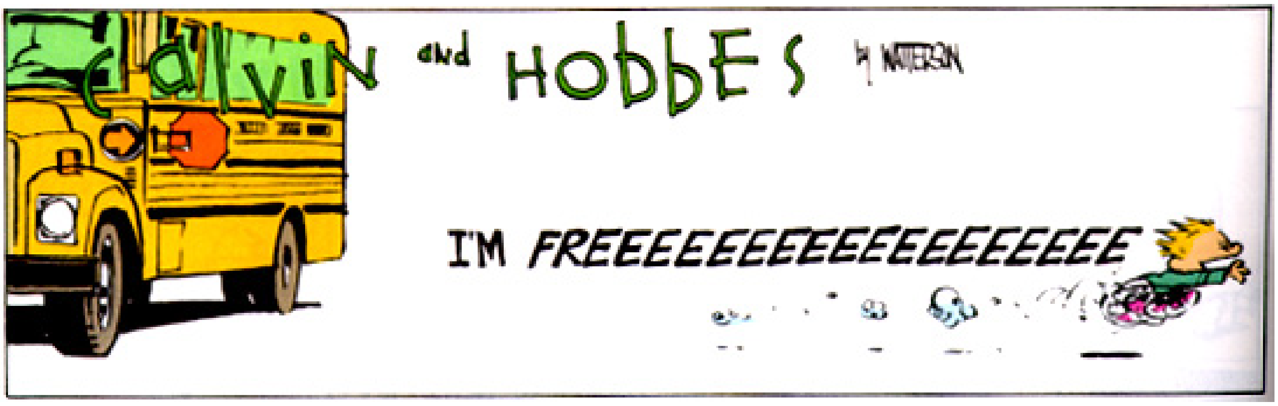
Name ______________________

ME161 Final. Thursday December 11, 2003 7:00-10:00 p.m.

I certify that I upheld the Stanford Honor code during this exam ______________________



- Print your name and sign the honor code statement
- You may use your course notes, homework, books, etc.
- Write your answers on this handout
- Where space is provided, **show your work to get credit**
- If necessary, attach extra pages for scratch work
- Best wishes for a fun vacation. Merry Christmas and Happy New Year!

Problem	Value	Score
1	2	
2	13	
3	17	
4	14	
5	10	
6	22	
Total	100	

Final.1 (2 pts.) Class participation. Fill in the blanks.

Go Stanford.

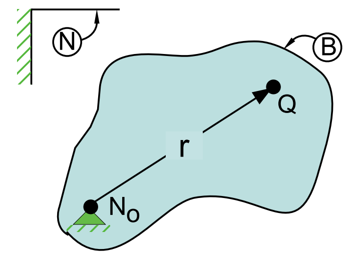
Beat Cal!

Final.2 (13 pts.) Miscellaneous

(a) **(3 pts.)** The equation $\mathbf{M} = I\boldsymbol{\alpha}$ is useful for analyzing three-dimensional rotational motions of a rigid body. **True/False**

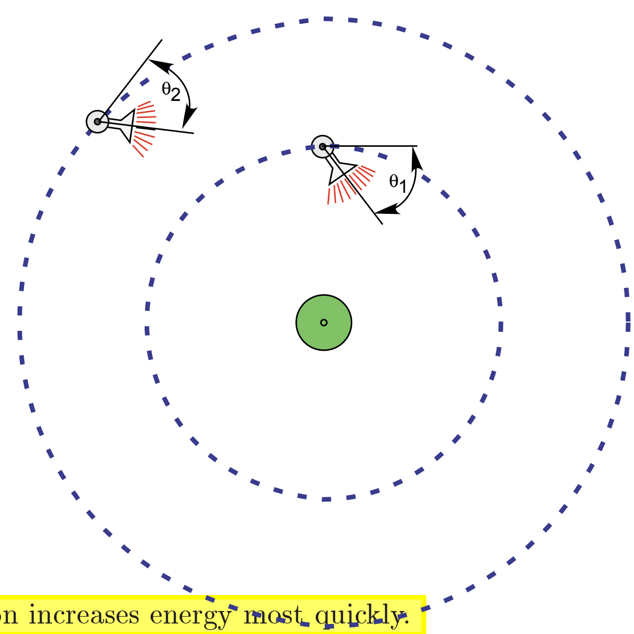
(b) **(4 pts.)**

Points N_o and Q are fixed on rigid body B. Point N_o is also fixed in reference frame N. Show that ${}^N\mathbf{v}^Q$ (the velocity of Q in N) can be written in terms of ${}^N\boldsymbol{\omega}^B$ (the angular velocity of B in N) and $\mathbf{r}$ (the position vector from N_o to Q). Explain each step in your mathematical proof with a brief phrase.



