

Helicopter retrieval equations of motion & simulation.

One way to generate an equation that governs the motion of fishermen in a rescue bucket (modeled as a particle Q) being retrieved with a relatively light straight cable B into a helicopter (modeled as a rigid body A) is to form the resultant of all forces on Q as $\mathbf{F}^Q = -m g \hat{\mathbf{a}}_y + T \hat{\mathbf{b}}_y$ and relate it to Q 's mass and acceleration in N via Newton's law as $\mathbf{F}^Q = m {}^N \mathbf{a}^Q$, i.e.,

$$-m g \hat{\mathbf{a}}_y + T \hat{\mathbf{b}}_y = m \left[\ddot{x} \hat{\mathbf{a}}_x + \ddot{y} \hat{\mathbf{a}}_y + (L \ddot{\theta} + 2 \dot{L} \dot{\theta}) \hat{\mathbf{b}}_x + (-\dot{L} + L \dot{\theta}^2) \hat{\mathbf{b}}_y \right]$$

Form a **single scalar** equation of motion written in terms of symbols in the table **except** T .¹

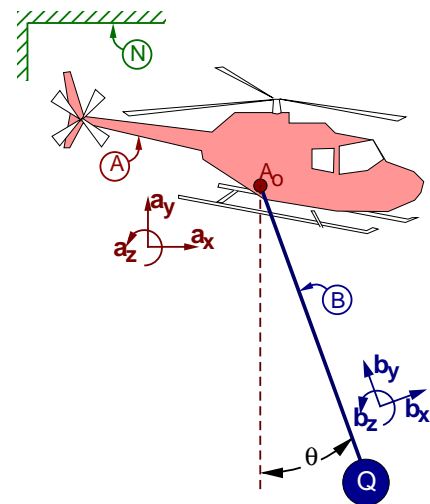
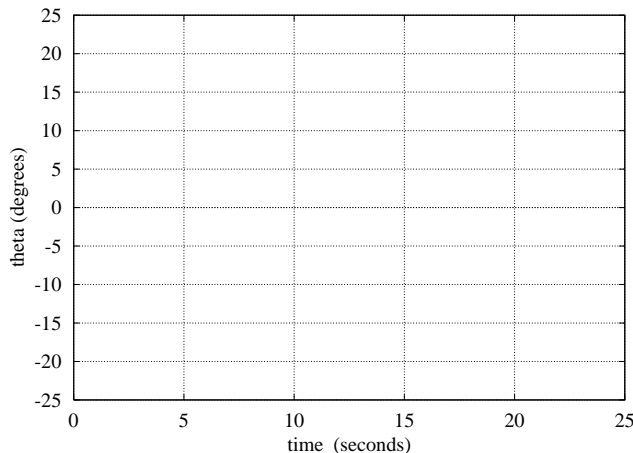
Result:

$$\boxed{\phantom{L \ddot{\theta} + 2 \dot{L} \dot{\theta} + g \sin(\theta) = 0}} = 0$$

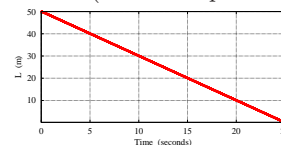
When the helicopter is **stationary** in N , Q 's motion is governed by

$$L \ddot{\theta} + 2 \dot{L} \dot{\theta} + g \sin(\theta) = 0$$

Plot θ vs. t for $0 \leq t \leq 24.92$ sec. Section 2.7 shows how to numerically solve ordinary differential equations.



$$L = 50 - 2t \quad (\text{constant speed retrieval})$$



Description	Symbol	Type	(Initial) value or specification
Angle between $\hat{\mathbf{a}}_y$ and $\hat{\mathbf{b}}_y$	θ	variable	$\theta(0) = 1^\circ \quad \dot{\theta}(0) = 0 \frac{\text{rad}}{\text{sec}}$
Distance between Q and A_o	L	specified	$L(t) = 50 - 2t$ meters
Earth's local gravitational acceleration	g	constant	$9.8 \frac{\text{m}}{\text{sec}^2}$

- I think the simulation results are intuitive. **True/False.**
- Similar results are associated with sucking spaghetti into your mouth. **True/False.**
- High-oscillation swings may be dangerous/fatal for the fishermen and helicopter. **True/False.**
- These high-oscillations may be related to problems encountered during a U.S. space-shuttle retrieval of a satellite in low-Earth orbit. **True/False.**
- I know how to specify a different function for $L(t)$ to control this system so $\theta(t)$ stays small (e.g., less than 5°). **True/False.**

¹The single scalar equation of motion should be a 2^{nd} -order ODE involving $\ddot{\theta}$.