

ME 106/227

Spring 2002

Assignment #1 – A Closer Look at the Simple Static Handling Model

Due Tuesday, April 9

Purpose of Assignment #1

Many characteristics of vehicles can be predicted fairly well using simple models. This assignment is intended to calibrate you to some of the terms and physical values associated with the simple static handling model. While the model itself is quite simple, realistic treatment of the tires requires a bit of complexity and some numerical analysis. Tackling this challenge provides some insight into what can and cannot be predicted with this model, how tire data is exchanged in industry and how MATLAB can be used to solve the nonlinear equations that arise from a tire model that covers the full range of vehicle performance.

General Comments

Attached to this assignment is the summary from Road&Track's test of four fairly similar vehicles: the BMW 330I, the Lexus IS 300, the Mercedes-Benz C320 and the Volvo S60 T5. All of the questions in this assignment refer to this data. Keep in mind that these are vehicles targeting a similar market segment and the quantitative handling differences among the four are much less than the differences between one of these vehicles and an economy compact or SUV.

A word of caution: One thing to get in the habit of doing right away is to constantly keep your units (degrees or radians? m/s/s or g's?) straight.

Another word of caution: Problems 1 and 2 are really very simple. Problem 3 requires some MATLAB coding and is best started prior to 2am the morning the assignment is due...

Problem 1: Appropriateness of Assumptions

First, let's see if the simple bicycle model would be a good assumption if we are trying to match the behavior of these vehicles on the skidpad. Since dimensions are similar, consider the BMW 330i for these questions.

- (1) What is the Ackermann angle (in degrees) for this car on the 200ft diameter skidpad used in testing? How much does the driver turn the steering wheel (in degrees) to produce this angle at the road wheels?
- (2) If the steering system of this car contained perfect Ackermann geometry, what would the inner and outer wheel angles be? Would ignoring the difference between inner and outer angles seem reasonable?
- (3) What speed (in mph) is the vehicle traveling when it hits its maximum lateral acceleration? What is it in m/s? What is the angular velocity at this point in deg/s? In rad/s?

Problem 2: Basic Handling Behavior

Using the simplest steady-state handling model, the understeer gradient should be determined from knowing the weight distribution and the cornering stiffness. While the weight

distribution is given the tire cornering stiffnesses are not (in fact, they never are). Assuming a ballpark figure of 1000 N/deg/tire (in other words, the front and rear axles each have assumed stiffnesses of 2000 N/deg), what is:

- (1) The understeer gradient of each vehicle (in deg/g), measured at the road wheel?
- (2) The understeer gradient of each vehicle (in deg/g), measured at the steering wheel? Note that you don't have enough information to calculate this for the Mercedes-Benz so just calculate it for the other three.
- (3) Take a look at the qualitative handling assessments given for the four vehicles. Based on your calculations, which vehicle would you expect to have more understeer than the Mercedes? Is that consistent with the handling assessment given? What vehicle would you expect to have less understeer than the Mercedes but qualitatively appears to have more?

For vehicles of this sort, understeer gradients in the approximate range of 2-3 deg/g at the road wheel might be expected. Your results should be lower than this – a sign that either our simple assumption that the tires have a cornering stiffness of 1000 N/deg is off or our simple model is simply too simple (or both). We'll investigate the tire properties now and model complexity in later assignments.

Problem 3: The Magic Formula Tire Model

Next we take a look at some of the variation in tire properties that can occur due to normal load and side forces beyond the initial linear region. We cannot directly investigate the differences between the vehicles tested since we do not have tire parameters for the exact tires used but we can get some idea of the order of magnitude of different effects.

Remember that the whole idea of cornering stiffness is an approximation, considering only the initial slope of the tire curve. To examine the handling characteristics of the vehicle over the full range from parking lot maneuvers to the friction limits it is important to include the full tire curve from the linear region to the point where saturation of the tire force occurs. This is complicated by the fact that tire curves are not constant but depend upon a number of factors including the vertical load. Luckily, there are a number of available tire models that handle load dependence, saturation and a wealth of other things, generally taking an empirical approach. One model that produces surprisingly good correlation with experiments is the "Magic Formula" tire model, so named since the formulation has no underlying physical rationale (this is often called the Pacejka tire model after one of its developers, Hans Pacejka). Pacejka parameters are a common means of communicating tire data between tire suppliers and vehicle manufacturers.

