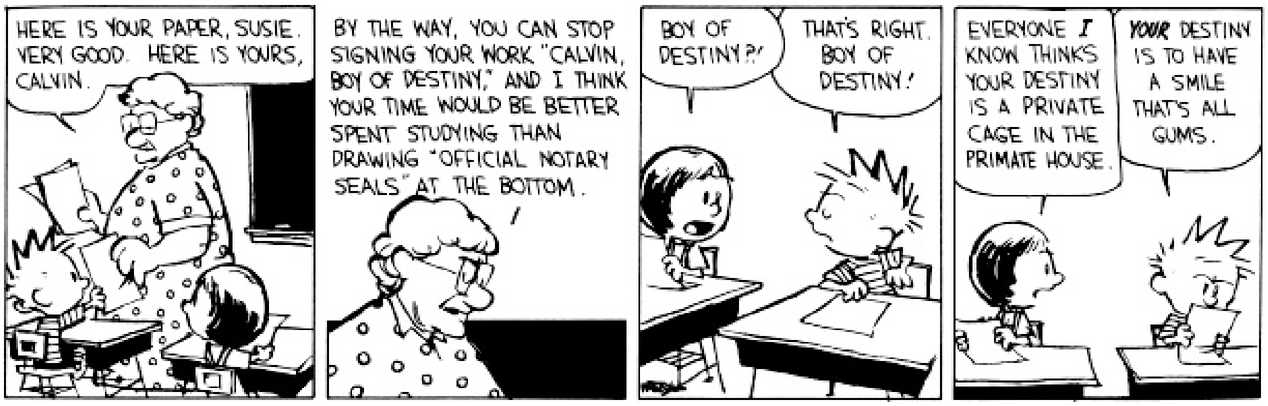
NAME ____________________

ME161 Midterm. Tuesday October 29, 2002 1:15-2:45 p.m.

I certify that I upheld the Stanford Honor code during this exam ____________________

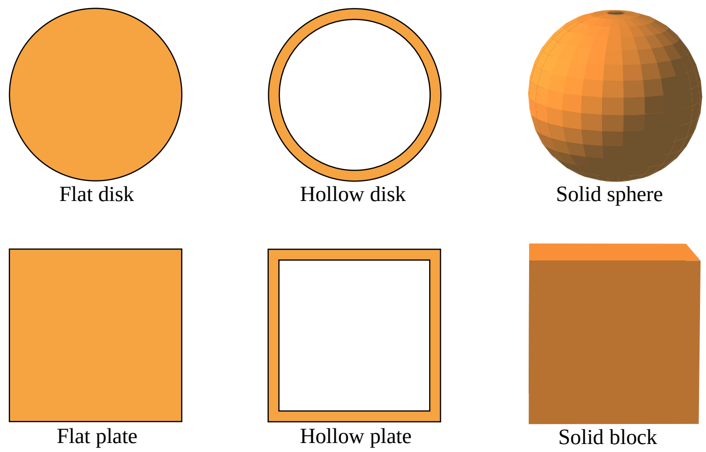


- Print your name and sign the honor code statement
- You may use your course notes, homework, books, etc.
- Write your answers on this handout
- Where space is provided, show your work to get full credit
- If necessary, attach extra pages for scratch work

Problem	Value	Score
1	33	
2	24	
3	18	
4	25	
Total	100	

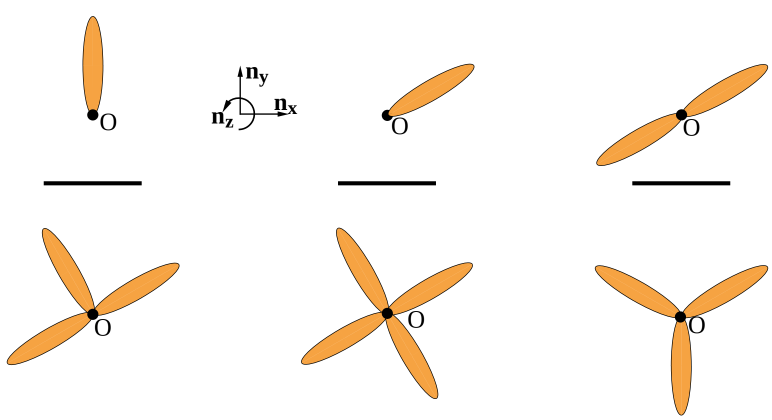
Midterm.1 (33 pts.) True/False, Multiple Choice, Definitions, etc.

