ME111
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Lecture #14
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Today's Topics

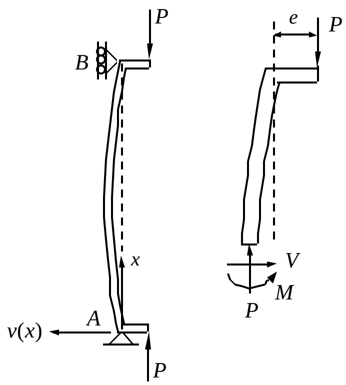
- Columns with eccentric loading.
- Secant column formula.

Reading Assignment

Juvinall, Section 5.13 - 5.15

14.1 Columns with Eccentric Loading

- The Euler and Johnson formulae govern columns which have perfectly concentric loading (ideal columns).
- Because of manufacturing tolerances, ideal columns are seldom found in practice.
- Most columns, either through design (in which a load eccentricity is deliberately introduced) or because of manufacturing defects (in which a load eccentricity is accidentally introduced), will be subject to eccentric axial loading.
- The load eccentricity can seriously degrade the column failure load below the ideal column estimate (Euler or Johnson formulae).



Equilibrium:

$\sum M_C = 0$:

$M(x) - P(e + v(x)) = 0$

14.2 Secant Column Formula

• Let's set up the mathematical problem:

Equilibrium:

$$M(x) = P(e + v(x))$$

Beam theory:

$$-\frac{M}{EI} = \frac{d^2 v}{dx^2}$$

Combining:

$$\frac{d^2 v}{dx^2} + \beta^2 v = -\beta^2 e, \qquad \beta^2 = \frac{P}{EI}$$

General solution:

$$v(x) = C_1 \sin \beta x + C_2 \cos \beta x - e$$

Boundary conditions:

$$v(0) = v(L) = 0$$

Integration constants:

$$C_1 = e \tan \frac{\beta L}{2}; \qquad C_2 = e$$

$$v(x) = e\left(\tan \frac{\beta L}{2} \sin \beta x + \cos \beta x - 1 \right)$$

Maximum deflection:

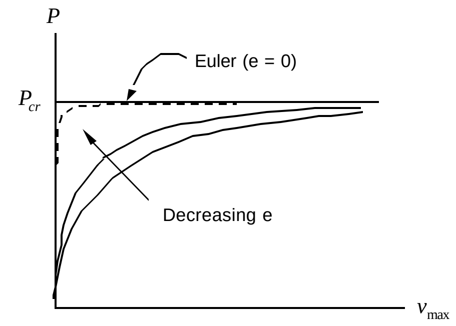
$$\begin{aligned} v_{\max} &= v(L/2) \\ &= e\left(\sec \frac{\beta L}{2} - 1 \right) \\ &= e\left(\sec\left(\frac{L}{2} \sqrt{\frac{P}{EI}} \right) - 1 \right) \end{aligned}$$

Recalling

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

We can write:

$$v_{\max} = e\left(\sec \frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} - 1 \right)$$



Let's examine the maximum stress:

Maximum moment is

$$M_{max} = P(e + v_{max}) = Pe\sec\frac{\beta L}{2}$$

Maximum stress is

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max}c}{I} = \frac{P}{A} + \frac{Pec}{I}\sec\frac{\beta L}{2} = \frac{P}{A}\left[1 + \frac{ec}{I/A}\sec\frac{\beta L}{2}\right]$$

Recall

$$\beta = \sqrt{\frac{P}{EI}}; \qquad \rho = \sqrt{\frac{I}{A}}$$

Giving

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{\rho^2}\sec\left(\frac{L}{2}\sqrt{\frac{P}{EI}}\right)\right]$$

Or

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{\rho^2}\sec\left(\frac{L}{2\rho}\sqrt{\frac{P}{EA}}\right)\right]$$

The last expression can also be expressed as:

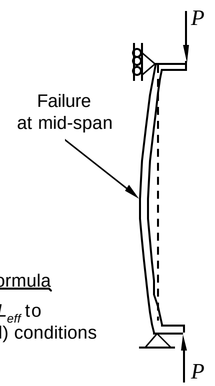
$$\frac{P}{A} = \frac{\sigma_{max}}{1 + \frac{ec}{\rho^2}\sec\left(\frac{L}{2\rho}\sqrt{\frac{P}{EA}}\right)}$$

