Prelab	Participation	Lab
$\surd+ \ \surd \ \surd-$ 0	$\surd+ \ \surd \ \surd-$ 0	$\surd+ \ \surd \ \surd-$ 0

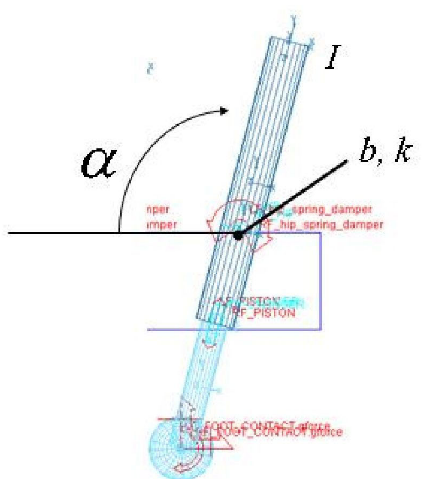
Name: ______________________________

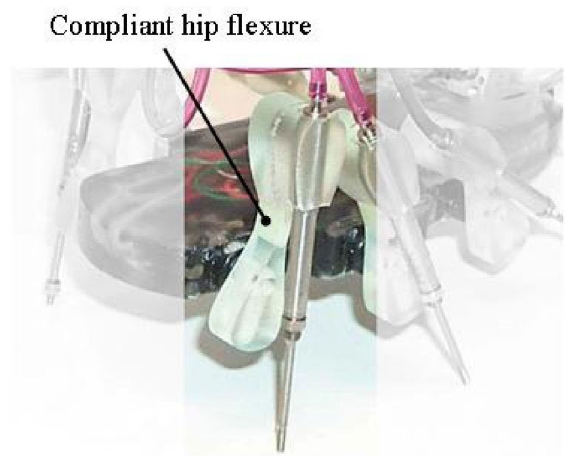
3 Lab 3: Rotational Systems

3.1 Introduction

The Sprawl family of robots has been designed over 10^+ years at Stanford University. They are small, hexapedal dynamic running robots capable of speeds of over 5 $\frac{\text{body-lengths}}{\text{second}}$ ($\approx$ 0.8 $\frac{\text{m}}{\text{s}}$). They operate without feedback control, instead relying on a well-tuned physical system (set of legs) to move quickly and stably, even over obstacles or rough terrain.

During this lab you will examine the leg's rotational design and how to tune the leg for the swing phase of locomotion. During normal operation, each full stride of the robot takes 80 ms. The leg is in contact with the ground (stance phase) for about 25 ms, allowing approximately 55 ms for the leg to retract and swing forward in preparation for the next step. Experience has shown that making the legs too stiff results in a slow robot, whereas making the legs too soft results in crashes. For this lab you will be experiment with a wooden "leg" that approximates a Sparwlita leg of a Sprawl robot.





$$I_{yy}\,\ddot{\theta} + b\,\dot{\theta} + k\,\theta - m\,g\,L\,\sin(\theta) = 0$$

3.2 PreLab: Working Model and brainstorming

1. Download the following Working Model simulations from the class website:
   MetronomeWnZeta.wm2d
   MetronomeIBK.wm2d
   MetronomeNonlinearAngle.wm2d
2. Run the Working Model simulations.
   Record results on the Working Model PreLab (see back of the book).

3.3 Experimental

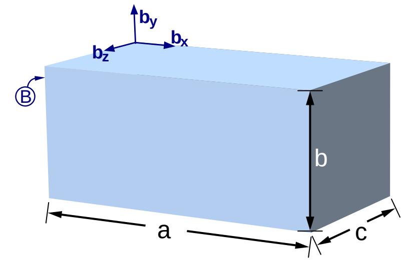
The system that you will be evaluating rotates using a compliant joint. The joint was designed and fabricated (from a soft grade of urethane) to be used as a leg flexure for Sprawlita. The leg flexure can be modeled as a pin joint with a built-in rotational spring and damper. It will be your job to experimentally determine the effective stiffness and damping of this joint.