(a) **(4 pts.)** Each object below has a uniform density and an equal mass. Answer the following questions about I_{zz}, the moment of inertia of each object about the line that passes through its mass center and is perpendicular to the plane of the paper.



- **(1 pt.)** Consider the first row of objects. The **flat disk/hollow disk/solid sphere** (circle one) has the **largest** value of I_{zz}, whereas the **flat disk/hollow disk/solid sphere** (circle one) has the **smallest** value of I_{zz}.
- **(1 pt.)** Consider the second row of objects. The **flat plate/hollow plate/solid block** has the **largest** value of I_{zz}, whereas the ____________ and ____________ (fill in the blanks) have **equal** values of I_{zz}.
- **(2 pts.)** Consider all the objects in both rows. The ____________ has the **largest** value of I_{zz}, whereas the ____________ has the **smallest** value of I_{zz}.

(b) **(3 pts.)** The figure below shows six objects, each with a uniform density. For each object, consider $I_{\mathbf{n}_x \mathbf{n}_y}$, the product of inertia of the object for lines that pass through point O and are parallel to $\mathbf{n}_x$ and $\mathbf{n}_y$. Below each object, mark whether the product of inertia is ***negative***, ***zero***, or ***positive***.



(c) **(2 pts.)** For both 2D and 3D analysis, the angular velocity of a rigid body B in a reference frame N can always be written in terms of the time-derivative of an angle θ and a unit vector $\mathbf{k}$ as ${}^{N}\boldsymbol{\omega}^{B} = \pm\,\dot{\theta}\,\mathbf{k}$ as long as θ and $\mathbf{k}$ are carefully chosen. **True/False**.

(d) **(5 pts.)** Put the following functions into amplitude/phase form by filling in the blanks.

$$\text{-}3 * \sin(2t) \; + \; 4 * \cos(2t) \;\; = \qquad\qquad * \sin(2t + \qquad\qquad)$$

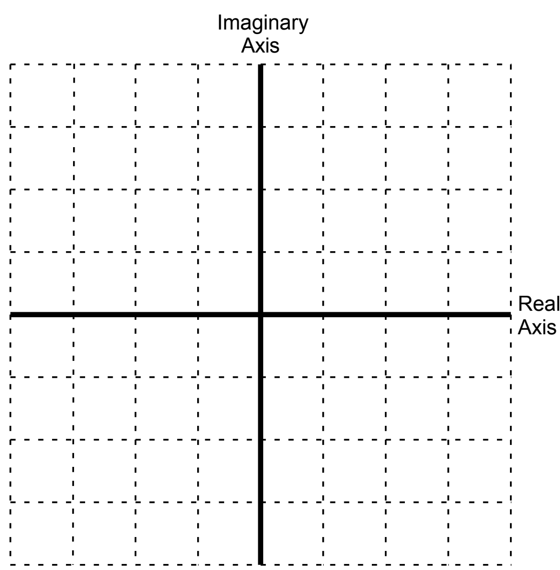
$$\text{-}3 * \sin(2t + \frac{\pi}{3}) \; + \; 4 * \cos(2t) \;\; = \qquad\qquad * \cos(2t + \qquad\qquad)$$

(e) **(5 pts.)** A system is governed by the differential equation

$$\ddot{y} \; + \; (k_d \; - \; 4)\,\dot{y} \; + \; 4\,y \; = \; 0$$

Draw a root locus versus k_d by filling in the table and completing the associated graph. Clearly mark the direction of increasing k_d. Do not worry if your root locus does not completely fit on the graph.

k_d	Pole location p_1	Pole location p_2
0		
2		
4		
6		
8		
10		



(f) **(1 pt.)** The range of values of k_d that stabilize $y(t)$ is

(g) **(1 pt.)** The value of k_d that results in a purely oscillatory solution for $y(t)$ is

(h) **(1 pt.)** $y(t)$ goes most quickly to zero when k_d=

(i) **(1 pt.)** The range of k_d that result in some oscillatory behavior of $y(t)$ is

(j) **(1 pt.)** The range of k_d that result in an overdamped solution for $y(t)$ is

(k) **(3 pts.)** Circle the phrase that *best* describes the associated behavior.

