

Prelab √+ √ √- 0	Participation √+ √ √- 0	Lab √+ √ √- 0
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Name: \_\_\_\_\_

## 2 Lab: 2<sup>nd</sup> Order Systems

### 2.1 Introduction

The first lab investigated motors and 1<sup>st</sup>-order ODEs. This lab looks at modeling vibrating systems and **2<sup>nd</sup>-order ODEs** of the form

$$m \frac{d^2 y}{dt^2} + b \frac{dy}{dt} + k y = 0$$

where the non-zero constants  $m$ ,  $b$ , and  $k$  are called the system's **effective mass**, **effective viscous damping**, and **effective spring constant**. Mathematically, the system behavior is more easily described with the following ODE which has only two constants, namely,  $\zeta$  and  $\omega_n$ .

$$\frac{d^2 y}{dt^2} + 2\zeta\omega_n \frac{dy}{dt} + \omega_n^2 y = 0$$

### 2.2 PreLab: Motor spin-down (analytical and Working Model)

1. Do the following homework problems (from the book):  
System response and damping.  
System response for various of  $\zeta$ .  
Calculating  $\zeta$  and  $\omega_n$  from empirical data for an underdamped 2<sup>nd</sup>-order ODE.
2. Download and run the following Working Model simulations from the class website:  
**[CarSuspensionWithWnAndZeta.wm2d](#)**  
**[CarSuspensionWithSpringDamper.wm2d](#)**  
Record results on the Working Model Car Suspension PreLab (see back of textbook).  
Complete the Working Model Car Suspension PreLab (from back of textbook)  
Show your lab-section TA your completed pre-lab at the start of lab.

### 2.3 Experimental

The purpose of this section is to give you a feel for how well real systems can be modeled with 2<sup>nd</sup>-order ODEs. First you will look at a simple system (a slinky) similar to that discussed in Chapter 7 of the textbook. Then you will characterize the physical parameters of the horizontal cart system that will be used in subsequent labs.

#### 2.3.1 Slinky Revisited. (Show data taken and calculations. Use SI units for all answers.)

Instead of a slinky, we will use a spring and relatively large mass. This will, hopefully, increase the accuracy of our assumption that the spring is massless. Before you begin the experiment, answer the following questions:

Knowing the (lumped) mass of the system is 150 g, determine the spring constant  $k$ .

**Result:**

$$k = \underline{\hspace{2cm}} \frac{\text{N}}{\text{m}}$$

Now calculate the system's (analytical) natural frequency  $\omega_n$ .

**Result:**

$$\omega_n = \underline{\hspace{2cm}} \frac{\text{rad}}{\text{sec}}$$

Use a stopwatch to determine the system's damped natural frequency  $\omega_d$ .

Hint: Taking an average of a few runs will probably improve your accuracy.

**Result:**

$$\omega_d = \underline{\hspace{2cm}} \frac{\text{rad}}{\text{sec}}$$

How well do the predicted and measured value agree? (Circle one)

- Extremely well (less than 0.1% deviation)
- Very well (less than 1% deviation)
- Good (less than 10% deviation)
- Sort of (less than 50% deviation)
- Poorly (less than 100% deviation)

Use a stopwatch to estimate the 1% settling time ( $t_{\text{settling}}$ ) for the system. Hint: Just eyeball it.

**Result:**

$$t_{\text{settling}} = \underline{\hspace{2cm}} \text{ sec}$$

Knowing  $t_s = \frac{4.6}{\zeta \omega_n}$ , find the value of  $\zeta$ .<sup>4</sup>

**Result:**

$$\zeta = \underline{\hspace{1.5cm}} \text{ no Units}$$

† What would the mass of the system need to be to get a settling time of less than 10 seconds? Remember that  $\zeta$  is a function of  $m$ ,  $b$ , and  $k$ .

**Result:**

$$m_{\text{new}} = \underline{\hspace{2cm}} \text{ kg}$$

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<sup>4</sup>The value of  $\zeta$  is calculated with an experimental value of  $t_s$  and a calculated value of  $\omega_n$ .

### 2.3.2 Estimation with an accelerometer

We use several pieces of equipment to measure and record the cart's horizontal acceleration, namely, we use an accelerometer, a microprocessor, a transceiver, and a computer.<sup>5</sup>

- **Accelerometer: ADXL 311 from Analog Devices**

The accelerometer is mounted on a cart and measures acceleration in up to three directions (we use data from only one direction). The accelerometer is relatively small and lightweight as compared to the cart - so its affect on the acceleration of the cart is negligible. The accelerometer's output signal is a linear  $0.3 \frac{\text{volts}}{g}$  signal over a range of  $\pm 2g$ . The accelerometer is designed to output 2.5 volts when there is **no** acceleration, but there is some variation from one accelerometer to the next.

