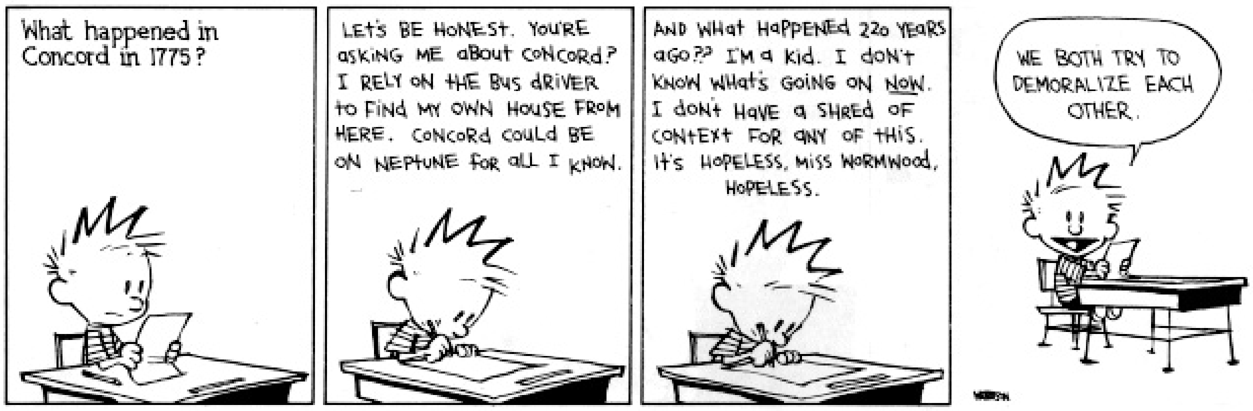
NAME ________________________________

ME161 Midterm. Tuesday October 28, 2003 1:15-2:45 p.m.

I certify that I upheld the Stanford Honor code during this exam ____________________

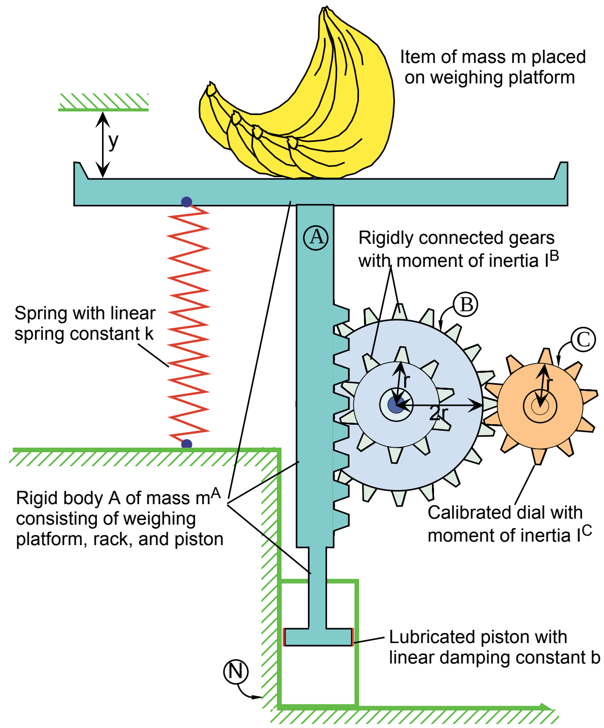


- Print your name and sign the honor code statement
- You may use your course notes, homework, books, etc.
- Write your answers on this handout
- Where space is provided, show your work to get full credit
- If necessary, attach extra pages for scratch work

Problem	Value	Score
1	2	
2	25	
3	22	
4	23	
5	28	
Total	100	

Midterm.1 (2 pts.)
In a sentence or two, describe one or two things you have enjoyed learning in ME161.
Result:

Midterm.2 (25 pts.) **Equation of motion for a commercial spring scale**



The figure to the right shows a spring scale whose weighing platform is supported by a linear translational spring with spring constant k and lubricated piston with linear viscous damping constant b. When an item (e.g., bananas) of mass m is weighed on Earth whose gravity is $9.8\ \frac{\text{m}}{\text{sec}^2}$, the platform is displaced vertically downward by a distance y which causes the inner gear of radius r of rigid body B to rotate counter-clockwise by an angle $\theta_B = y/r$. To magnify the rotation of B, a calibrated gear C of radius r is meshed with B's outer gear of radius $2r$.