The original form of the Magic Formula for side force as a function of slip angle is:

$$F_y = D \sin(C \tan^{-1}(B\Phi))$$

$$\Phi = (1 - E)\mathbf{a} + (E/B) \tan^{-1}(B\mathbf{a})$$

where each of the coefficients in the model is described as a function of vertical load (and can be modified to also become a function of camber angle):

$$C = 1.30$$

$$D = a_1 F_z^2 + a_2 F_z$$

$$BCD = a_3 \sin(a_4 \tan^{-1}(a_5 F_z))$$

$$E = a_6 F_z^2 + a_7 F_z + a_8$$

The coefficient D represents the peak side force while the product of coefficients BCD represents the cornering stiffness. One set of data (**which requires vertical force in kN and slip angle in degrees and gives side force in N**) is:

$$\begin{aligned} a_1 &= -30.0 & a_5 &= 0.3 \\ a_2 &= 1011 & a_6 &= 0 \\ a_3 &= 1078 & a_7 &= -0.354 \\ a_4 &= 1.82 & a_8 &= 0.707 \end{aligned}$$

(1) Once we put empirical data into a formula like this it is easy to lose sight of what is going on. To prevent this, plot the following to get an idea of the tire behavior predicted by this model:

- (A) Plot side force as a function of slip angle for normal loads of 2, 4, 6 and 8kN and slip angles ranging from -45 to 45 degrees. As written, the magic formula uses the convention that a positive slip angle produces a positive side force. Go ahead and plot it this way for now, we'll change it in part (2) to prevent confusion.
 - (B) Plot the cornering stiffness (BCD) in Newtons/degree as a function of normal load from 0 to 8kN. Calculate the stiffnesses of the front and rear tires of the Mercedes E320 and BMW 330i assuming these tires are put on each vehicle. If we assume that the load dependence of the tires in this data set is typical, would the change in cornering stiffness with vertical load be sufficient to explain the difference in understeer behavior between the BMW and the Mercedes-Benz or would these vehicles have to have very different tires to explain the difference?
 - (C) Plot the peak side force divided by the normal load (D/F_z) as a function of normal load from 0 to 8kN. Calculate this value for the front and rear tires of the Mercedes E320 and the Volvo. Based on this information, what maximum value of lateral acceleration could each of these vehicles achieve with these tires assuming the simple lateral handling model? Which car can achieve the higher lateral acceleration? Is this consistent with the results reported? Which tire would lose traction first for each vehicle, the front or rear?
- (2) With this model, we can calculate a handling diagram which gives steer angle versus lateral acceleration for a constant radius test (as described on pages 227-9 of Gillespie). To do this, sweep through lateral accelerations from 0 to 0.7g, figure out the force on each axle for the vehicle to be in equilibrium, then get the slip angles corresponding to that force from the Magic Formula model (remember that the values given are per tire, not per axle). This requires numerical iteration to solve the above equations for slip angle given side force. A MATLAB script and functions to do this are available on the class web site. From there, you should be able to get the steer angle for each vehicle on the 200 ft. radius curve as a function of lateral velocity.
- (A) Plot this handling curve for the Mercedes together with the line representing the constant understeer gradient you obtained in part (1) of Problem 2 (this line should have the value of the Ackerman angle at zero g and a slope equal to the understeer gradient).
 - (B) A common rule of thumb is that the linear tire approximation holds up to about 0.4 or 0.5g. Do your results agree with this rule?
 - (C) Does the Mercedes become more understeering or less understeering at higher lateral accelerations?

(see notes on reverse side)

Important note: There are a number of issues that arise with using the magic formula in the form used above for the class. The first is the entry of normal loads in kN, not N. The other is the fact that it uses the opposite sign convention as we use in class. The script and functions provided to you on the web site take both of these things into account. All forces are in N and all sign conventions match the class notes (which follow ISO and the new SAE standards). All of these functions are documented so you can use the MATLAB 'help' command to get the proper syntax.

Another important note: This part of the assignment is designed to get you used to using the magic formula tire model and thinking about changes with load. If it doesn't seem like this model has given you any great revelations, wait until next week. Remember, we have not yet considered anything having to do with the suspension and that probably influences handling in some way....