- **Breadboard signal processing**

The accelerometer's signal is filter by a low-pass filter to remove high-frequency noise in the signal. Op-amps are used as a buffer to supply additional current and avoid unwanted voltage drops.

- **Arduino UNO microprocessor:**

The microprocessor's A/D port receives an analog voltage signal from the breadboard in a specified range (i.e., continuous voltages from 0 volts to 5 volts). The A/D port samples the analog signal at 1000 Hz (i.e., at 1 ms intervals). The 10 bit A/D converter on the microprocessor changes the 0 to 5 volt analog signal to bits (ones and zeros) that represent  $2^{10} = 1024$  integer values (e.g., 5 volts converts to 1024 and 2.5 volts converts to 512). The Arduino transmits this data to the computer via serial communication.

- **Computer:**

The computer receives bits from its serial port and uses the Arduino software program to translate the bits to integer numbers which are then printed to the screen. The numbers displayed on the computer screen are integers from 0 to 1024.

Data acquisition proceeds in a manner similar to Lab 1.<sup>6</sup>

1. If necessary, login to the lab computer. Username: me161student  
Password: 1euler1. Ensure the domain is ENGR
2. Power the Arduino by plugging-in (in order):
  - (a). 12 Volt adaptor (between the board and wall socket)
  - (b). USB cable (between the board and the computer)
3. From the desktop, navigate to the Lab2 folder and open Lab2.ino
4. Under Tools  $\rightarrow$  ports, select something other than COM1, COM2, or COM3 (the USB port can be enumerated to anything other than these)
5. Click the magnifying glass button (or type Ctrl+Shift+m) to open the serial monitor
6. On the serial monitor screen, a menu should appear. Enter "a" to start reporting data.
7. Have one group member pull the cart to one side and then release it.
8. Enter "a" or "r" to stop recording data after the cart has stopped.
9. Plot the data (e.g., using Excel, MATLAB<sup>®</sup>, or PlotGenesis).
10. Email the data files and/or graphs to yourself and your group members.
11. Pass in a printed graph of the cart's acceleration ( $\frac{m}{s^2}$ ) vs. time (sec) with your lab.
12. Ensure the power to the board is off and the setup is neat for the next lab.

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<sup>5</sup>Most accelerometers do not come assembled with a microprocessor, transceiver, and computer.

<sup>6</sup>Although the output signal can be twice-integrated to generate position data, this is unnecessary for present purposes.

### 2.3.3 Questions - SHOW YOUR WORK

Use your graph to determine numerical values for period of vibration and decay ratio.<sup>7</sup>

**Result:**

$$\tau_{\text{period}} = \text{_____secs} \qquad \text{decayRatio} = \text{_____ no units}$$

Calculate numerical values for the system's natural frequency and damping ratio.

**Result:**

$$\omega_n = \text{_____} \frac{\text{rad}}{\text{sec}} \qquad \zeta = \text{_____ no Units}$$

List three things that could improve the accuracy of these values.

- 1.
- 2.
- 3.

**Now add the mass to the platform and repeat the experiment.**

1. Pass in a new graph of the cart's acceleration ( $\frac{m}{s^2}$ ) vs. time (sec) with your lab.
2. Calculate the new natural frequency and damping ratio

**Result:**

$$\omega_n = \text{_____} \frac{\text{rad}}{\text{sec}} \qquad \zeta = \text{_____ no Units}$$

3. The formula  $\omega_n = \text{_____}$  shows adding mass **increases/decreases** natural frequency.
4. The formula  $\zeta = \text{_____}$  shows adding mass **increases/decreases** damping ratio.

† What are the physical parameters (m,b,k) for this system without the added mass? Show your work on a separate page. You will use these values again in later labs!

**Result:**

$$m = \text{_____ kg} \qquad b = \text{_____} \frac{\text{N sec}}{\text{m}} \qquad k = \text{_____} \frac{\text{N}}{\text{m}}$$

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<sup>7</sup>The textbook chapter on time-specifications for *2<sup>nd</sup>-order ODEs* is helpful for determining period of vibration and decay ratio.