The combined mass of the rigid body A consisting of the weighing platform, rack, and piston is m^A. The moments of inertia of B and C about their individual axes of rotation (which are fixed in N) are denoted I^B and I^C, respectively.

Find the differential equation governing $y(t)$, cast it in the form $(m+m_e)\,\ddot{y} + b_e\,\dot{y} + k_e\,y = g(t)$, and express m_e, b_e, k_e, and $g(t)$ in terms of m, m^A, I^B, I^C, r, b, k, etc.

Result:

m_e	b_e	k_e	$g(t)$

Midterm.3 (22 pts.) **Determining mass, damping, and spring constants from empirical data**

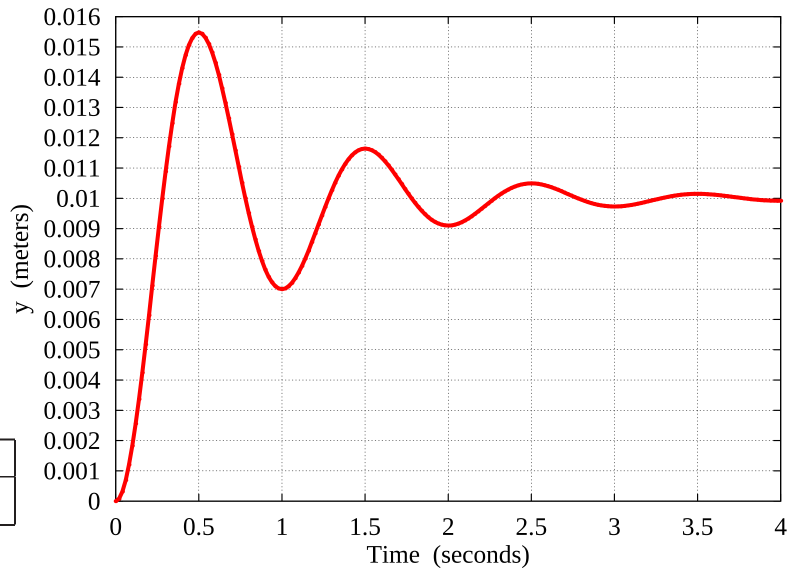
The graph to the right shows the time-response of the system described in problem 2 when an object of mass m=2 kg is placed on the scale. Knowing that the equation governing $y(t)$ is

$$(m+m_e)\,\ddot{y} \;+\; b_e\,\dot{y} \;+\; k_e\,y \;=\; 19.6$$

determine numerical values for m_e, b_e, and k_e.

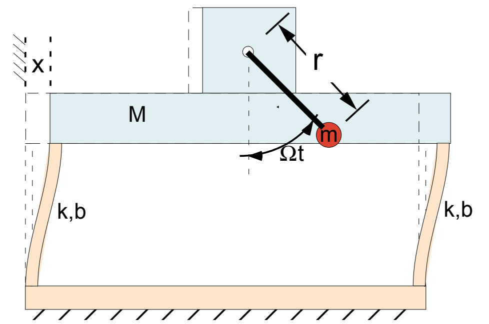
Result:

m_e	b_e	k_e
kg	$\frac{\text{n*sec}}{\text{m}}$	$\frac{\text{n}}{\text{m}}$



Midterm.4 (23 pts.) **Dynamic response for air-conditioner on a building**

The schematic to the right shows an air conditioner that is bolted to the roof of a one story building. The air conditioner's motor is unbalanced and its eccentricity is modeled as a particle of mass m attached to the distal end of a rigid rod of length r. When the motor spins with angular speed Ω, it causes the building's roof of mass M to vibrate. The stiffness and material damping in *each* column that supports the roof is modeled as a linear horizontal spring and linear horizontal damper. The equation governing the horizontal displacement x of the roof of the building is



$$(M+m)\,\ddot{x} \;+\; 2\,b\,\dot{x} \;+\; 2\,k\,x \;=\; m\,r\,\Omega^2\,\sin(\Omega t)$$