Mathematical statement	Reasoning
${}^N\mathbf{v}^Q \triangleq \frac{{}^N d\,\mathbf{r}}{dt}$	Definition of velocity of Q in N
$= \frac{{}^B d\,\mathbf{r}}{dt} + {}^N\boldsymbol{\omega}^B \times \mathbf{r}$	Golden rule for vector differentiation
$= {}^N\boldsymbol{\omega}^B \times \mathbf{r}$	$\mathbf{r}$ does not change in magnitude or direction in B

(c) **(4 pts.)**
To thrust a satellite from low circular orbit about Earth to a higher circular orbit, an impulse is provided at two instants. The first impulse can be directed radially outward from Earth, tangential to the satellite's circular orbit, or directed at some angle θ_1 from the satellite's orbital tangent. The second impulse is applied at apogee (when the satellite is furthest away from Earth), and is directed at an angle θ_2 from the satellite's orbital tangent.
Using your engineering insights, provide values for θ_1 and θ_2 that minimize the amount of fuel required to move the satellite from low circular orbit to high circular orbit. Explain your reason for choosing these values.



Result:

$\theta_1 = 0^\circ$ $\qquad$ $\theta_2 = 0^\circ$

Reason: Putting force in the direction of the motion increases energy most quickly.

(d) **(2 pts.)** Put the following functions into amplitude/phase form by filling in the phase.[40]

$$-\sin(2t) = \sin(2t + \pi)$$
$$-\cos(2t) = \sin(2t + \tfrac{-\pi}{2})$$

[40] In amplitude/phase form, the amplitude is non-negative.

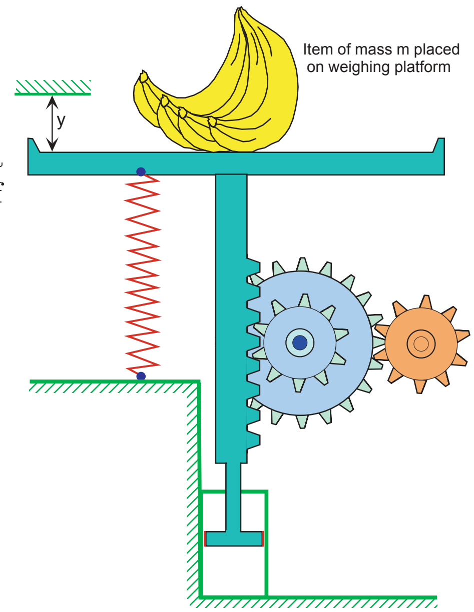
Final.3 (17 pts.) **Finding mass, damping, and spring constants to meet design specifications**

One task an engineer performs is sizing springs, dampers, etc., to meet design specifications. Knowing that the equation governing the deflection $y(t)$ for the spring scale shown to the right is

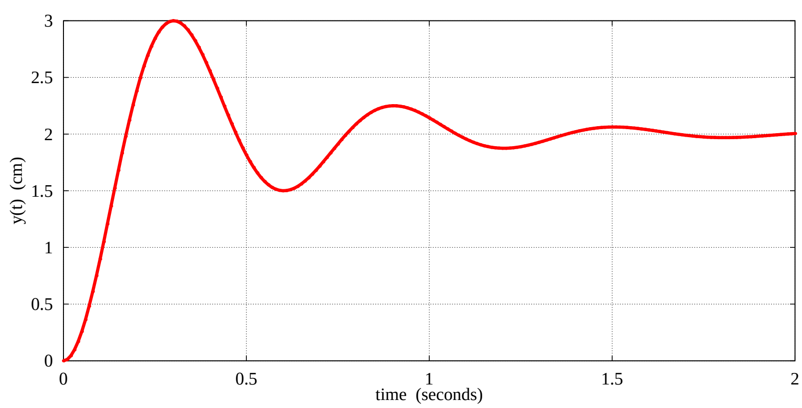
$$(m+m_e)\,\ddot{y} \;+\; b_e\,\dot{y} \;+\; k_e\,y \;=\; 9.8\,m$$

determine numerical values for m_e, b_e, and k_e so that

- the scale deflects y=2 cm when m=10 kg
- the scale settles to 1% of its final value within 2 seconds
- the maximum overshoot of $y(t)$ from its final value is 50%



To ensure you understand the design specifications, first make a *rough* sketch of $y(t)$ for $0 \le t \le 2$ sec and clearly identify each of the design specifications on the graph. Assume $y(0)$=0 and $\dot{y}(0)$=0. **(5 pts.)**

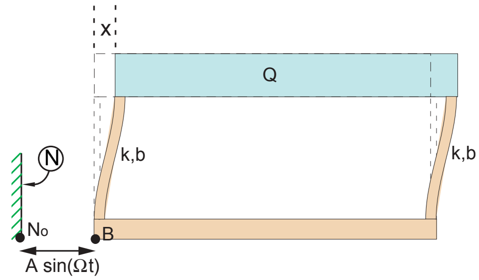


(12 pts.) **Result:**

m_e	b_e	k_e
32.90 kg	197.57 $\frac{\text{n*sec}}{\text{m}}$	4900 $\frac{\text{n}}{\text{m}}$

