ME 106/227
Spring 2002
Assignment #2 – Race Car Set-up
Due Tuesday, April 17

Purpose of Assignment #2

The simple handling model discussed last week suggests that a vehicle's understeering or oversteering characteristics are determined by the cornering stiffness of the tires and the weight distribution. While these are indeed important factors, they are not the only factors. The roll characteristics of the suspension (roll center height, roll stiffness, track width) also play a major role (so to speak) and roll of the vehicle can influence things like camber and steering directly. In this assignment, we will look at how these factors influence static handling and the peak lateral acceleration that a car can produce in a corner. We'll also take a quick first look at the data you obtained in the lab. For the purposes of this assignment, we'll assume that we can change things like stiffness, roll center height and roll steer. Next week, we will match these characteristics with specific suspension and steering design parameters and take a closer look at the lab data.

A more complete version of the magic formula:

This week we examine the Magic Formula tire model with the addition of camber angle. This is slightly larger than last week, but reduces to the same equations when the camber angle is set equal to zero. In this model, camber alters the cornering stiffness and causes horizontal and vertical shifts of the tire curves away from the origin. The equations are:

$$F_Y = D\sin(C\tan^{-1}(B\Phi)) + S_v$$

$$\Phi = (1-E)(a + S_h) + (E/B)\tan^{-1}(B(a + S_h))$$

where each of the coefficients in the model is described as a function of vertical load and the camber angle (or more appropriately the inclination angle γ of the tire):

$$C = 1.30$$

$$D = a_1 F_z^2 + a_2 F_z$$

$$BCD = a_3 \sin(a_4 \tan^{-1}(a_5 F_z))(1 - a_{12}|g|)$$

$$E = a_6 F_z^2 + a_7 F_z + a_8$$

$$S_h = a_9 g$$

$$S_v = (a_{10} F_z^2 + a_{11} F_z) g$$

The coefficient D represents the peak side force while the product of coefficients BCD represents the cornering stiffness. One set of data (**which requires vertical force in kN, slip angle in degrees and inclination angle in degrees and gives side force in N**) is:

$a_1 = -30.0$	$a_5 = 0.3$	$a_9 = 0.028$
$a_2 = 1011$	$a_6 = 0$	$a_{10} = 0$
$a_3 = 1078$	$a_7 = -0.354$	$a_{11} = 14.8$
$a_4 = 1.82$	$a_8 = 0.707$	$a_{12} = 0.022$

Problem 1 – Let's get rolling

While the basic function you used in the homework last week took the normal loads on the inside and outside tires as inputs, we assumed these were the same. With the information from class this week, we can now add roll and put together a more complete model that will demonstrate the effects of stabilizer bars, spring stiffnesses and roll center heights.

(1) Using MATLAB, develop a program that calculates the steer angle required for any level of lateral acceleration. Assume that we will be looking at a vehicle in left-hand turn so that the steering angle is positive and the lateral acceleration is positive. The flow of the program should parallel the development in Gillespie and in class but substitute this tire model. The structure should look like:

$$\begin{gathered}\text{Calculate roll angle for this lateral acceleration}\\ \downarrow \\ \text{Calculate normal load on all 4 tires} \\ \downarrow \\ \text{Get slip angles at front and rear axles} \\ \downarrow \\ \text{Calculate steering angle}\end{gathered}$$

(2) Debugging is a critical skill in developing analytical tools of this sort. After all, if you aren't confident in the model you have, you can't be confident in the design advice it gives you. Use the following parameter set and assume the vehicle has a 55/45 weight distribution and no rear roll bar. For Problems 1 and 2, assume zero inclination angles on both tires.

$$\begin{aligned}
&m = 1646kg\\
&L = 2.72m\\
&h_{cg} = 0.6m\\
&h_f = h_r = 0.1m\\
&t_f = 1.46m\\
&t_r = 1.49m\\
&K_{ff} = 350 Nm/\deg \text{ (springs only)}\\
&K_{ff} = 800 Nm/\deg \text{ (springs and anti-roll bar)}\\
&K_{fr} = 350 Nm/\deg \text{ (springs only)}
\end{aligned}$$

Do the following to check that your model is working correctly:

(A) What is the roll rate of this vehicle based upon the formula we derived in class? Plot the roll angle versus lateral acceleration. Is the slope the same as the predicted roll rate?

(B) Put the c.g. of the vehicle and the roll center heights on the ground. What should the load transfer from side to side be in this case? Is it? Plot the steer angle (in degrees) versus lateral acceleration (in g's) curve for the vehicle. Do your results agree with what you obtained last week?

(C) Put the c.g. height and roll center heights back to their original values. Plot the four tire normal loads (in Newtons) as a function of lateral acceleration (in g's). Do the inside and outside forces always sum to the original load on that axle?

(D) Plot inside and outside slip angles and the steer angle (in degrees) versus lateral acceleration (in g's) from 0 to the point that the vehicle loses traction.