3.3.1 Analytical determination of moment of inertia of a solid block

The moment of inertia of a rigid body about a point is defined with a line (axis). Changing the specified point on the body or the line's orientation changes the associated moment of inertia.

The moment of inertia of a uniform rectangular block about the y-axis passing through the block's center of mass is:

$$I_{yy} \;=\; \frac{1}{12}\, m\,(a^2 \,+\, c^2)$$



Mass of block m = _____ kg	Note: The object's mass in grams is written on the object	
Length of block a = _____ m	Height of block b = _____ m	Width of block c = _____ m

Does I_{yy} depend on the block's height? **Yes/No**.
Explain: Moment of inertia depends on mass-distribution **away from/along** a line.

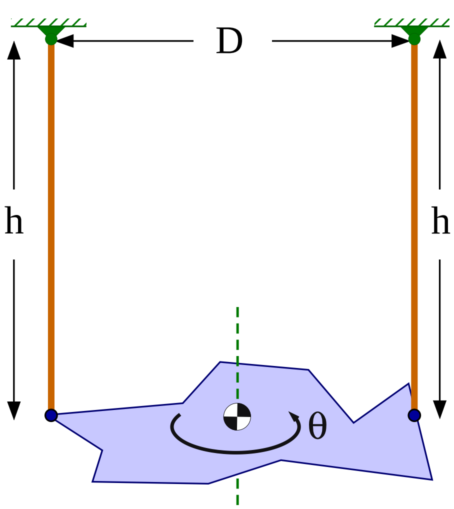
3.3.2 Methods for determining moments of inertia

Moments of inertia play a central role in the rotational motions of objects. List a few methods for determining (e.g., calculating, measuring, or approximating) moments of inertia of large irregularly-shaped objects such as an airplane or automobile.

- Compute:
- Experiment:
- Approximate:

3.3.3 Experimental apparatus

h	Length of each suspension wire
D	Distance between (2 parallel) wires
$m\,g$	Weight of the object
I_{yy}	Mass moment of inertia about y-axis
T	Tension in string
θ	Angle of rotation



As the object rotates in the horizontal plane, it **moves up and down**. The linearized equation of motion is[a]

$$\ddot{\theta} \;+\; \frac{m\,g\,D^2}{4\,I_{yy}\,h}\,\theta \;=\; 0$$

[a] One way to form the linearized equation of motion is to first generate the nonlinear equation (which is not simple) and then linearize the nonlinear equation about $\theta = 0$, $\dot{\theta} = 0$, $\ddot{\theta} = 0$.

Using our knowledge of ODEs and rearranging to solve for the moment of inertia gives

$$\omega_n = \sqrt{\frac{m\,g\,D^2}{4\,\mathrm{I}_{yy}\,h}} \quad \Rightarrow \quad \tau_{\text{period}} = \frac{2\,\pi}{\omega_n} = 2\,\pi\sqrt{\frac{4\,\mathrm{I}_{yy}\,h}{m\,g\,D^2}} \quad \Rightarrow \quad \mathrm{I}_{yy} = \frac{m\,g\,D^2}{16\,\pi^2\,h}\,\tau_{\text{period}}{}^2$$

Record measurements of the bifilar pendulum to the right. Hang the block on two strings (ensure they are **equal length** and **parallel**). Use a stopwatch to determine the pendulum's period of vibration. Provide an initial **small angle** of $\theta(t=0) \approx 30°$.

Mass of block	m =	_____ kg
Length of the wires	h =	_____ m
Distance between wires	D =	_____ m
Period of vibrations	τ_{period} =	_____ sec
Moment of inertia	I_{yy} =	_____ $\mathrm{kg\,m^2}$

How well do the predicted and measured value agree? (Circle one)

Note: If you got more than 10% deviation, you probably did something wrong.