- A pole in the right half plane is **stable/neutrally stable/unstable**
- As a pole moves to the left, it is more **stable/unstable/oscillatory/damped/fast**
- As a pole moves away from the origin (in any direction), it is more **stable/unstable/oscillatory/**

(l) **(6 pts.)** Each graph below shows $y(t)$ versus t for a specific value of k_d [with $y(0)$=1 and $\dot{y}(0)$=0]. Below each graph, fill in the appropriate value of k_d, i.e., k_d=0, 2, 4, 6, 8, or 10.

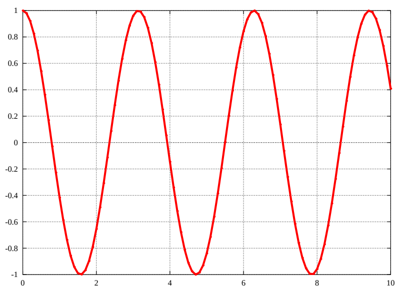


Figure 22.32: k_d=

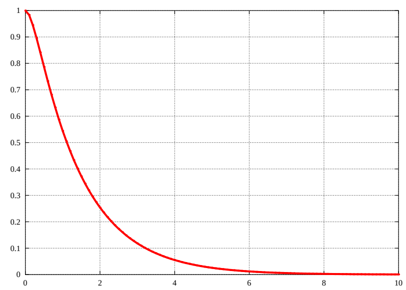


Figure 22.33: k_d=

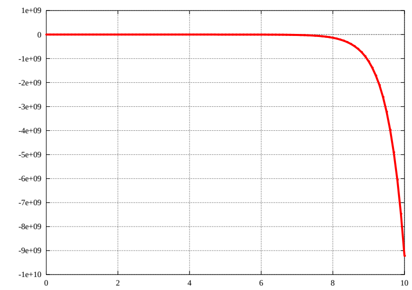


Figure 22.34: k_d=

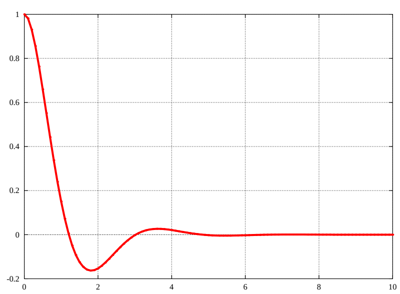


Figure 22.35: k_d=

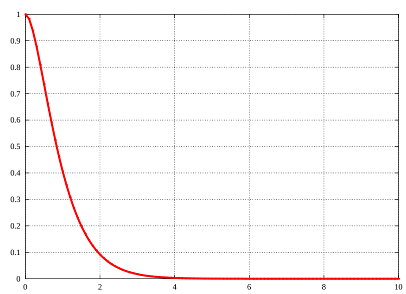


Figure 22.36: k_d=

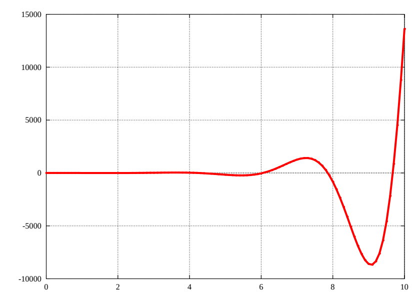


Figure 22.37: k_d=

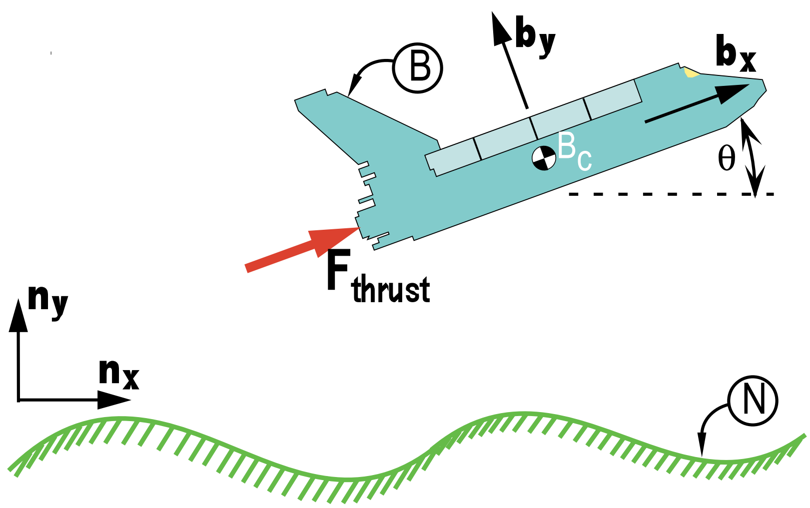
Figure 22.38: Time response of y for various values of k_d

