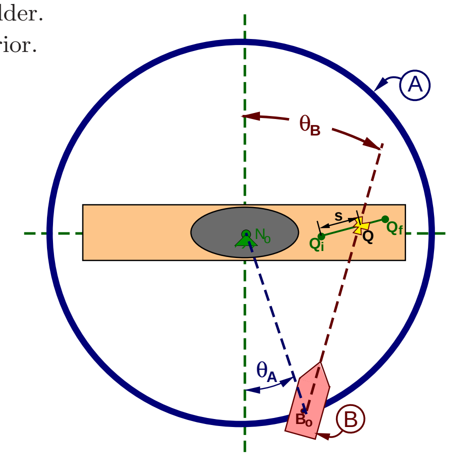
## 6.4 Radiation beam targeting a cancer cell (two-axis machine).

The following shows a radiation machine consisting of a rigid emitter $B$ and a circular gimbal $A$. Emitter $B$ is connected by a small one-axis revolute "targeting" motor to gimbal $A$ at point $B_o$. Gimbal $A$ rotates relative to the treatment room $N$ because of a **constant-speed** revolute motor that connects $A$ to $N$ at point $N_o$ and causes $B_o$ to move with **constant-speed** in a circle centered at point $N_o$ of $N$. Note: Both revolute motor axes are perpendicular to the circle traced out by $B_o$.

A thin radiation ray is emitted from $B_o$ to target a cancer cell $Q$ that moves (e.g., because of respiration and circulation) along a line that passes through points $Q_i$ and $Q_f$ of a patient who is otherwise motionless in a treatment room $N$. Note: Line segment $\overline{Q_i\,Q_f}$ is perpendicular to the revolute motors' axes.

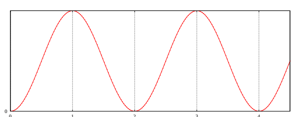
| Description | Symbol | Type | Value |
|---|---|---|---|
| Horizontally-right measure of $Q_i$'s position vector from $N_o$ | $x_i$ | Constant | 10 cm |
| Vertically-upward measure of $Q_i$'s position vector from $N_o$ | $y_i$ | Constant | 1 cm |
| Horizontally-right measure of $Q_f$'s position vector from $N_o$ | $x_f$ | Constant | 20 cm |
| Vertically-upward measure of $Q_f$'s position vector from $N_o$ | $y_f$ | Constant | 2 cm |
| Patient respiration rate (15 breaths/minute) | $\omega_r$ | Constant | $\frac{2\,\pi}{4}\ \frac{\text{rad}}{\text{sec}}$ |
| Radius of circular gimbal $A$ | $R$ | Constant | 40 cm |
| Angle between downward vertical and $B_o$'s position vector from $N_o$ | $\theta_A$ | Specified | $\frac{2\,\pi}{360}\,t\ \frac{\text{rad}}{\text{sec}}$ |
| Angle between upward vertical and $Q$'s position vector from $B_o$ | $\theta_B$ | Variable | |

Horizontally-right is generally directed from the patient's left to right shoulder.
Vertically-upward is generally directed from the patient's posterior to anterior.
$\theta_A$ is **counter-clockwise positive** whereas $\theta_B$ is **clockwise positive**.




The cancer cell moves from $Q_i$ to $Q_f$ with displacement

$$s \;=\; \frac{A}{2}\,[1 \;-\; \cos(\omega_r\,t)]$$



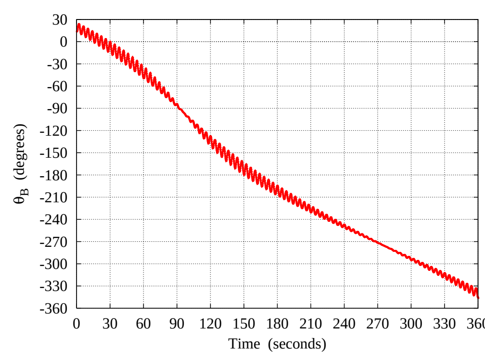
where $A$, the distance between $Q_i$ and $Q_f$, is

$$A \;=\; \sqrt{(x_f - x_i)^2 \;+\; (y_f - y_i)^2}$$

Determine $\theta_B$ at $t = 0$ and $t = 60$ sec (3 significant digits).
Plot $\theta_B{}^\circ$ for $0 \le t \le 360$ sec with data every 0.25 sec.
See Section 2.5.5 and ensure $\theta_B$ is continuous to reflect a real motor.

**Result:**

$\theta_B|_{t=0} \;=\;$ 13.7 $^\circ$ $\qquad\qquad$ $\theta_B|_{t=60} \;=\;$ -49.6 $^\circ$




Maximum torque specification helps properly size a motor. Knowing the targeting motor's torque is $I\,\ddot{\theta}_B$ $(I \,=\, 2\,\text{g}\,\text{m}^2)$ determine its maximum absolute value ($1^+$ significant digit).

**Result:**

$|\mathbf{T}_{\text{max}}| \;=\;$ 0.83 milliNewton m

An alternate treatment strategy is to lock $\theta_A$ and $\theta_B$ so radiation targets $Q$ at the midpoint of line segment $\overline{Q_i\,Q_f}$. The sensitivity and ease of targeting depends on the angle between the radiation and $\overline{Q_i\,Q_f}$. Determine sets of values for $\theta_A$ and $\theta_B$ to deliver radiation perpendicular or parallel to $\overline{Q_i\,Q_f}$ ($3^+$ significant digits). Complete the table with $0^\circ \le \theta_A \le 360^\circ$ and $-360^\circ \le \theta_B \le 0^\circ$.

**Result:**

| Solution # | **Perpendicular** to $\overline{Q_i\,Q_f}$ | | **Parallel** to $\overline{Q_i\,Q_f}$ | |
|---|---|---|---|---|
| 1 | $\theta_A = 27.9^\circ$ | $\theta_B = -5.71^\circ$ | $\theta_A =$ 95.7 $^\circ$ | $\theta_B =$ -95.7 $^\circ$ |
| 2 | $\theta_A =$ 163.6 $^\circ$ | $\theta_B =$ -185.7 $^\circ$ | $\theta_A =$ 275.7 $^\circ$ | $\theta_B =$ -275.7 $^\circ$ |