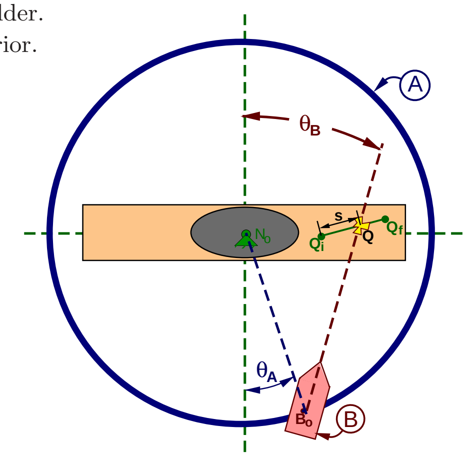
3.7 Radiation beam targeting a cancer cell (two-axis machine).

The following shows a radiation machine consisting of a rigid emitter B and a circular gimbal A. Emitter B is connected by a small one-axis revolute "targeting" motor to gimbal A at point B_o. Gimbal A rotates relative to the treatment room N because of a **constant-speed** revolute motor that connects A to N at point N_o and causes B_o to move with **constant-speed** in a circle centered at point N_o of N. Note: Both revolute motor axes are perpendicular to the circle traced out by B_o.

A thin radiation ray is emitted from B_o to target a cancer cell Q that moves (e.g., because of respiration and circulation) along a line that passes through points Q_i and Q_f of a patient who is otherwise motionless in a treatment room N. Note: Line segment $\overline{Q_i\,Q_f}$ is perpendicular to the revolute motors' axes.

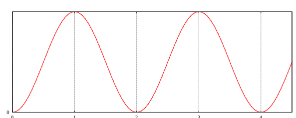
Description	Symbol	Type	Value
Horizontally-right measure of Q_i's position vector from N_o	x_i	Constant	10 cm
Vertically-upward measure of Q_i's position vector from N_o	y_i	Constant	1 cm
Horizontally-right measure of Q_f's position vector from N_o	x_f	Constant	20 cm
Vertically-upward measure of Q_f's position vector from N_o	y_f	Constant	2 cm
Patient respiration rate (15 breaths/minute)	ω_r	Constant	$\frac{2\,\pi}{4}\ \frac{\text{rad}}{\text{sec}}$
Radius of circular gimbal A	R	Constant	40 cm
Angle between downward vertical and B_o's position vector from N_o	θ_A	Specified	$\frac{2\,\pi}{360}\ t\ \frac{\text{rad}}{\text{sec}}$
Angle between upward vertical and Q's position vector from B_o	θ_B	Variable	

Horizontally-right is generally directed from the patient's left to right shoulder.
Vertically-upward is generally directed from the patient's posterior to anterior.
θ_A is **counter-clockwise positive** whereas θ_B is **clockwise positive**.



The cancer cell moves from Q_i to Q_f with displacement

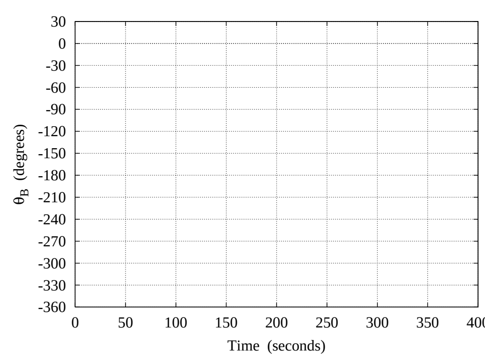
$$s \;=\; \frac{A}{2}\,[1 \;-\; \cos(\omega_r\,t)]$$



where A, the distance between Q_i and Q_f, is

$$A \;=\; \sqrt{(x_f - x_i)^2 \;+\; (y_f - y_i)^2}$$

Determine θ_B at $t = 0$ and $t = 60$ sec (3 significant digits).
Plot $\theta_B{}^\circ$ for $0 \le t \le 360$ sec with data every 0.25 sec.
See Section 2.5.4 and ensure θ_B is continuous to reflect a real motor.



Result:

$\theta_B|_{t=0} \;=\;$ ____ $^\circ$ $\qquad$ $\theta_B|_{t=60} \;=\;$ ____ $^\circ$

Maximum torque specification helps properly size a motor. Knowing the targeting motor's torque is $2\,\ddot{\theta}_B$ milliNewton m, determine its maximum absolute value (1 significant digit).

Result:

$|\mathbf{T}_{\text{max}}| \;=\;$ ____ milliNewton m

The sensitivity and ease of targeting depends on the angle between the radiation and line segment $\overline{Q_i\,Q_f}$. Determine sets of values for θ_A and θ_B when Q is at the midpoint of $\overline{Q_i\,Q_f}$ to deliver radiation perpendicular or parallel to $\overline{Q_i\,Q_f}$ (3^+ significant digits). Complete the table with $0^\circ \le \theta_A \le 360^\circ$ and -$360^\circ \le \theta_B \le 0^\circ$.

Result:

Solution #	Perpendicular to $\overline{Q_i\,Q_f}$		Parallel to $\overline{Q_i\,Q_f}$	
1	$\theta_A =$ ____ $^\circ$	$\theta_B =$ ____ $^\circ$	$\theta_A =$ ____ $^\circ$	$\theta_B =$ ____ $^\circ$
2	$\theta_A =$ ____ $^\circ$	$\theta_B =$ ____ $^\circ$	$\theta_A =$ ____ $^\circ$	$\theta_B =$ ____ $^\circ$