Midterm.2 (24 pts.) Equations of motion for an aircraft

The figure below shows an aircraft B flying over Earth N (considered to be a Newtonian reference frame). The aircraft's orientation and speed depends on engine thrust, aerodynamics, and gravity. To design a robust flight controller, one needs the equations that govern the aircraft's motion.

To facilitate the analysis, right-handed sets of unit vectors $\mathbf{n}_x$, $\mathbf{n}_y$, $\mathbf{n}_z$ and $\mathbf{b}_x$, $\mathbf{b}_y$, $\mathbf{b}_z$ are fixed in N and B, respectively, with $\mathbf{n}_x$ horizontally right, $\mathbf{n}_y$ vertically upward, $\mathbf{n}_z$=$\mathbf{b}_z$, and $\mathbf{b}_x$ in the aircraft's forward direction. The following table of symbols is available to assist in the analysis.

Quantity	Symbol	Type
Local gravitational constant	g	constant
Mass of B	m	constant
Moment of inertia of B about B_c (B's mass center) for $\mathbf{b}_z$	I_{zz}	constant
Measure of the aerodynamic forces on B in $\mathbf{b}_x$ direction	Fx_{aero}	specified
Measure of the aerodynamic forces on B in $\mathbf{b}_y$ direction	Fy_{aero}	specified
Measure of the aerodynamic torque on B in $\mathbf{b}_z$ direction	T_{aero}	specified
Thrust from jet engine in the $\mathbf{b}_x$ direction	F_{thrust}	specified
Angle that $\mathbf{b}_x$ makes with the local horizontal	θ	dependent variable
Measure of B's velocity in $\mathbf{b}_x$ direction	v_x	dependent variable
Measure of B's velocity in $\mathbf{b}_y$ direction	v_y	dependent variable
Time	t	independent variable



(a) **(3 pts.)** Form ${}^{\text{b}}R^{\text{n}}$, the rotation matrix that relates $\mathbf{b}_x$, $\mathbf{b}_y$, $\mathbf{b}_z$ to $\mathbf{n}_x$, $\mathbf{n}_y$, $\mathbf{n}_z$.
Result:

${}^{\text{b}}R^{\text{n}}$	$\mathbf{n}_x$	$\mathbf{n}_y$	$\mathbf{n}_z$
$\mathbf{b}_x$			
$\mathbf{b}_y$			
$\mathbf{b}_z$			

(b) **(2 pts.)** Find ${}^{N}\boldsymbol{\omega}^{B}$, the angular velocity of B in N, and express it in terms of $\mathbf{b}_x$, $\mathbf{b}_y$, $\mathbf{b}_z$.
Result:

$${}^{N}\boldsymbol{\omega}^{B} =$$

(c) **(2 pts.)** Write the *complete* definition of ${}^{N}\boldsymbol{\alpha}^{B}$, the angular acceleration of B in N, and then express it in terms of $\mathbf{b}_x$, $\mathbf{b}_y$, $\mathbf{b}_z$.

Result:

$$ {}^{N}\boldsymbol{\alpha}^{B} \triangleq \qquad\qquad = $$

(d) **(5 pts.)** The velocity of B_c in N is ${}^{N}\mathbf{v}^{B_c} = v_x \mathbf{b}_\text{x} + v_y \mathbf{b}_\text{y}$. Write the *complete* definition of ${}^{N}\mathbf{a}^{B_c}$, (the acceleration of B_c in N), and express it in terms of $\mathbf{b}_\text{x}$, $\mathbf{b}_\text{y}$, $\mathbf{b}_\text{z}$.
Result:

$$ {}^{N}\mathbf{a}^{B_c} \triangleq \qquad\qquad = $$

(e) **(2 pts.)** The aerodynamic forces acting on B can be replaced with a force $Fx_{aero}\,\mathbf{b}_\text{x} + Fy_{aero}\,\mathbf{b}_\text{y}$ applied at B_c together with a couple of torque $T_{aero}\,\mathbf{b}_\text{z}$. Considering aerodynamic forces, thrust, and gravitational forces, find $\mathbf{F}^B$, the resultant of all forces on B.