Final.4 (14 pts.) **Dynamic response of a building in an earthquake**

The schematic to the right shows a building whose base vibrates because of an earthquake. The earth's horizontal motion is modeled as $A\sin(\Omega t)$ where A is the magnitude of the ground motion and Ω is the earthquake's frequency. When the earth moves, it causes the building's roof of mass m to vibrate. The stiffness and material damping in *each* column that supports the roof is modeled as a linear horizontal spring (k) and linear horizontal damper (b).



The equation governing the horizontal displacement x of the building's roof is

$$m\ddot{x} + 2b\dot{x} + 2kx = m\Omega^2 \sin(\Omega t)$$

For certain values of m, b, and k, this ODE simplifies to

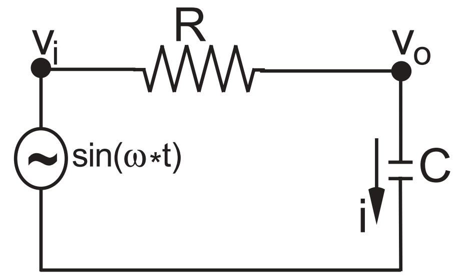
$$\ddot{x} + 0.02\dot{x} + 100x = 1\,\mathrm{x}\,10^{-5}\,\Omega^2 \sin(\Omega t)$$

Comment on the effect of varying m, b, k, A, and Ω on the magnitude of the inhomogeneous forcing function [i.e., the coefficient $1\,\mathrm{x}\,10^{-5}\,\Omega^2$ of $\sin(\Omega t)$ in the previous equation]; the magnifying effect of harmonic forcing near the natural frequency; and the magnitude of $x_{ss}(t)$. Fill in each element in the following two tables by writing − (decreases), **0** (no effect), + (increases), or ? (if it may decrease *or* increase). For the first table, assume the earthquake's frequency is approximately 11 $\frac{\text{rad}}{\text{sec}}$, and for the second table, assume $\Omega \approx 9\,\frac{\text{rad}}{\text{sec}}$.

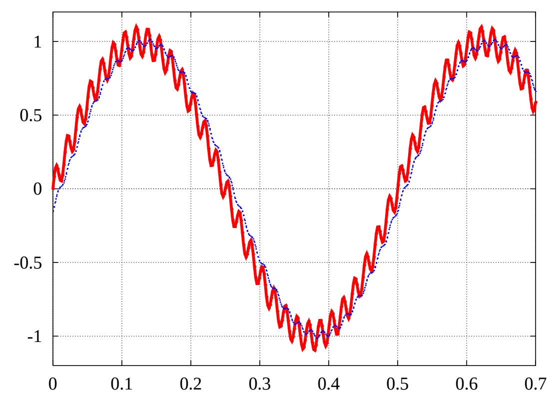
$\Omega \approx 11\,\frac{\text{rad}}{\text{sec}}$	**Effect on forcing function**	**Effect on harmonic forcing**	**Effect on $\lvert x_{ss}(t)\rvert$**
Increasing A (the earthquake's amplitude)	+	**0**	+
Increasing Ω to 11.5 $\frac{\text{rad}}{\text{sec}}$	+	−	?
Decreasing Ω to 10.5 $\frac{\text{rad}}{\text{sec}}$	−	?	?
Adding mass to the roof (increasing m)	−	−	−
Removing mass from the roof	+	+	+
Stiffening the support columns (increasing k)	**0**	?	?
Adding damping to the columns (increasing b)	**0**	−	−

$\Omega \approx 9\,\frac{\text{rad}}{\text{sec}}$	**Effect on forcing function**	**Effect on harmonic forcing**	**Effect on $\lvert x_{ss}(t)\rvert$**
Increasing A (the earthquake's amplitude)	+	**0**	+
Increasing Ω to 9.5 $\frac{\text{rad}}{\text{sec}}$	+	?	?
Decreasing Ω to 8.5 $\frac{\text{rad}}{\text{sec}}$	−	−	−
Adding mass to the roof (increasing m)	−	?	?
Removing mass from the roof	+	−	?
Stiffening the support columns (increasing k)	**0**	−	−
Adding damping to the columns (increasing b)	**0**	−	−

Final.5 (10 pts.) Design of a low-pass filter



The electrical circuit to the right is an RC (resistor-capacitor) circuit that acts as a filter. The input voltage signal v_i is a known (specified) function of time t, and the output quantity of interest is $v_o(t)$, the voltage across the capacitor.



