

VinU Mini-Lecture
Introduction to Optimization
Homework 1
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Problem 1

(Textbook Section 2.8 problem 3) An oil refinery has two sources of crude oil: a light crude that costs \$ 35/barrel and a heavy crude that costs \$ 30/barrel. The refinery produces gasoline, heating oil, and jet fuel from crude in the amounts per barrel indicated in the following table:

	Gasoline	Heating Oil	Jet Fuel
Light Crude	0.3	0.2	0.3
Heavy Crude	0.3	0.4	0.2

The refinery has contracted to supply 900,000 barrels of gasoline, 800,000 barrels of heating oil, and 500,000 barrels of jet fuel. The refinery wishes to find the amounts of light and heavy crude to purchase so as to be able to meet its obligations at minimum cost.

Formulate this problem as a linear program.

Problem 2

Consider a school district with N neighborhoods, M schools, and G grades at each school. Each school j has a capacity of C_{jg} for grade g . In each neighborhood i , the student population of grade g is S_{ig} . Finally, the distance of school j from neighborhood i is d_{ij} . Formulate a linear programming problem whose objective is to assign all students to schools, while minimizing the total distance travelled by all students. (You may ignore the fact that numbers of students must be integer.)

Problem 3

Consider the LP below:

$$\begin{array}{ll} \max & 3x_1 + x_2 \\ \text{s.t.} & x_1 + x_2 \leq 4 \\ & 2x_1 - x_2 \geq -1 \\ & x_1 \leq 3 \\ & x_2 \leq 3 \\ & x_1 \geq 0 \\ & x_2 \geq 0 \end{array}$$

1. Plot the feasible region.
2. Write down the corner points of the feasible region.

Problem 4

Label the followings statements as True or False. For true provide reason and for false either provide reason or a counter example.

1. Scale the right-hand-side coefficients does not change the optimal solution.
2. Multiply both sides of an equality constraint by a constant does not change the optimality but the new optimal solution should be scaled accordingly.
3. Reorder the constraints (together with their right-hand-side coefficients) does not change the optimality