Prelab	Participation	Lab
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Name: ____________________________

6 Lab: Motor Constants

Motors are useful for actuating and controlling systems. Understanding how they work is helpful for designing and building real systems. Two quantities that characterize a motor's function are the ***motor torque constant*** k_m and the ***motor voltage constant*** k_v that appear in the following formulas

$$T_m = k_m \, i_m \qquad\qquad v_m = k_v \, \omega_m$$

where T_m is the motor torque, i_m is the current passing through the motor, v_m is motor back-EMF voltage, and ω_m is the motor's angular speed.

The values of these constants are usually found on a ***motor specification sheet*** that is experimentally determined by the motor's manufacturer. The motors and encoders used in this lab were purchased second-hand. Since the motor constants are not known, we will run an experiment to determine their values.

6.1 PreLab

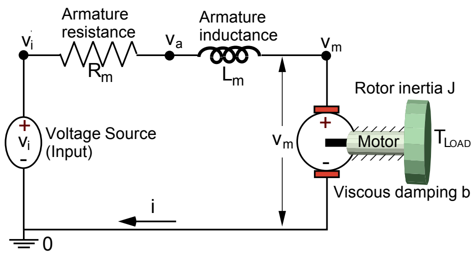
Soon, you will do your own MIPSI lab. You will choose a physical system, and use the MIPSI technique, i.e., **M**odel, **I**dentifiers, **P**hysics, **S**implify and Solve, and **I**nterpret your results. To prepare for the future MIPSI lab, brainstorm two physical systems that have an interesting question to be answered and are not a Ph.D. dissertation.

Short system description and question to be answered   Rough system schematic

Short system description and question to be answered   Rough system schematic

6.2 Experimental

A schematic of an ideal DC motor is shown to the right. This motor model includes the voltage supplied to the motor (v_i), the motor's coil resistance (R_m), the motor's inductance (L_m), and the motor's back-EMF (v_m). The ODE that relates the motor's angular speed ω_m to the input voltage v_i is[a]



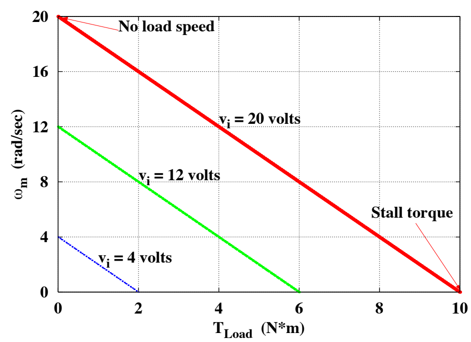
[a] This electro-mechanical ODE is in the homework.

$$L_m\, J\, \ddot{\omega}_m \;+\; (L_m\, b + R_m\, J)\, \dot{\omega}_m \;+\; (R_m\, b + k_m\, k_v)\, \omega_m \;=\; k_m\, v_i \;-\; R_m\, T_{Load} \;-\; L_m\, \dot{T}_{Load}$$

Shown to the right is the linear relationship between the motor's steady-state angular speed ω_m and a constant value of T_{Load} (when v_i is constant).

Notice that increasing the input voltage v_i increases the line's offset.

Note: The slope of this line is called the motor's ***speed-torque gradient constant***, k_{gradient}.



6.2.1 Stall torque

Write the ODE governing T_{Load} when both v_i and ω_m are **constant** and determine the steady-state part of T_{Load} as a function of ω_m and v_i when both v_i and ω_m are **constant**.
Result:

$$L_m\, \dot{T}_{Load} \;+\; R_m\, T_{Load} \;=\; k_m\, v_i \;-\; (R_m\, b + k_m\, k_v)\, \omega_m$$

$$T_{Load}\Big|_{\substack{v_i \text{ is constant} \\ \omega_m \text{ is constant}}} \;=\; \boxed{\qquad\qquad}\, \omega_m \;+\; \boxed{\qquad}\, v_i$$

Solve for k_m in terms of the steady-state value of T_{Load} when v_i is **constant** and the motor is ***stalled***, i.e., $\omega_m(t) = 0$.

$$k_m \;=\; \boxed{\qquad}\, T_{Load}$$

While the motor is ***stalled***, use the multi-meter to measure the resistance of the motor coil at several angular positions.

$$\text{Average Resistance} \quad R_m \;\approx\; \boxed{\qquad} \text{ Ohms}$$

Using the motor attached to the cart via the wire, measure the ***stall torque*** for an appropriate range of v_i from 3 to 5 volts. Assume the spring is linear. Note the length of the moment arm. **Do not stall the motor for long** or it will overheat and burn out.

Approx. v_i	Measured v_i	Cart Displacement	Force	Torque	k_m
3.0					
4.0					
5.0					
				Average k_m	

6.2.2 No-load angular speed

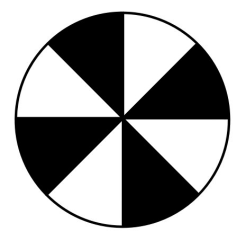
Solve for k_v in terms of the steady-state value of ω_m when v_i is **constant** and there is ***no load*** on the motor, i.e., $T_{Load}(t) = 0$.

$$k_v =$$

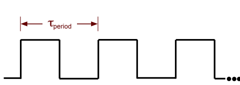
6.2.3 Estimation with an Encoder

We use an several pieces of equipment to measure and record the motor's angular speed,

- **Encoder:**
  Our optical quadrature encoder determines our motor's rotational speed by detecting alternating light and dark patterns on a disk. For example, the encoder on the right shows 8 transitions (from light to dark or vice-versa). A quadrature encoder has the ability to detect both angular speed **and** direction. Our encoder has 1000 transitions (500 black sections and 500 white sections) and counts 1000 $\frac{\text{tics}}{\text{rev}}$.



- **Oscilloscope:**
  The oscilloscope receives a digital signal from the encoder (i.e., bits of one and zero) and displays them to the screen. For example, the oscilloscope pattern to the right shows bits of one ("high bits") and bits of zero ("low bits"). The value τ_{period} is the time interval from a transition from low to high to the next transition from low to high, e.g., measured in microseconds (10^{-6} seconds).



Using a motor without load, measure the steady-state value of ω_m (use the oscilloscope and a single output channel of the encoder) for an appropriate range of v_i from 0 to 6 volts.

$$\omega_m = \frac{1 \text{ interval}}{\tau_{\text{period}}\ \mu\text{sec}} * \frac{1 \text{ rev}}{500 \text{ intervals}} * \frac{10^6\ \mu\text{sec}}{1 \text{ sec}} * \frac{2\,\pi \text{ rad}}{1 \text{ rev}} \approx \frac{12566 \text{ rad}}{\tau_{\text{period}} \text{ sec}}$$

Complete the following table assuming $b \approx 0$.

Approx. v_i	Measured v_i	Period X	Angular speed ω_m ($\frac{rad}{s}$)	k_v
2.0				
4.0				
6.0				
			Average k_v	

†In lab 1 we found that these motors are dominated by Coulomb friction. Based on the data just taken, find a better estimate for k_v by including a friction loading term, T_f. Also find the magnitude of the frictional loading T_f in N m.

6.2.4 Comparison and verification

Determine the percent error[11] in the difference in your estimates for k_m and k_v.

$$\frac{k_m - k_v}{k_v} * 100 = \boxed{} \%$$

- If k_m and k_v differ more than 10%, why do think that this is the case?

- Which estimate do you think is more accurate and why?

- What measurement would you perform to improve your accuracy?

[11]The definition of "volt" unit leads to (for an ideal DC motor) $k_m = k_v$ when SI units are used.