$$ \mathbf{F}^B = $$

(f) **(7 pts.)** Using your knowledge of vector algebra to simplify your work, form a set of scalar equations that suffice to predict the motion of the aircraft, i.e., differential equations for $v_x(t)$, $v_y(t)$, and $\theta(t)$. Put the equations in standard form.
Result:

(g) **(1 pt.)** Classify the previous equations by picking the relevant qualifiers from the list below.

Uncoupled	**Linear**	**Homogeneous**	**Constant-coefficient**	**1st-order**	**Ordinary**	**Algebraic**
Coupled	**Nonlinear**	**Inhomogeneous**	**Variable-coefficient**	**2nd-order**	**Partial**	**Differential**

(h) **(2 pts.)** Given initial values of v_x, v_y, θ, and $\dot{\theta}$; numerical values for m, g, and I_{zz}; and the functional form of Fx_{aero}, Fy_{aero}, T_{aero}, and F_{thrust}, how would you simulate the motion of the aircraft, i.e., how would you solve the equations of motion to find $v_x(t)$, $v_y(t)$, and $\theta(t)$.
Result:

Midterm.3 (18 pts.) Equations of Motion for Control of an Antenna

The figure below shows a small motor A which rotates a pulley B which in turn rotates a second pulley C. The purpose of this mechanism is to control the orientation of an antenna which is rigidly welded to C. The following table of symbols is available to assist in the analysis.

Quantity	Symbol	Type
Radius of A and inner wheel of B	r	constant
Relevant moment of inertia of A and its motor about A_o	I^A	constant
Relevant moment of inertia of B about B_o	I^B	constant
Relevant moment of inertia of C and antenna about C_o	I^C	constant
Linear torsional viscous damper for A	b_A	constant
Linear torsional viscous damper for B	b_b	constant
Linear torsional viscous damper for C	b_C	constant
Motor torque on A	T_m	specified
Angle between a line fixed in A and line fixed in N	θ_A	dependent variable
Angle between a line fixed in B and line fixed in N	θ_B	dependent variable
Angle between a line fixed in C and line fixed in N	θ_C	dependent variable
Time	t	independent variable

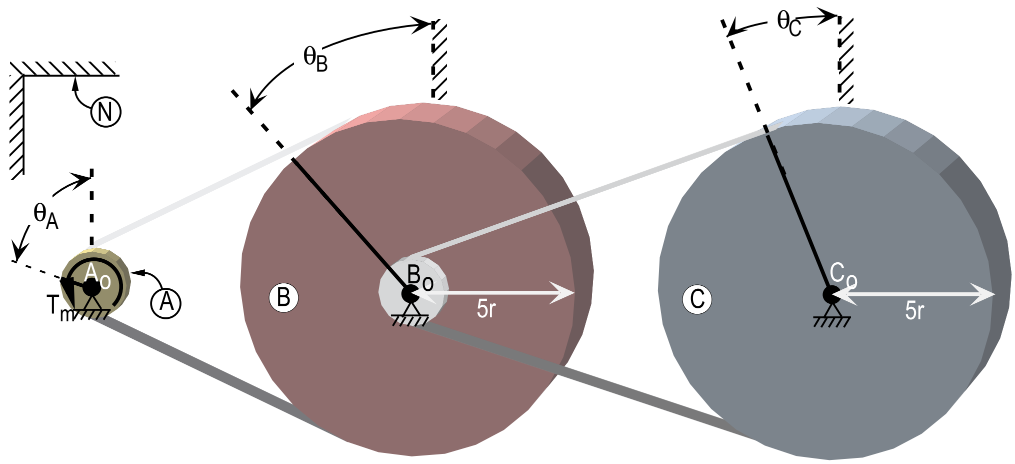


Figure 22.39: Motor with belt-drive controlling antenna orientation

(a) **(2 pts.)** Form an expression for K, the kinetic energy of the entire system in a Newtonian reference frame N, and express it in terms of symbols in the table and their time-derivatives.
Result:

$$K \; =$$

(b) **(4 pts.)** Ignoring the flexibility of the pulley belts, form an expression for P, the power of the system in N, and express it in terms of T_m, b_A, b_B, b_C, $\dot{\theta}_A$, $\dot{\theta}_B$, and $\dot{\theta}_C$.
Result:

$$P \; =$$

(c) **(5 pts.)** Form an equation of motion for this system. Use an inextensible model for the pulley belts, i.e., $\dot{\theta}_A$=5$\dot{\theta}_B$ and $\dot{\theta}_B$=5$\dot{\theta}_C$, to express your results without θ_A, θ_B, or their derivatives.
Result:

(d) **(1 pt.)** Cast the equation of motion into the form $m_e \ddot{\theta}_C + b_e \dot{\theta}_C + k_e \theta_C = g(t)$ and determine m_e, b_e, k_e, $g(t)$ in terms of numbers, k_p, and k_d.
Result:

m_e	b_e	k_e	$g(t)$

(e) **(2 pts.)** The motor and A are very small as compared to the larger pulley C and its attached antenna, i.e., $I^A < 0.01 * I^C$. In view of this, an engineer tells you to simplify the model of the system and ignore I^A. Do you think this is a good idea or not? Explain.
Result:

(f) **(4 pts.)** At a certain instant of time (t=0), θ_C=0, $\dot{\theta}_C$=0.2 rad/sec, and the motor is off (T_m=0). Find the solution for $\theta_C(t)$ and express it in terms of m_e, b_e, k_e, and t.
Result:

$\theta_C(t) \ =$

Midterm.4 (25 pts.) Motion Control of an Antenna

The equation governing θ_C (henceforth designated as θ) in an antenna system similar to the one depicted in Figure 22.39 is a function of the motor torque T_m. The equations of motion and the feedback control law for T_m have the form

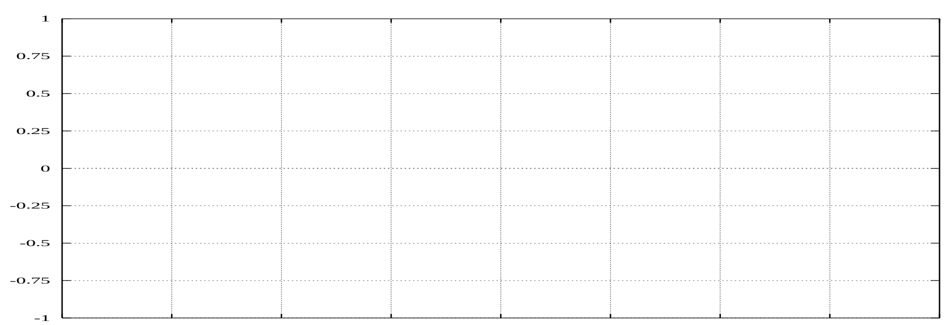
$$40\,\ddot{\theta} \;+\; 200\,\dot{\theta} \;=\; T_m \qquad\qquad T_m \;=\; \text{-}k_p\,\theta \;+\; \text{-}k_d\,\dot{\theta}$$

(a) **(2 pts.)** Cast the equation of motion into the form $m_e\,\ddot{\theta} \;+\; b_e\,\dot{\theta} \;+\; k_e\,\theta \;=\; g(t)$ and determine m_e, b_e, k_e, $g(t)$ in terms of numbers, k_p, and k_d.
Result:

m_e	b_e	k_e	$g(t)$

(b) **(8pts.)** Find values of k_p and k_d such that the decay ratio *decayRatio*=0.75 and the settling time $t_{settling}$=0.1 secs
Result:

(c) **(5pts.)** Taking $\theta(0)$=1.0 rad and $\dot{\theta}(0)$=0 rad/sec, create a rough sketch of the first few periods of the response.
Result:



(d) **(10pts.)** Determine the effect of increasing each of the feedback-control constants on the various quantities listed on the left-hand side of the table below. Only consider control constants that produce an *underdamped* system. Fill in the table with one of the following: "smaller", "unchanged", "larger', or "need more information".

Effect on various quantities	Effect of increasing k_p	Effect of increasing k_d
Decay ratio		
Damping ratio ζ		
Natural frequency ω_n		
Damped natural frequency ω_d		
Period of vibration τ_{period}		
Maximum overshoot M_p		
Settling time $t_{settling}$		