \\ & \quad \mathbf{x} \geq \mathbf{0}. \end{array}$$

Show how to convert this problem to a linear programming problem.