For certain values of M, m, b, k, and r, this ODE simplifies to

$$\ddot{x} \;+\; 0.02\,\dot{x} \;+\; 100\,x \;=\; 1\,\text{x}\,10^{-5}\;\Omega^2\,\sin(\Omega t)$$

(a) **(5 pts.)** While the air conditioner is off (Ω=0), the side of the building is pushed with a big gust of wind. Determine $x(t)$ in response to initial values of $x(0)$=0 and $\dot{x}(0)$=5.

$x(t)$

(b) **(9 pts.)** Fill in numerical values in the following expressions for $x_{ss}(t)$, the steady-state part of $x(t)$.

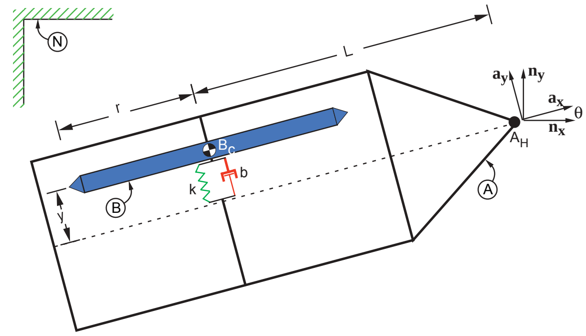
Ω	$x_{ss}(t)$
1	$*\sin(\quad t+\quad)$
10	$*\sin(\quad t+\quad)$
100	$*\sin(\quad t+\quad)$

(c) **(9 pts.)** The building's occupants complain that the roof shakes too much and propose several remedies. Comment on the effect each remedy has on the magnitude of $x_{ss}(t)$.

Remedy when Ω=12 $\frac{\text{rad}}{\text{sec}}$	**Effects of remedy on $\lvert x_{ss}(t)\rvert$**
Balancing the motor	
Changing the motor speed Ω	
Adding or removing mass from the roof (changing M)	
Stiffening the columns that support the roof (changing k)	
Adding damping to the columns and roof (changing b)	

Midterm.5 (28 pts.) Dynamic response of a single-wheel trailer

Although it is well know that single-wheel trailers sometimes behave poorly, it is not always clear why they do so. One possibility is that tire flexibility and loose wheel mounting give rise to unstable behavior. To explore this concept, the linearized equation of motion for the system depicted to the right was formed, and after assuming a solution of the form $C\,e^{p\,t}$, the equation governing values of p was found to be



$$p^3 \;+\; 5.06\,p^2 \;+\; (50.6{+}3.4{*}v)\,p \;+\; 33.7{*}v \;=\; 0$$

where v measures the speed of the hitch-point in the $\mathbf{n}_\mathrm{x}$ direction.

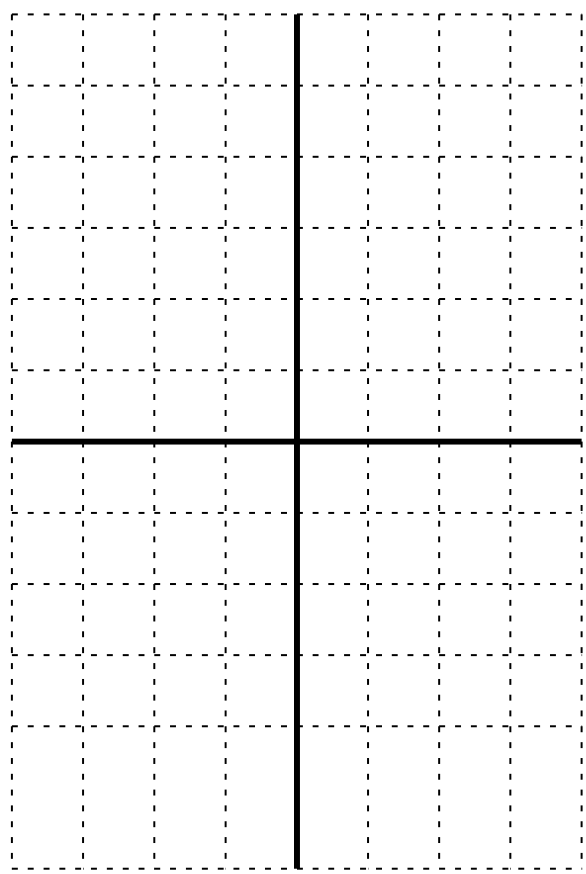
(a) **(1 pt.)** Classify the equation governing p by picking the relevant qualifiers from the list.