Let's use this result to consider the condition when failure will first occur. For a ductile material, this will happens when the max stress reaches the yield strength, and for a brittle material when the max stress reaches the fracture strength:

$$\sigma_{max} \to S_{yc}; \quad P \to P_y$$

(assuming a ductile material for the moment). This now gives a failure condition as:

$$\frac{P_y}{A} = \frac{S_{yc}}{1 + \frac{ec}{\rho^2}\sec\left(\frac{L_{eff}}{2\rho}\sqrt{\frac{P_y}{EA}}\right)}$$



- This is known as the secant column formula
- Notice that we have replaced L with L_{eff} to account for the columns support (end) conditions

Define the eccentricity ratio to be

$$E_r = \frac{ec}{\rho^2}$$

Then the secant formula can be written as

$$\frac{P_y}{A} = \frac{S_{yc}}{1 + E_r \sec\left(\frac{L_{eff}}{2\rho}\sqrt{\frac{P_y}{EA}}\right)}$$

For concentrically loaded columns, it is recommended to take

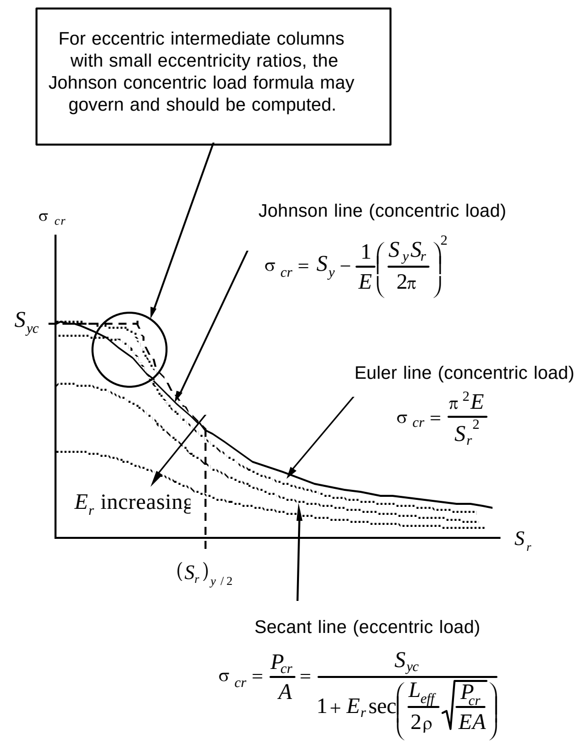
$$E_r = 0.025$$

as an estimate of the actual eccentricity introduced by manufacturing tolerances.

14.3 Factor of Safety

Define the factor of safety as:

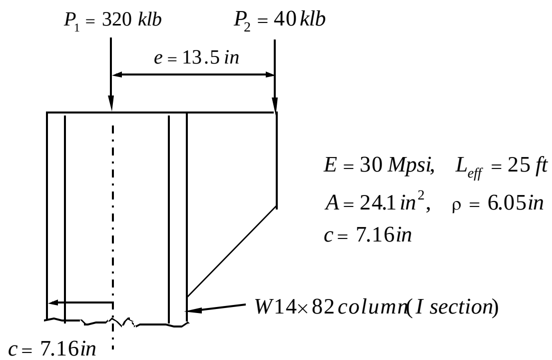
$$N_b = \frac{P_y}{P_{design}}$$



Example 14.1

A column is 25 ft long and is pinned at its ends, and supports a centrally applied load P_1 = 320 klb and an eccentrically applied load P_2 = 40 klb.

(a) What us the maximum compressive stress in the column?
(b) If S_y = 42 kpsi, what is the factor of safety against failure?
(c) Ignoring the eccentricity, what is the factor of safety against buckling?



Example 14.2

A column with the cross-section shown has the following data:

$$S_y = 20\,kpsi,\quad E = 10\,Mpsi,\quad L_{eff} = 120\,in$$

(a) Determine the critical concentric load.
(b) Determine the critical eccentric load if $E_r = \dfrac{ec}{\rho^2} = 1$.