- Extremely well (less than 0.1% deviation)
- Very well (less than 1% deviation)
- Good (less than 10% deviation)
- Sort of (less than 50% deviation)
- Poorly (less than 100% deviation)

Using the parallel axis theorem and the moment of inertia determined via the bifilar pendulum experiment, calculate the wooden leg's (B) moment of inertia about the joint.

Hint: Use the parallel axis theorem, B's moment of inertia about its center of mass B_{cm}, the mass of the block, and d, the distance from the pivot to B_{cm}.

$$\mathrm{I}_{yy}^{B/joint} = \mathrm{I}_{yy}^{B/B_{\text{cm}}} + m\,\text{distance}^2 = \underline{\hspace{2em}}\ \mathrm{kg\,m^2}$$

3.3.4 Characterization of the Compliant Joint

Carefully assemble the leg by sliding the joint into the base. Tape the accelerometer to the top of the leg. When you displace the leg, please do so gently. Keep in mind that our calculations use a ***small angle approximation***. For this part of the lab we again use an ADXL 311 accelerometer from Analog Devices. The data acquisition proceeds in a manner similar to previous labs, sampling again at 1 kHz.

1. If necessary, login to the lab computer. Username: me161student
   Password: 1euler1. Ensure the domain is ENGR
2. Power the Arduino by plugging-in (in order):
   (a). 12 Volt adaptor (between the board and wall socket)
   (b). USB cable (between the board and the computer)
3. From the desktop, navigate to the Lab2 folder and open Lab2.ino
4. Under Tools → ports, select something other than COM1, COM2, or COM3
   (the USB port can be enumerated to anything other than these)
5. Click the magnifying glass button (or type Ctrl+Shift+m) to open the serial monitor
6. On the serial monitor screen, a menu should appear. Enter "a" to start reporting data.

7. Have one group member pull the leg slightly to one side and then release.
8. Enter "a" or "r" to stop recording data after the cart has stopped.
9. Plot the data (e.g., using Excel, MATLAB®, or PlotGenesis).
10. Email the data files and/or graphs to yourself and your group members.
11. Pass in a printed graph of the cart's acceleration ($\frac{m}{s^2}$) vs. time (sec) with your lab.
12. Ensure the power to the board is off and the setup is neat for the next lab.

3.3.5 Questions

When the small angle approximation $\sin(\theta) \approx \theta$ is used, the equation of motion for small values of θ is

$$\mathrm{I}_{yy}\,\ddot{\theta} \;+\; b\,\dot{\theta} \;+\; (k - m\,g\,L)\theta \;=\; 0$$

Form symbolic expressions for the natural frequency ω_n and damping ratio ζ associated with the previous ODE.

$\omega_n \;=\;$ $\qquad\qquad$ $\zeta \;=\;$

Use the graph you generated in Section 3.3.4 to determine numerical values for the period of vibration and the decay ratio. (Show work!)

$\tau_{\text{period}} \;=\;$ _____secs $\qquad$ decayRatio $=$ _____NoUnits

Determine numerical values for the natural frequency and damping ratio of the system.

$\omega_n \;=\;$ _____ $\frac{\text{rad}}{\text{sec}}$ $\qquad$ $\zeta \;=\;$ _____ no units

Determine numerical values for this system's physical parameters I_{yy}, b, and k.

$\mathrm{I}_{yy} \;=\;$ _____ $\mathrm{kg\,m^2}$ $\qquad$ $b \;=\;$ _____ N m sec $\qquad$ $k \;=\;$ _____ $\frac{\text{N m}}{\text{rad}}$

† Using your value for the wooden leg's mass moment of inertia (I_{yy}) and experimentally determined value of the flexure stiffness k determine the damping constant b, needed to ensure a settling time of less than 55 ms.

$k \;=\;$ _____ $\frac{\text{N}}{\text{m}}$