Uncoupled	**Linear**	**Homogeneous**	**Algebraic**
Coupled	**Nonlinear**	**Inhomogeneous**	**Differential**

(b) **(3 pts.)** By inspecting the coefficients of p, it is possible to know certain facts about p_1, p_2, and p_3 (the values of p that satisfy the equation), and to know that the solution $C_1\,e^{p_1\,t} + C_2\,e^{p_2\,t} + C_3\,e^{p_3\,t}$ is **stable/unstable** (circle one) when $v < 0$ because (explain)

(5 pts.) Calculate the missing values of p_1, p_2, and p_3 in the table below. Sketch the root locus for $0 \le v \le 30$ and extrapolate for $v < 0$ and $v > 30$. Use arrows to show the direction of increasing v.

v	p_1	p_2, p_3
0		
5	-2.75	-1.15 ± 7.74 i
10	-4.17	-0.45 ± 8.98 i
15	-4.99	-0.03 ± 10.06 i
20	-5.55	+0.25 ± 11.01 i
25	-5.97	+0.46 ± 11.87 i

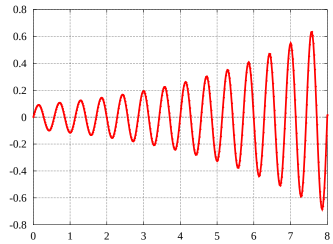


(c) **(10 pts.)** Complete the following with a value or range of values of v from the previous table.

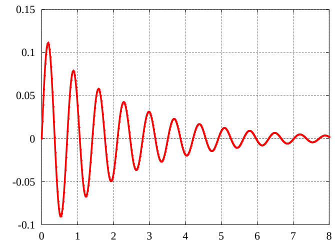
Statement	Value(s) of v
Solution is stable	
Solution is neutrally stable	and
Solution is unstable	and
Solution is most oscillatory	$v =$
Solution is least oscillatory	$v =$
Solution is most stable	$v =$
Solution is least stable	$v =$
Fastest speed of response	$v =$
Slowest speed of response	$v =$
$e^{p\,t} \to 0$ most quickly	$v =$

(d) **(3 pts.)** Your younger sister is taking the car with attached trailer for a ride. What do you tell her about safe speeds and conditions when driving with the trailer?
Result:

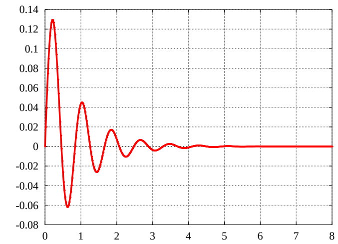
(e) **(6 pts.)** For each graph shown below, circle the associated value of v.



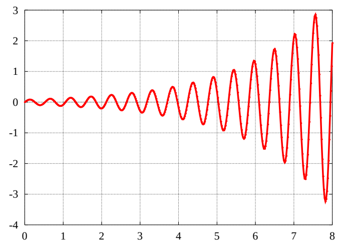
$v=$ **0/5/10/15/20/25**



$v=$ **0/5/10/15/20/25**



$v=$ **0/5/10/15/20/25**



$v=$ **0/5/10/15/20/25**

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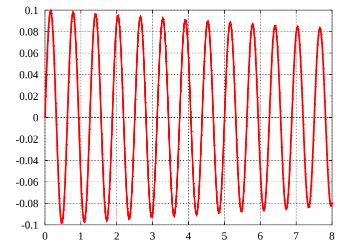

$v=$ **0/5/10/15/20/25**