Note on data presentation: The function that you are using returns 1e6 to let you know that no physical value of slip produces the side force you require. This is clearly not a physical value of slip and none of your plots should show such a ridiculous value. You should plot only up to the point where one of the axles loses grip and not beyond (some plots may show "recovery" when both axles have passed the friction limits, but this again is not physical). In the problems that follow, calculate and express peak lateral accelerations to 0.01g.

Problem 2 – Springs and Stabilizer bars

Inspired by a particularly thrilling lecture in vehicle dynamics, you decide to devote your weekends to racing and want to adapt your car to the purpose. You decide to get a handle on the characteristics of your car by running it through the simulator developed above. Assume that your test track has a radius of 50m.

(1) Using the steer angle versus lateral acceleration plot that you generated above, answer the following questions. Does the car understeer or oversteer in the linear region? What is the understeer gradient (approximated from the graph)? Does it display limit understeer or limit oversteer? What is the peak lateral acceleration? What limits it?

(2) You're bumming. This doesn't seem like it is going to win you many autocrosses. Figuring that the front anti-roll bar might be causing you problems, you decide to remove it. Plot the new steering angle required as a function of the lateral acceleration. Does the car understeer or oversteer in the linear region? What is the understeer gradient (approximated from the graph)? Does it display limit understeer or limit oversteer? What is the peak lateral acceleration? What is the limiting factor?

(3) Next you decide to put the anti-roll bar you removed from the front on the rear axle. Does the car understeer or oversteer in the linear region? What is the understeer gradient (approximated from the graph)? Does it display limit understeer or limit oversteer? What is the peak lateral acceleration? What is the limiting factor? Does this seem like a good modification?

(4) Instead of removing bars, you next decide to buy new anti-roll bars of greater stiffness so that you have stiffnesses of 800 Nm/deg on both the front and the rear. Does the car understeer or oversteer in the linear region? What is the understeer gradient (approximated from the graph)? Does it display limit understeer or limit oversteer? What is the peak lateral acceleration? What is the limiting factor?

(5) Assume that you have a total roll stiffness of 1700 Nm/deg to allocate as you wish between the axles. What allocation would you choose to maximize (loosely speaking – I'm not looking for a numerical optimization here) speed through the turn? Plot steer angle versus lateral acceleration. What is the peak lateral acceleration? What is your new roll rate? Is that reasonable or have you made the car too stiff with this change?

(6) If you could make a suspension change as well that would alter the roll center height (while leaving the c.g. height unchanged) would you want to lower it or raise it if your intention was to reduce roll and load transfer? Show mathematically why this is the case.

Just as a point of interest, if you were to get seriously into racing, you would want to be able to consider several other modifications as well that might come in handy on race day. One of these is changing the tire pressure, which alters the tire characteristics (cornering stiffness and peak). Another is to add wedge, cranking up the pre-load on a rear spring to create a diagonal imbalance in the tire normal force distribution. In many racing circuits, there are also suspension modifications that can be made. In NASCAR, the rear roll center height can be adjusted by changing the track rod of the suspension. The key to understanding the challenge of racing is to see that the tire properties change as the tires heat up and wear. Since your front tires are steering while your rear tires carry all of the tractive forces in acceleration, they wear at different – and rather unpredictable – rates. Thus, the tricky balancing act you have done on this assignment only holds for one point in time. As a crew chief, you need to make the call on how much to change each parameter so that the car will run as well as it can between pit stops. You also have rather imprecise data from which to work if you have any data at all. If you make the wrong call… well, that's racing, and you'll get 'em next week.

Problem 3 – Roll and camber change

In the previous problem, we assumed that the tires remained perfectly horizontal while the vehicle was cornering. This is never true since the camber angle of each tire changes as the suspension deflects. For all of these problems, use the set up of Problem 2 part (4).

(1) To get a feel for what camber does, plot the baseline tire curve for a normal load of 4kN over a slip angle range of –20 to 20 degrees together with the curves representing a +4 degree inclination angle and a –4 degree inclination angle.
(2) Assume that you have +4 degree inclination angles on your inside and outside tires. Do these represent positive or negative camber on each tire? Plot the steering angle versus lateral acceleration curve for this case. What is the understeer gradient now? The peak lateral acceleration? Have either of these changed at all from Problem 2 part (4)?
(3) Repeat part (2) with a –4 degree inclination angle on each tire.
(4) In reality, camber changes with roll. Assume that the inside and outside camber angles can be written in terms of roll as:

$$g_i = e_{camber} f$$
$$g_o = -e_{camber} f$$

for some camber gain e_{camber}. Plot steering angle versus lateral acceleration for the cases where the camber gain is +0.75 deg/deg and –0.75 deg/deg. Estimate the understeer gradients and peak lateral accelerations. As a performance enthusiast, which would you prefer in a suspension?

Problem 4 – A handling curve from your test data

Reduce the data you obtained in the lab to give a steer angle versus lateral acceleration plot for your constant radius understeer gradient test. From this plot, calculate the initial understeer gradient of the vehicle. For what range of lateral acceleration do you results look reasonably linear?