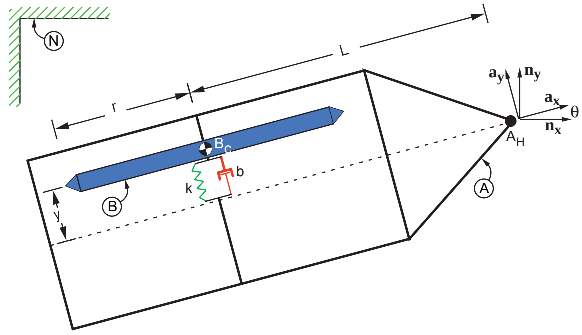
The input signal v_i comes from a sensor that is "noisy" because it transmits ambient 60 Hz frequencies which need to be filtered out. In other words, the sensor transmits a signal such as the thick solid-line shown to the right.
Knowing that C=1milliFarad, find a value for R so that the filter's bandwidth is 15 Hz and the output signal $v_o(t)$ looks like the thin dotted-line shown to the right

Result:

$$R = \boxed{10.6} \text{ Ohms}$$

Final.6 (22 pts.) Linearization, state-space, and stability of a single-wheel trailer

Although it is well know that single-wheel trailers sometimes behave poorly, it is not always clear why they do so. One possibility is that tire flexibility and loose wheel mounting give rise to unstable behavior. To explore this concept, the following identifiers are useful.



Quantity	Identifier	Type
Distance from A_H (hitch point) to the wheel's axle	L	constant
Wheel radius	r	constant
Linear spring constant modeling tire flexibility	k	constant
Linear damping constant modeling tire flexibility	b	constant
Mass of B	m	constant
Moment of inertia of A about A_H for $\mathbf{a}_z$	I^A	constant
Moment of inertia of B about B_c for $\mathbf{a}_y$	I^B	constant
Moment of inertia of B about B_c for *any* line perpendicular to $\mathbf{a}_y$	J^B	constant
$\mathbf{n}_x$ measure of the velocity of A_H in N	v	constant
Angle between $\mathbf{n}_x$ and $\mathbf{a}_x$	θ	dependent variable
Distance between B_c and the line parallel to $\mathbf{a}_x$ that passes through A_H	y	dependent variable
Time	t	independent variable

The equations governing this system's motion can be written in terms of $z_1 \triangleq m + \frac{I^B}{r^2}$ as

$$\dot{y} \;-\; v\,\sin(\theta) \;-\; L\,\dot{\theta} \;=\; 0$$
$$(I^A + J^B + z_1\,y^2)\,\ddot{\theta} \;+\; L\,z_1\,y\,\dot{\theta}^2 \;+\; [b\,L^2 + 2\,V\,z_1\,y\,\sin(\theta)]\,\dot{\theta} \;+\; L\,[k\,y + b\,V\,\sin(\theta)] \;=\; 0$$

(a) **(4 pts.)** Assuming y, θ, and $\dot{\theta}$ are small, linearize the nonlinear differential equations.
Result:

$$\dot{y} \;-\; v\,\theta \;-\; L\,\dot{\theta} \;=\; 0$$
$$(I^A + J^B)\,\ddot{\theta} \;+\; b\,L^2\,\dot{\theta} \;+\; L\,(k\,y + b\,V\,\theta) \;=\; 0$$

(b) **(3 pts.)** After solving the last equation for $\ddot{\theta}$, cast the previous set of equations into the state-space form $\dot{Y} = A\,Y$ where $Y \triangleq \begin{bmatrix} y \\ \theta \\ \dot{\theta} \end{bmatrix}$, and find each element of A.

Result:

$$\begin{bmatrix} \dot{y} \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & v & L \\ 0 & 0 & 1 \\ \frac{-k\,L}{I^A + J^B} & \frac{-b\,L\,v}{I^A + J^B} & \frac{-b\,L^2}{I^A + J^B} \end{bmatrix} \begin{bmatrix} y \\ \theta \\ \dot{\theta} \end{bmatrix}$$

(c) **(12 pts.)** The solution to the previous set of equations has the form $Y(t) = U * e^{\lambda t}$ where λ is a constant and U is a *non-zero* 3×1 matrix of constants. Find the polynomial equation which governs the value of λ.

Result:

$$\lambda^3 \;+\; \frac{b\,L^2}{I^A + J^B} * \lambda^2 \;+\; \frac{L\,(b\,v + k\,L)}{I^A + J^B} * \lambda \;+\; \frac{k\,L\,v}{I^A + J^B} \;=\; 0$$

(d) **(3 pts.)** For a certain trailer, the polynomial equation governing λ is

$$\lambda^3 \;+\; 5.06\,\lambda^2 \;+\; (50.6 + 3.4 * v)\,\lambda \;+\; 33.7 * v \;=\; 0$$

How does one determine the values of v which result in an unstable trailer motion?

Result:

Draw a root-locus of λ versus v.